Integration Exam Questions Mark Scheme Topic

Test and Revision

Date:

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Mark Scheme

Q1.

Question	Scheme for Substitution		Marks	AOs
	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • Using a valid substitution $u=...$, changing the terms to u's • integrating and using appropriate limts .		M1	3.1a
	Substitution $u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ oe	Substitution $u=x+2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=1$ oe	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A(u^2\pm2)u^2\mathrm{d}u$	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A(u\pm2)\sqrt{u}\mathrm{d}u$	M1	1.1b
	$=Pu^5\pm Qu^3$	$=Su^{\frac{5}{2}}\pm Tu^{\frac{3}{2}}$	dM1	2.1
	$=\frac{4}{5}u^5-\frac{8}{3}u^3$	$=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$	A1	1.1b
	Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correct way around	ddM1	1.1b
	$=\frac{32}{15}(2+\sqrt{2})$ *		A1*	2.1
			(7)	
				(7 marks)

M1: For attempting to integrate using substitution. Look for

- terms and limits changed to u's. Condone slips and errors/omissions on changing $\mathrm{d}x \to \mathrm{d}u$
- attempted multiplication of terms and raising of at least one power of u by one. Condone slips
- Use of at least the top correct limit. For instance if they go back to x's the limit is 2

B1: For substitution it is for giving the substitution and stating a correct $\frac{\mathrm{d}x}{\mathrm{d}u}$

Eg, $u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ or equivalent such as $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2\sqrt{x+2}}$

M1: It is for attempting to get all aspects of the integral in terms of 'u'.
All terms must be attempted including the $\mathrm{d}x$. You are only condoning slips on signs and coefficients
dM1: It is for using a correct method of expanding and integrating each term (seen at least once) . It is dependent upon the previous M
A1: Correct answer in x or u See scheme
ddM1: Dependent upon the previous M, it is for using the correct limits for their integral, **the correct way around**

A1*: Proceeds correctly to $=\frac{32}{15}(2+\sqrt{2})$. **Note that this is a given answer**

There must be at one least correct intermediate line between $\left[\frac{4}{5}u^5-\frac{8}{3}u^3\right]_{\sqrt{2}}^{2}$ and $\frac{32}{15}(2+\sqrt{2})$

Question Alt	Scheme for by parts	Marks	AOs
	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,dx$ Award for • using by parts the correct way around • and using limits	M1	3.1a
	$\int(\sqrt{x+2})\,dx = \frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
	$\int 2x\sqrt{x+2}\,dx = Ax(x+2)^{\frac{3}{2}} - \int B(x+2)^{\frac{3}{2}}(dx)$	M1	1.1b
	$= Ax(x+2)^{\frac{3}{2}} - C(x+2)^{\frac{5}{2}}$	dM1	2.1
	$= \frac{4}{3}x(x+2)^{\frac{3}{2}} - \frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
	Uses limits 2 and 0 the correct way around	ddM1	1.1b
	$= \frac{32}{15}(2+\sqrt{2})$	A1*	2.1
		(7)	

M1: For attempting using by parts to solve It is a problem- solving mark and all elements do not have to be correct.

- the formula applied the correct way around. You may condone incorrect attempts at integrating $\sqrt{x+2}$ for this problem solving mark
- further integration, again, this may not be correct, and the use of at least the top limit of 2

B1: For $\int(\sqrt{x+2})\,dx = \frac{2}{3}(x+2)^{\frac{3}{2}}$ oe May be awarded $\int_0^2 2x\sqrt{x+2}\,dx \to x^2 \times \frac{2(x+2)^{\frac{3}{2}}}{3}$

M1: For integration by parts the right way around. Award for $Ax(x+2)^{\frac{3}{2}} - \int B(x+2)^{\frac{3}{2}}(dx)$

dM1: For integrating a second time. Award for $Ax(x+2)^{\frac{3}{2}} - C(x+2)^{\frac{5}{2}}$

A1: $\frac{4}{3}x(x+2)^{\frac{3}{2}} - \frac{8}{15}(x+2)^{\frac{5}{2}}$ which may be un simplified

ddM1: Dependent upon the previous M, it is for using the limits 2 and 0 the **correct way around**

A1*: Proceeds to $= \frac{32}{15}(2+\sqrt{2})$. **Note that this is a given answer.**

At least one correct intermediate line must be seen. (See substitution). You would condone missing dx's

(Q13 9MA0/01, June 2018)

Q2.

Question	Scheme	Marks	AOs
(a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

(a)

M1: $\int \frac{2}{(3x-k)}\,dx = A\ln(3x-k)$ Condone a missing bracket

A1: $\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k - k \to A\ln 8k$

dM1: For substituting k and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

A1: Uses correct ln work and notation to show that I $= \frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of k)

(b)

M1: $\int \frac{2}{(2x-k)^2}\,dx = \frac{C}{(2x-k)}$

dM1: For substituting k and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

A1: Shows that it is inversely proportional to k Eg proceeds to the answer is of the form A/k with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)} + \frac{1}{(k)} = \left(-\frac{1}{3}+1\right)\times\frac{1}{k} \propto \frac{1}{k}$

If the calculation is performed it must be correct.

Do not isw here. They should know when they have an expression that is inversely proportional to k.

You may see substitution used but the mark is scored for the same result. See below

$u = 2x - k \to \left[\frac{C}{u}\right]$ for M1 with limits $3k$ and k used for dM1

(**Q07 9MA0/01, June 2018**)

Q3.

Question	Scheme	Marks	AOs
(a)	$(\mathrm{f}'(x)=)\ \dfrac{\lambda x(1+x^2)^2-\mu x(1-x^2)(1+x^2)}{(1+x^2)^4}$	M1	1.1b
	$=\dfrac{-2x(1+x^2)^2-4x(1-x^2)(1+x^2)}{(1+x^2)^4}$	A1	1.1b
	e.g. $\dfrac{-2x(1+x^2)-4x(1-x^2)}{(1+x^2)^3}=0$ $\Rightarrow(1+x^2)+2(1-x^2)=0$ $\Rightarrow 3-x^2=0$ or e.g. $=\dfrac{-2x(1+x^2)\left[(1+x^2)+2(1-x^2)\right]}{(1+x^2)^4}$ $=\dfrac{-2x(1+x^2)(3-x^2)}{(1+x^2)^4}$	dM1	3.1a
	e.g. $3-x^2=0$ or $2x(1+x^2)(x^2-3)=0$ $\Rightarrow x=\ldots\Rightarrow y=\ldots$	ddM1	2.1
	$\left(-\sqrt{3},\ -\dfrac{1}{8}\right)$	A1	2.3
		(5)	

Notes	
(a)	**There may be other valid ways to differentiate or to solve the resulting equation.**
M1:	Attempts the quotient rule to achieve $\dfrac{\pm\lambda x(1+x^2)^2 \pm \mu x(1-x^2)(1+x^2)}{(1+x^2)^4}$ but condone $\dfrac{\pm\lambda x(1+x^2)^2 \pm \mu x(1-x^2)(1+x^2)}{(1+x^2)^2}$ provided an incorrect quotient rule is not seen. Alternatively, attempts the product rule on $f(x)=(1-x^2)(1+x^2)^{-2}$ to achieve $\pm\lambda x(1+x^2)^{-2} \pm \mu x(1-x^2)(1+x^2)^{-3}$ There may be attempts at splitting the numerator to $f(x)=(1+x^2)^{-2}-x^2(1+x^2)^{-2}$ which should differentiate to $\pm\lambda x(1+x^2)^{-3} \pm \mu x(1+x^2)^{-2} \pm \gamma x^3(1+x^2)^{-3}$ In **all** cases, do not penalise signs of λ, μ and/or γ unless an incorrect quotient rule or an incorrect product rule is stated. Any occurrences of $(1+x^2)^2$ may be replaced with $1+2x^2+x^4$ There is no need for a LHS e.g. $f'(x)=$ to be present so ignore an incorrect LHS. Invisible brackets may be recovered/implied by later work.
A1:	Correct differentiation. May be unsimplified. Ignore absence of (or incorrect) LHS. The correct derivative using the quotient rule is $=\dfrac{-2x(1+x^2)^2-4x(1-x^2)(1+x^2)}{(1+x^2)^4}$ o.e. using the product rule is $-2x(1+x^2)^{-2}-4x(1-x^2)(1+x^2)^{-3}$ o.e. and splitting the numerator is $-4x(1+x^2)^{-3}-2x(1+x^2)^{-2}+4x^3(1+x^2)^{-3}$ o.e. You may need to check carefully for equivalent derivatives and ISW after a correct expression is seen. Note that some candidates may set = 0 at the start and may omit the denominator as a result – send to review if you are unsure in such cases.
dM1:	Attempts to reduce the expression to a suitable form so that the roots can be found, either by cancelling a factor of $(1+x^2)$ and simplifying the other brackets e.g. $\dfrac{-2x(1+x^2)-4x(1-x^2)}{(1+x^2)^3} \rightarrow$ e.g. $\dfrac{-6x+2x^3}{(1+x^2)^3}$ or $\dfrac{2x(x^2-3)}{(1+x^2)^3}$ or $2x(x^2-3)$ $(=0)$ or by attempting to take x, $(1+x^2)$ or $\pm 2x(1+x^2)$ out as a factor and simplify the other brackets. If they only take out a factor of x then they must have $x(\pm Ax^4 \pm Bx^2 \pm C)$

e.g. $\dfrac{-2x(1+x^2)\left[(1+x^2)+2(1-x^2)\right]}{(1+x^2)^4} \rightarrow$ e.g. $\dfrac{-2x(1+x^2)(3-x^2)}{(1+x^2)^4}$ or $\dfrac{x(2x^4-4x^2-6)}{(1+x^2)^4}$

Depends on the first M mark.
Note: they might attempt to expand the brackets first, or set = 0 and multiply through by $(1+x^2)^k$ and/or divide through by x or $2x$ at any stage.

Do not be concerned if any factors of $(1+x^2)$, x or 2 go "missing" during their work.

The = 0 does not explicitly need to be seen.

Look for $\pm\dfrac{2x(1+x^2)(Ax^2+B)}{(1+x^2)^k}$ or $\pm\dfrac{(1+x^2)(Ax^3+Bx)}{(1+x^2)^k}$ or $\pm\dfrac{x(Ax^4+Bx^2+C)}{(1+x^2)^k}$ where

- k may be 0, 1, 2, 3 or 4
- A and B (and C if present) are non-zero
- any of $2x$ or $(1+x^2)$ in the numerator and/or $(1+x^2)^k$ in the denominator may be absent

ddM1: Attempts to solve their numerator set = 0 using a valid non-calculator method **and** uses a solution for x to find a corresponding value for y.
The substitution may be implied by their value of y but, if not, the substitution must be seen.

- For either Ax^2+B or Ax^3+Bx, we require $A \neq B$ and $A \times B < 0$ From either expression they can write down their $x = \pm\sqrt{\dfrac{-B}{A}}$ without working.
- For Ax^4+Bx^2+C they must show a valid non-calculator method for solving the quartic, treating it as a quadratic in x^2 and using the usual rules for solving a quadratic algebraically, e.g. $a = x^2 \rightarrow 2a^2-4a-6(=0) \rightarrow (a+1)(2a-6)(=0) \rightarrow x = \pm\sqrt{3}$. They must reach a value for x (and not just x^2) as well as finding a value for y.

Dependent on both previous method marks. They must use a value for x that is not –1, 0 or 1.

A1: Deduces the correct **exact** coordinates for P $\left(-\sqrt{3}, -\dfrac{1}{8}\right)$ o.e. $\left(-\sqrt{3}, -0.125\right)$

Requires all the previous marks to have been scored.

Condone $x = -\sqrt{3}$, $y = -0.125$ or e.g. $-\sqrt{3}, -\dfrac{1}{8}$

If there is more than one pair of coordinates given, then the correct coordinates must be clearly selected or any others clearly rejected.

Ignore any mistakes that occur if they multiply out the denominator $(1+x^2)^4$ after differentiation.

Question	Scheme		Marks	AOs
(b)	$x=-1 \Rightarrow \alpha=-\frac{\pi}{4}$ and $x=1 \Rightarrow \beta=\frac{\pi}{4}$		B1	2.2a
	$\frac{dx}{d\theta}=\sec^2\theta$		B1	1.1b
	$\int\frac{1-x^2}{(1+x^2)^2}dx=\int\frac{1-\tan^2\theta}{(1+\tan^2\theta)^2}\text{"}\sec^2\theta\text{"}\,d\theta$ or $=\int\frac{1-\tan^2\theta}{\sec^4\theta}\text{"}\sec^2\theta\text{"}\,d\theta$		M1	1.1b
	$=\int(1-\tan^2\theta)\cos^2\theta\,d\theta$ $=\int\left(1-\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta\,d\theta$ $=\int(\cos^2\theta-\sin^2\theta)\,d\theta$	$=\int\frac{2-1-\tan^2\theta}{\sec^2\theta}d\theta$ $=\int\left(\frac{2}{\sec^2\theta}-\frac{1+\tan^2\theta}{\sec^2\theta}\right)d\theta$ $=\int(2\cos^2\theta-1)\,d\theta$	dM1	3.1a
	$=\int\cos2\theta\,d\theta$ *		A1*	2.1
			(5)	

Notes

(b) **Mark parts (b) and (c) together.**

B1: Deduces the correct limits for the integral in θ which may be seen separately as side working or within their integral work. This mark cannot be scored working in degrees.
Allow $\frac{3\pi}{4}$ instead of $-\frac{\pi}{4}$ but not decimal approximations.

B1: $\frac{dx}{d\theta}=\sec^2\theta$ or equivalent e.g. $\frac{dx}{d\theta}=1+x^2$ or $\frac{\cos^2\theta--\sin^2\theta}{\cos^2\theta}$ (coming from $x=\frac{\sin\theta}{\cos\theta}$) but must be correct, so not e.g. $\frac{dx}{d\theta}=\sec^2 x$ unless recovered.

M1: Makes a complete attempt at using the substitution $x=\tan\theta$
Requires:

- An attempt at using $\frac{dx}{d\theta}$ to replace dx with $d\theta$ either way round, so if $\frac{dx}{d\theta}=g(\theta)$ allow either $dx=g(\theta)\{d\theta\}$ or $dx=\frac{1}{g(\theta)}\{d\theta\}$. $\frac{dx}{d\theta}$ must be a function of θ and not a constant.
- All terms in x replaced with $\tan\theta$ (or $1+x^2$ with $\sec^2\theta$)

Condone the absence of $d\theta$ but dx must no longer be present.
Condone if they fail to square the denominator or e.g. a slip in missing a θ
Use of notation such as $d(\tan\theta)$ is correct but does not score the M1 until replaced with $\sec^2\theta\{d\theta\}$ or their derivative of $\tan\theta$ (which must not be a constant multiple of $\tan\theta$).

dM1: Uses trigonometric identities e.g. $\pm1\pm\tan^2\theta=\pm\sec^2\theta$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\sec\theta=\frac{1}{\cos\theta}$, $\pm\cos^2\theta\pm\sin^2\theta=\pm1$ to simplify the integral to an expression of the form $\pm a\sin^2\theta\pm b\cos^2\theta\pm c$ where one of a, b or c may be 0.
The algebra should essentially be correct but condone e.g. sign slips or errors collecting terms.

Alternatively, e.g. $\int \frac{1-\tan^2\theta}{1+\tan^2\theta}\,d\theta = \int \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}\,d\theta = \int \frac{\cos^2\theta-\sin^2\theta}{1}\,d\theta$

They must have replaced dx with dθ the correct way round for this mark, so if $\frac{dx}{d\theta} = g(\theta)$

then they must have $dx = g(\theta)\{d\theta\}$ and **not** $dx = \frac{1}{g(\theta)}\{d\theta\}$

It is acceptable to replace e.g. $\tan^2\theta\cos^2\theta$ with $\sin^2\theta$

Dependent on the previous method mark.

A1*: cso Arrives at $\int \cos 2\theta\,d\theta$ with sufficient working shown and no incorrect work ignoring limits. The final line must be fully correct, including the integral sign and dθ (ignoring limits). Note that this may be seen in part (c) and may score the mark.

All trigonometric identities must be fully correct.

Condone one or two slips in a missing θ but not frequent omissions.

Condone missing integral signs in their intermediate work, but it must be present on the final line.

dx must be replaced with dθ at some stage before the final line but does not need to be present in every intermediate line of working.

Condone notational errors throughout e.g. $\sin\theta^2$ provided they are recovered.

This mark is independent from any work to do with limits, i.e., B0B1M1dM1A1* is possible.

Question	Scheme	Marks	AOs
(c)	$\int \cos 2\theta \, d\theta = \frac{1}{2}\sin 2\theta$	M1	1.1b
	$\left[\frac{1}{2}\sin 2\theta\right]_{"-\frac{\pi}{4}"}^{"\frac{\pi}{4}"} = \frac{1}{2}\sin\left(2\times\frac{\pi}{4}\right) - \frac{1}{2}\sin\left(2\times -\frac{\pi}{4}\right)$	dM1	1.1b
	$=1$	A1	2.1
		(3)	
			(13 marks)

Notes

(c) **Mark parts (b) and (c) together.**

M1: $\int \cos 2\theta \, d\theta \to \pm\frac{1}{2}\sin 2\theta \, \{+c\}$

dM1: Substitutes their changed limits (not 1 and -1) into $\pm\frac{1}{2}\sin 2\theta$ and subtracts either way round.

May be implied e.g. $\frac{1}{2}+\frac{1}{2}$ provided they have the correct limits.

Dependent on the previous method mark.

A1: Area of 1 found following correct work. This mark requires clear substitution of the correct limits into $\frac{1}{2}\sin 2\theta$ the correct way round. If they have the limits the wrong way round and achieve an answer of -1 they cannot just make the answer positive for this mark.

They may use limits from 0 to $\frac{\pi}{4}$ and multiply their result by 2.

$\frac{3\pi}{4}$ may be used in place of $-\frac{\pi}{4}$ as the lower limit which is acceptable.

Condone any spurious integral symbol accompanying $\frac{1}{2}\sin 2\theta$

The substitution may be implied by e.g. $\frac{1}{2}--\frac{1}{2}$ or $\frac{1}{2}\sin\left(\frac{\pi}{2}\right)-\frac{1}{2}\sin\left(-\frac{\pi}{2}\right)$ but must follow sight of $\frac{1}{2}\sin 2\theta$

Condone use of limits -45 and 45 (degrees) the correct way round for full marks in part (c). Note that use of a calculator on the original integral will give the correct answer. An answer of 1 scores no marks without evidence of scoring both method marks above including the substitution.

(**Q15 9MA0/02, June 2025**)

Q4.

Question	Scheme	Marks	AOs
(a)	$\frac{dH}{dt}=\frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H}dH=\int \frac{\cos 0.25t}{40}dt$	M1	3.1a
	$\ln H=\frac{1}{10}\sin 0.25t(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t=0, H=5 \Rightarrow c=\ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right)=\frac{1}{10}\sin 0.25t \Rightarrow H=5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1}=5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t=\frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

(a)

M1: Separates the variables to reach $\int \frac{1}{H}dH=\int \frac{\cos 0.25t}{40}dt$ or equivalent.

The integral signs need to be present on both sides and the dH AND dt need to be in the correct positions.

M1: Integrates both sides to reach $\ln H = A\sin 0.25t$ or equivalent with or without the + c

A1: $\ln H=\frac{1}{10}\sin 0.25t+c$ or equivalent with or without the + c. Allow two constants, one either side

If the 40 was on the lhs look for $40\ln H=4\sin 0.25t+c$ or equivalent.

dM1: Substitutes $t=0, H=5 \Rightarrow c=..$ There needs to have been a single "$+c$" to find.

It is dependent upon the previous M mark. You may allow even if you don't explicitly see "$t=0, H=5$" as it may be implied from their previous line

If the candidate has attempted to change the subject and made an error/ slip then condone it for this M but not the final A. Eg. $40\ln H=4\sin 0.25t+c \Rightarrow\Rightarrow H^{40}=e^{4\sin 0.25t}+e^{c} \Rightarrow 5^{40}=1+e^{c} \Rightarrow c=...$

Also many students will be attempting to get to the given answer so condone the method of finding c =... These students will lose the A1* mark

A1*: Proceeds via $\ln H=\frac{1}{10}\sin 0.25t+\ln 5$ or equivalent to the given answer $H=5e^{0.1\sin 0.25t}$ with at least one correct intermediate line and no incorrect work.

DO NOT condone c's going to c's when they should be e^{c} or A

Accept as a minimum $\ln H = \frac{1}{10}\sin 0.25t + \ln 5 \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + \ln 5}$ or $H = e^{\frac{1}{10}\sin 0.25t} \times e^{+\ln 5}$ before sight of the given answer

If the only error was to omit the integration signs on line 1, thus losing the first M1, allow the candidate to have access to this mark following a correct intermediate line (see above).

If they attempt to change the subject first then the constant of integration must have been adapted if the A1* is to be awarded. $\ln H = \frac{1}{10}\sin 0.25t + c \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + c} \Rightarrow H = Ae^{\frac{1}{10}\sin 0.25t}$

The dM1 and A1* under this method are awarded at virtually the same time.

Also, for the final two marks, you may see a proof from $\int_0^H \frac{40}{H}\, dH = \int_5^t \cos 0.25t\, dt$

....

There is an alternative via the use of an integrating factor.

....

(b)

B1: States that the maximum height is 5.53 m Accept $5e^{0.1}$ Condone a lack of units here, but penalise if incorrect units are used.

(c)

M1: For identifying that it would reach the maximum height for the 2nd time when $0.25t = \frac{5\pi}{2}$ or 450

A1: Accept awrt 31.4 or 10π Allow if units are seen

(**Q10 9MA0/01, June 2018**)

Q5.

Question	Scheme	Marks	AOs
(a)	$\frac{dV}{dh}=200$ oe e.g. $\frac{dh}{dV}=\frac{1}{200}$	B1	1.1b
	$\left(\frac{dh}{dt}=\right)\frac{dh}{dV}\times\frac{dV}{dt}=\frac{1}{200}\times\frac{k}{\sqrt{h}}$	M1	3.1a
	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}$ *	A1*	2.1
		(3)	
(b)	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}\Rightarrow\int h^{\frac{1}{2}}\,dh=\int\lambda\,dt\Rightarrow ...h^{\frac{3}{2}}=\lambda t\{+c\}$	M1	1.1b
	$\frac{2}{3}h^{\frac{3}{2}}=\lambda t\{+c\}$ oe e.g. $\frac{h^{\frac{3}{2}}}{\frac{3}{2}}=\lambda t\{+c\}$	A1	1.1b
	$\frac{2}{3}(1.44)^{\frac{3}{2}}=\lambda\times 0+c\Rightarrow c=1.152\left(=\frac{144}{125}\right)$	dM1	3.4
	$\frac{2}{3}(3.24)^{\frac{3}{2}}=\lambda\times 8+"1.152"\Rightarrow\lambda=0.342\left(=\frac{171}{500}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	
	(b) Alternative:		
	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}\Rightarrow\frac{dt}{dh}=\frac{\sqrt{h}}{\lambda}\Rightarrow t=...h^{\frac{3}{2}}(+c)$	M1	1.1b
	$t=\frac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe	A1	1.1b
	$0=\frac{2(1.44)^{\frac{3}{2}}}{3\lambda}+c$ **and** $8=\frac{2(3.24)^{\frac{3}{2}}}{3\lambda}+c$ $\Rightarrow\lambda=...\left(\frac{171}{500}\right)$ **or** $c=...\left(-\frac{64}{19}\right)$	dM1	3.4
	$\Rightarrow\lambda=...\left(\frac{171}{500}\right)$ **and** $c=...\left(-\frac{64}{19}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	
(c)	$5^{\frac{3}{2}}=0.513t+1.728\Rightarrow t=...$	M1	3.4
	$(t=)$ awrt18.4 min	A1	3.2a
		(2)	
			(10 marks)

Notes

(a)

B1: For $\frac{dV}{dh} = 200$ stated or used – may be implied by their chain rule attempt

M1: Requires:

- $\frac{dV}{dh} = p, \ \ p > 1$
- $\frac{dV}{dt} = \pm\frac{k}{\sqrt{h}}$ or e.g. $\frac{dV}{dt} = \pm\frac{1}{k\sqrt{h}}$ (or a suitable letter for k, which may be λ, but must **not** be a number)
- **application** of the correct chain rule $\left(\frac{dh}{dt} = \right)\frac{dh}{dV} \times \frac{dV}{dt}$ or any equivalent with $\frac{dV}{dt} = \pm\frac{k}{\sqrt{h}}$ or $\pm\frac{1}{k\sqrt{h}}$ and their $\frac{dV}{dh}$ correctly placed. So $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{200k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores M0 as $\frac{dh}{dV}$ is incorrectly placed.

A1*: A rigorous argument with all steps shown and simplifies to achieve $\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$ with no errors.

Do not allow the use of λ for both constants. Allow use of e.g. $\frac{dV}{dt} = \pm\frac{1}{k\sqrt{h}}$ for full marks.

e.g. $\frac{dV}{dh} = 200$, $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{\lambda}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores B1M1A0* *unless* e.g. "let $\lambda = \frac{\lambda}{200}$ " seen.

Allow correct work leading to e.g. $\frac{dh}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}} \to \frac{\lambda}{\sqrt{h}}$ or $\frac{dh}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$ so $\lambda = \frac{k}{200}$

There must be an attempt to link the $\frac{dh}{dt}$ with the $\frac{\lambda}{\sqrt{h}}$ which may be missing an = sign.

Allow an argument with $\frac{dV}{dt} = -\frac{k}{\sqrt{h}}$ e.g. $\frac{dV}{dh} = 200$, $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times -\frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$

Withhold this A mark if there are notational errors e.g. $\frac{dV}{dt} = 200$, $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores B1(implied)M1A0*

(b) Note that some candidates may work with e.g. $\lambda = \frac{k}{200}$ or e.g. $\lambda = 200k$ which is acceptable.

Candidates who do not have a λ e.g. assume $\frac{dh}{dt} = \frac{1}{200\sqrt{h}}$ or e.g. $\frac{dh}{dt} = \frac{200}{\sqrt{h}}$ then only the first 2 method marks are available (see note below). Condone use of other variables if the intention is clear but the final answer must be in terms of h and t.

M1: Separates the variables and integrates to obtain an equation of the form $..h^{\frac{3}{2}} = \lambda t\{+c\}$ oe

The constant of integration is not needed for this mark.

A1: $\frac{2}{3}h^{\frac{3}{2}} = \lambda t(+c)$ oe. The constant of integration is not needed for this mark.

Condone spurious notation for this intermediate mark e.g. integral signs left in after integrating.

dM1: Substitutes $t = 0$ and $h = 1.44$ and attempts to find c.

It is dependent on the previous method mark.

Do not be concerned with the "processing" to find "c" as long as they are using $t = 0$ and $h = 1.44$

May be implied by their value of c.

ddM1: Substitutes $t = 8$ and $h = 3.24$ and their c and attempts to find λ. Do not be concerned with the "processing" to find λ as long as they are using $t = 8$ and $h = 3.24$.

It is dependent on both previous method marks.

A1: Correct equation in the correct form from correct work. $h^{\frac{3}{2}} = 0.513t + 1.728$ or $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$

Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

Note candidates who do not have a λ e.g. assume $\frac{dh}{dt} = \frac{1}{200\sqrt{h}}$ or e.g. $\frac{dh}{dt} = \frac{200}{\sqrt{h}}$ can use either $t = 0$ and $h = 1.44$ or $t = 8$ and $h = 3.24$ to find their constant of integration.

(b)Alternative:

M1: Finds the reciprocal of both sides and integrates to obtain an equation of the form $t = ...h^{\frac{3}{2}}(+c)$

A1: $t = \frac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe. The constant of integration is not needed for this mark.

dM1: Substitutes $t=0$ and $h=1.44$ **and** substitutes $t=8$ and $h=3.24$ **and** attempts to find λ **or** c.
It is dependent on the previous method mark.
Do not be concerned with the "processing" to find λ or c as long as they are using $t=0$ and $h=1.44$ and $t=8$ and $h=3.24$ and reach a value for λ or c. May be implied by their value(s).

ddM1: Complete attempt to find λ **and** c. **It is dependent on both previous method marks.**
Do not be concerned with the "processing" to find λ and c as long as they are using $t=0$ and $h=1.44$ and $t=8$ and $h=3.24$.

A1: Correct equation in the correct form. $h^{\frac{3}{2}} = 0.513t + 1.728$ or $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$
Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

<u>Special Case:</u>
Some candidates are using the given equation in part (b) to find the value of A and the value of B using the given conditions. May score a maximum of 00110. This should be marked as follows:

M0A0: (No attempt to integrate)
M1: Substitutes $t = 0$ and $h = 1.44$ to find a value for B
dM1: Substitutes $t = 8$ and $h = 3.24$ with their value of B to find a value for A
A0: Since they have not used the given model.
(Allow full recovery in (c) if this equation is correct)

(c)

M1: Attempts to substitute $h = 5$ into their equation which must be of the form $h^{\frac{3}{2}} = At + B$ or possibly a rearranged equation e.g. $h^{\frac{1}{2}} = \sqrt[3]{At+B}$ with values of A and B leading to a value for t.
Do not be concerned about the processing as long as they use $h = 5$ and obtain a value for t even if t is negative.

A1: Awrt 18.4 minutes **following a correct equation in (b)**.
The units are required but allow e.g. min, mins, but not just 18.4 and not m (which means metres)
Allow e.g. 18 minutes 25 seconds or 18 mins 26 secs
Note this may follow A0 in part (b) as they may have rearranged incorrectly in (b) but use a correct equation in (c) e.g. $\frac{2}{3}h^{\frac{3}{2}} = \frac{171}{500}t + \frac{144}{125}$, $h^{\frac{1}{2}} = \sqrt[3]{\frac{513}{1000}t + \frac{216}{125}}$ or may come from the special case.
Apply isw following a correct time and units, e.g., 18.4 followed by 18 mins.

(Q11 9MA0/02, June 2023)

Q6.

Question	Scheme	Marks	AOs
(a)	$\dfrac{1}{V(25-V)}=\dfrac{P}{V}+\dfrac{Q}{25-V}$ e.g. $1=P(25-V)+QV$ $V=0$ or $V=25$ leading to $P=\ldots$ or $Q=\ldots$	M1	1.1b
	$\dfrac{1}{V(25-V)}=\dfrac{1}{25V}+\dfrac{1}{25(25-V)}$	A1	1.1b
		(2)	

Notes

(a)

M1: Sets $\dfrac{1}{V(25-V)}=\dfrac{P}{V}+\dfrac{Q}{25-V}$ and uses a correct method to identify the value of at least one constant.

Do **not** condone incorrect work e.g. $\dfrac{1}{V(25-V)}=\dfrac{P}{V}+\dfrac{Q}{25-V}\Rightarrow 1=PV+Q(25-V)$ etc.

this scores M0

A1: Correct partial fractions in any form e.g.

$\dfrac{1}{25V}+\dfrac{1}{25(25-V)},\ \dfrac{1}{25V}+\dfrac{1}{625-25V},\ \dfrac{1/25}{V}+\dfrac{1/25}{(25-V)},\ \dfrac{1}{25V}-\dfrac{1}{25(V-25)},\ \dfrac{1}{25}\left(\dfrac{1}{V}+\dfrac{1}{25-V}\right)$ etc.

Note that this mark is not just for the correct constants, it is for the correctly written fractions either seen in part (a) **or used in part (b).** Allow 0.04 for $\dfrac{1}{25}$.

Correct partial fractions only scores both marks.

If the correct fractions are obtained following incorrect work score M0A0 but allow full recovery in the rest of the question.

(b) Way 1	$\int \frac{1}{V(25-V)}\mathrm{d}V = \int \frac{1}{25V} + \frac{1}{25(25-V)}\mathrm{d}V = \frac{1}{25}\ln V - \frac{1}{25}\ln(25-V)$	M1	3.1a
	$\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V) = \frac{1}{10}t(+c)$	A1ft	1.1b
	$t=0, V=20 \Rightarrow \frac{1}{25}\ln 20 - \frac{1}{25}\ln(25-20) = c\left(\Rightarrow c = \frac{1}{25}\ln 4\right)$	M1	3.4
	$V = 24 \Rightarrow t = \frac{2}{5}\ln 24 - \frac{2}{5}\ln(25-24) - \frac{2}{5}\ln 4$	dM1	3.1b
	= 43 (or exact $24\ln 6$)	A1	3.2a
		(5)	
	Alternative for the final 3 marks:		
	$\left[\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V)\right]_{20}^{24} = \left[\frac{1}{10}t\right]_0^T \Rightarrow \frac{1}{25}\ln 24 - \frac{1}{25}\ln 4 = \frac{1}{10}T$	M1	3.4
	$T = \frac{10}{25}\ln 24 - \frac{10}{25}\ln 4 = \ldots$	dM1	3.1b
	= 43 (or exact $24\ln 6$)	A1	3.2a
(c)	$\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V) = \frac{1}{10}t + \frac{1}{25}\ln 4$ $\ln V - \ln(25-V) = 2.5t + \ln 4$ $\ln\frac{V}{4(25-V)} = 2.5t \Rightarrow \frac{V}{4(25-V)} = \mathrm{e}^{2.5t}$	M1	2.1
	$\Rightarrow V = 4\mathrm{e}^{2.5t}(25-V) \Rightarrow V + 4V\mathrm{e}^{2.5t} = 100\mathrm{e}^{2.5t} \Rightarrow V = \ldots$	M1	2.1
	$\Rightarrow V = \frac{100\mathrm{e}^{2.5t}}{1+4\mathrm{e}^{2.5t}} = \frac{100}{\mathrm{e}^{-2.5t}+4}$	A1	1.1b
		(3)	
(d)	25 (microlitres)	B1	2.2a
	Since e.g. As $t \to \infty$, $\mathrm{e}^{\text{"}-2.5\text{"}t} \to 0$	B1	2.4
		(2)	
			(12 marks)

Notes

(b) **Mark (b) and (c) together**

M1: Realises that $\int \frac{\ldots}{V(25-V)}(\mathrm{d}V)$ is required and reaches the form $p\ln\alpha V \pm q\ln\beta(25-V)$ (or e.g. $p\ln\alpha V \pm q\ln\beta(V-25)$) or equivalent for this integration e.g. $p\ln 25V - q\ln(625-25V)$ with p and q non-zero.

But note that $\int \frac{\ldots}{V(25-V)}\mathrm{d}V = \ln V(25-V)$ does not score this mark unless we see an attempt to integrate the partial fractions first.

Condone missing brackets e.g. around the $V-25$ for this mark

Note that the rhs may be incorrect or missing for this mark.

A1ft: Fully correct <u>equation</u> following through their P and Q. The "+ c" is **not** required here. You may need to check carefully when awarding this mark as there will be various alternative correct (or correct ft) forms e.g. these are correct (for correct PF's):

$\frac{1}{25}\ln 25V - \frac{1}{25}\ln(625-25V) = \frac{1}{10}t(+c)$, $\ln V - \ln(25-V) = 2.5t(+c)$,

$\frac{2}{5}\ln 25V - \frac{2}{5}\ln(25-V) = t(+c)$, $\frac{2}{5}\ln 5V - \frac{2}{5}\ln(125-5V) = t(+c)$

In general look for an equation of the form $P\ln \alpha V - Q\ln \beta(25-V) = \frac{1}{10}t(+c)$ or a multiple of this equation. Do **not** condone missing brackets unless they are implied by later work e.g. $\ln 25 - V$ for $\ln(25-V)$

Allow ( ) or | | around the arguments of the ln's and condone "log" for ln.

M1: States or uses $t = 0$ and $V = 20$ consistently leading to a constant of integration which may be simplified or unsimplified. May be implied by their constant so may need to be checked.
This mark is not formally dependent but depends on having made some attempt to integrate both sides, however poor.

dM1: States or uses $V = 24$ and proceeds to find a value for t (even if $t < 0$). You do not need to check the processing provided they reach a value for t. **Depends on the previous method mark** and depends on an attempt to integrate both sides however poor.
May be implied by their value for t so may need to be checked.

A1: Correct value of 43 or awrt 43.0 or exact value of 24ln6.
Units are not required but if any are given it must be minutes or condone "m".
Note that in hours the time is 0.7167037877… and scores A0

Alternative for final 3 marks:

M1: $\left[\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V)\right]_{20}^{24} = \left[\frac{1}{10}t\right]_0^T \Rightarrow \frac{1}{25}\ln 24 - \frac{1}{25}\ln 4 = \frac{1}{10}T$

Applies the limits 20 and 24 to lhs and 0 to "T" or e.g. "t" on rhs
This mark is not formally dependent but depends on having made some attempt to integrate both sides, however poor.

dM1: $T = \frac{10}{25}\ln 24 - \frac{10}{25}\ln 4 = ...$ Solves to find "T". You do not need to check the

dM1: $T = \frac{10}{25}\ln 24 - \frac{10}{25}\ln 4 = ...$ Solves to find "T". You do not need to check the

processing provided they reach a value for t/T. **Depends on the previous method mark** and depends on having made some attempt to integrate both sides however poor.

A1: Correct value of 43 or awrt 43.0 or exact value of 24ln6.
Units are not required but if any are given it must be minutes or condone "m".
Note that in hours the time is 0.7167037877… and scores A0

<u>Note</u>

$$\int \frac{1}{25V} + \frac{1}{25(25-V)}\,\mathrm{d}V = \int \frac{1}{25V} - \frac{1}{25V-625}\,\mathrm{d}V = \frac{1}{25}\ln V - \frac{1}{25}\ln(25V-625) = \frac{1}{10}t(+c)$$

is also correct integration and scores M1A1

The subsequent marks are also available as described above and could lead to the correct answer if the limits and constant of integration are dealt with correctly.
Use review for any examples like these if you are unsure but generally apply the MS as above.

(c) The marks in (c) depend on having integrated their partial fractions to obtain an equation of the form $\pm...\ln...V \pm ...\ln...(25-V) = \pm kt \pm c$, $k, c \neq 0$ and … are non-zero constants or equivalent if they have already attempted to eliminate the ln's in (b)

e.g. $\frac{...V}{...(25-V)} = e^{\pm kt \pm c}$ oe

M1: Uses fully correct log work, having obtained a constant of integration, to eliminate all the ln's including from e.g. $e^{\ln 4}$. We condone sign or coefficient slips only.

M1: Proceeds from an **equation of the form** $\frac{...V}{...(25-V)} = ...e^{...t}$ oe using correct algebra to

$V = ...$ e.g. $\frac{...V}{...(25-V)} = ...e^{...t} \Rightarrow ...V = ...(25-V)...e^{...t} \Rightarrow (...\pm...)V = ...e^{...t} \Rightarrow V = ...$

Condone sign/coefficient slips only.

A1: Correct expression not just values for the constants.

(d)

B1: Correct value of 25 seen
Allow e.g. < 25 or „ 25
Condone "> 25" but the following mark is then not available

B1: Depends on a correct final equation in any form in (c) e.g. $V = \frac{100e^{2.5t}}{1+4e^{2.5t}}$ oe **and one of**:

- Considers the behaviour as $t \to \infty$ e.g. states that as $t \to \infty$, $e^{-2.5t} \to 0$ (condone "= 0") oe
- $V < 25$ as $\ln(25 - V)$ is not possible when $V … 25$
- Verifies the 25 using a value of t, $t … 9$

Using the differential equation:

B1: Correct value of 25 seen
Allow e.g. "< 25" or „ 25
Condone "> 25" but the following mark is then not available

B1: E.g. when $V = 25$, $\frac{dV}{dt} = 0$ or $\frac{dV}{dt} < 0$ if $V > 25$

(Q12 9MA0/02, June 2024)

Q7.

Question	Scheme	Marks	AOs
(a)	e.g. $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x+4}+\frac{B}{x-2} \Rightarrow 3kx-18 \equiv A(x-2)+B(x+4)$ **or** $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x-2}+\frac{B}{x+4} \Rightarrow 3kx-18 \equiv A(x+4)+B(x-2)$	B1	1.1b
	$6k-18=6B \Rightarrow B=...$ or $-12k-18=-6A \Rightarrow A=...$ **or** $3kx-18 \equiv (A+B)x+4B-2A \Rightarrow A+B=3k,\ -18=4B-2A$ $\Rightarrow A=...$ or $B=...$	M1	1.1b
	$\frac{2k+3}{x+4}+\frac{k-3}{x-2}$	A1	1.1b
		(3)	
(b)	$\int\left(\frac{"2k+3"}{x+4}+\frac{"k-3"}{x-2}\right)\mathrm{d}x = ...\ln(x+4)+...\ln(x-2)$	M1	1.2
	$("2k+3")\ln(x+4)+("k-3")\ln(x-2)$	A1ft	1.1b
	$("2k+3")\ln(5)-("k-3")\ln(5) \Rightarrow ("k+6")\ln 5 = 21 \Rightarrow k=...$	dM1	3.1a
	$(k=)\frac{21}{\ln 5}-6$	A1	2.2a
		(4)	
		(7 marks)	

Notes

(a)

B1: Correct form for the partial fractions and sets up the correct corresponding identity which may be implied by two equations in A and B if they are comparing coefficients.

M1: Either

- substitutes $x=2$ or $x=-4$ in an attempt to find A or B in terms of k
- expands the rhs, collects terms and compares coefficients in an attempt to find A or B in terms of k

Or may be implied by one correct fraction (numerator **and** denominator)

You may see candidates substituting two other values of x and then solving simultaneous equations.

A1: Achieves $\frac{2k+3}{x+4}+\frac{k-3}{x-2}$ with no errors. Must be the correct partial fractions not just for correct numerators. May be seen in (b). Correct answer implies B1M1A1. One correct fraction only B0M1A0

(b)

M1: Attempts to find $\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) dx$. Score for either $\frac{...}{x+4} \to ...\ln(x+4)$ or $\frac{...}{x-2} \to ...\ln(x-2)$

Allow the … to be in terms of k or just constants but there must be no x terms.

Condone invisible brackets for this mark.

A1ft: $("2k+3")\ln|x+4| + ("k-3")\ln|x-2|$

but condone round brackets e.g. $("2k+3")\ln(x+4) + ("k-3")\ln(x-2)$ or equivalent e.g.

$("2k+3")\ln(x+4) + ("k-3")\ln(2-x)$

Follow through their partial fractions with numerators which must both be in terms of k.

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of k. To score this mark they must have attempted to integrate their partial fractions, substituted in the correct limits, subtracted either way round, set = 21 and attempted to solve to find k. Condone omission of the terms containing ln(1) or ln(-1).

Note that e.g. ln(-5) or ln(5) must be seen but may be disregarded **after** substitution and subtraction.

Do not be concerned with the processing as long as they proceed to $k = \ldots$

Condone if they use x instead of k after limits have been used as long as the intention is clear.

A1: Deduces $(k =)\frac{21}{\ln 5} - 6$ or exact equivalent e.g. $\frac{21 - 6\ln 5}{\ln 5}, \frac{21 - 3\ln 25}{\ln 5}$.

Allow recovery from expressions that contain e.g. ln(−5) as long as it is dealt with subsequently.

Also allow recovery from invisible brackets. Condone $x = \frac{21}{\ln 5} - 6$

Some candidates may use substitution in part (b) e.g.

$$\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) dx = \int \left(\frac{"2k+3"}{x+4} \right) dx + \int \left(\frac{"k-3"}{x-2} \right) dx$$

$$u = x+4 \Rightarrow \int \left(\frac{"2k+3"}{x+4} \right) dx = \int \left(\frac{"2k+3"}{u} \right) du = ...\ln u$$

$$u = x-2 \Rightarrow \int \left(\frac{"k-3"}{x-2} \right) dx = \int \left(\frac{"k-3"}{u} \right) du = ...\ln u$$

Score **M1** for integrating at least once to an appropriate form as in the main scheme e.g. …ln$u$

A1ft: For $("2k+3")\ln|u| + ("k-3")\ln|u|$

but condone $("2k+3")\ln u + ("k-3")\ln u$ which may be seen separately

Follow through their "A" and "B" in terms of k.

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of k. To score this mark they must have attempted to integrate their partial fractions using substitution, substituted in the correct changed limits and subtracts either way round, set = 21 and attempted to solve to find k. Do not be concerned with processing as long as they proceed to $k = \ldots$ Condone omission of terms which contain e.g. ln(1) or ln(-1).

Note that e.g. ln(-5) or ln(5) must be seen but may be disregarded **after** substitution and subtraction.

$$\left[(2k+3)\ln u\right]_1^5 + \left[(k-3)\ln u\right]_{-5}^{-1} = 21 \Rightarrow (2k+3)\ln 5 - (2k+3)\ln 1 + (k-3)\ln 1 - (k-3)\ln 5 = 21$$

$$\Rightarrow (2k+3)\ln 5 - (k-3)\ln 5 = 21 \Rightarrow (k+6)\ln 5 = 21 \Rightarrow k = \ldots$$

A1: $k = \dfrac{21}{\ln 5} - 6$ or exact equivalent e.g. $\dfrac{21-6\ln 5}{\ln 5}$, $\dfrac{21-3\ln 25}{\ln 5}$, $21\log_5 e - 6$.

Allow recovery from expressions that contain e.g. ln(−5) as long as it is dealt with subsequently.

Also allow recovery from invisible brackets.

(Q10 9MA0/02, June 2023)

Q8.

Question	Scheme	Marks	AOs
(a)	Attempts $y.\dfrac{dx}{dt} = (2\sin 2t + 3\sin t) \times 16\sin t\cos t$ and uses $\sin 2t = 2\sin t\cos t$	M1	2.1
	Correct expanded integrand. **Usually** for one of $(R) = \int 48\sin^2 t\cos t + 16\sin^2 2t\,dt$; $(R) = \int 48\sin^2 t\cos t + 64\sin^2 t\cos^2 t\,dt$; $(R) = \int 24\sin 2t\sin t + 16\sin^2 2t\,dt$	A1	1.1b
	Attempts to use $\cos 4t = 1 - 2\sin^2 2t = \left(1 - 8\sin^2 t\cos^2 t\right)$	M1	1.1b
	$R = \int_0^a 8 - 8\cos 4t + 48\sin^2 t\cos t\,dt$ *	A1*	2.1
	Deduces $a = \dfrac{\pi}{4}$	B1	2.2a
		(5)	
(b)	$\int 8 - 8\cos 4t + 48\sin^2 t\cos t\,dt = 8t - 2\sin 4t + 16\sin^3 t$	M1 A1	2.1 1.1b
	$\left[8t - 2\sin 4t + 16\sin^3 t\right]_0^{\frac{\pi}{4}} = 2\pi + 4\sqrt{2}$	M1 A1	2.1 1.1b
		(4)	
		(9 marks)	
Notes:			

(a) Condone work in another variable, say $\theta \leftrightarrow t$ if used consistently for the first 3 marks

M1: For the key step in attempting $y.\frac{dx}{dt} = (2\sin 2t + 3\sin t) \times 16\sin t\cos t$ with an attempt to use

$\sin 2t = 2\sin t\cos t$ Condone slips in finding $\frac{dx}{dt}$ but it must be of the form $k\sin t\cos t$

E.g. I $y.\frac{dx}{dt} = (2\sin 2t + 3\sin t) \times k\sin t\cos t = (4\sin t\cos t + 3\sin t) \times k\sin t\cos t$

E.g. II $y.\frac{dx}{dt} = (2\sin 2t + 3\sin t) \times k\sin t\cos t = (2\sin 2t + 3\sin t) \times \frac{k}{2}\sin 2t$

A1: A correct (expanded) integrand in t. Don't be concerned by the absence of $\int$ or dt or limits

$(R) = \int \underline{\underline{48\sin^2 t\cos t + 16\sin^2 2t}}\, dt$ or $(R) = \int \underline{\underline{48\sin^2 t\cos t + 64\sin^2 t\cos^2 t}}\, dt$

but watch for other correct versions such as $(R) = \int \underline{\underline{24\sin 2t\sin t + 16\sin^2 2t}}\, dt$

M1: Attempts to use $\cos 4t = \pm 1 \pm 2\sin^2 2t$ to get the integrand in the correct form.

If they have the form $P\sin^2 2t$ it is acceptable to write $P\sin^2 2t = \frac{P}{2}(\pm 1 \pm \cos 4t)$

If they have the form $Q\sin^2 t\cos^2 t$ sight and use of $\sin 2t$ and/or $\cos 2t$ will usually be seen first.

There are many ways to do this, below is such an example

$$Q\sin^2 t\cos^2 t = Q\left(\frac{1-\cos 2t}{2}\right)\left(\frac{1+\cos 2t}{2}\right) = Q\left(\frac{1-\cos^2 2t}{4}\right) = Q\left(\frac{1}{4} - \frac{\cos^2 2t}{4}\right) = Q\left(\frac{1}{4} - \frac{1+\cos 4t}{8}\right)$$

Allow candidates to start with the given answer and work backwards using the same rules. So expect to see $\cos 4t = \pm 1 \pm 2 \times \sin^2 2t$ or $\cos 4t = \pm 2 \times \cos^2 2t \pm 1$ before double angle identities for $\sin 2t$ or $\cos 2t$ are used.

A1*: Proceeds to the given answer with correct working. The order of the terms is not important. Ignore limits for this mark. The integration sign and the dt must be seen on their final answer. If they have worked backwards there must be a concluding statement to the effect that they know that they have shown it. The integration sign and the dt must also be seen

E.g. Reaches $\int 48\sin^2 t\cos t + 64\sin^2 t\cos^2 t\, dt$

Answer is

$$\int 8 - 8\cos 4t + 48\sin^2 t\cos t\, dt$$

$$= \int 8 - 8\left(1 - 2\sin^2 2t\right) + 48\sin^2 t\cos t\, dt$$

$$= \int 16\sin^2 2t + 48\sin^2 t\cos t\, dt$$

$$= \int 64\sin^2 t\cos^2 t + 48\sin^2 t\cos t\, dt$$

which is the same, ✓

B1: Deduces $a = \frac{\pi}{4}$. It may be awarded from the upper limit and can be awarded from (b)

(b)

M1: For the key process in using a correct approach to integrating the trigonometric terms.
May be done separately.
There may be lots of intermediate steps (e.g. let $u = \sin t$).
There are other more complicated methods so look carefully at what they are doing.

$\int 8-8\cos 4t+48\sin^2 t\cos t \, dt = ... \pm P\sin 4t \pm Q\sin^3 t$ where P and Q are constants

A1: $\int 8-8\cos 4t+48\sin^2 t\cos t \, dt = 8t-2\sin 4t+16\sin^3 t(+c)$

If they have written $16\sin^3 t$ as $16\sin t^3$ only award if further work implies a correct answer.
Similarly, $8t$ may be written as $8x$. Award if further work implies $8t$, e.g. substituting in their limits.
Do not penalise this sort of slip at all, these are intermediate answers.

M1: Uses the limits their a and 0 where $a = \frac{\pi}{6}, \frac{\pi}{4}$ or $\frac{\pi}{3}$ in an expression of the form $kt \pm P\sin 4t \pm Q\sin^3 t$ leading to an exact answer. Ignore evidence at lower limit as terms are 0

A1: CSO $2\pi+4\sqrt{2}$ or exact **simplified** equivalent such as $2\pi+\frac{8}{\sqrt{2}}$ or $2\pi+\sqrt{32}$.

Be aware that $\int_0^{\frac{\pi}{4}} 8-8\cos 4t+48\sin^2 t\cos t \, dt = 8t \underset{=}{+} \lambda\sin 4t+16\sin^3 t(+c)$ would lead to the correct answer but would only score M1 A0 M1 A0

(Q16 9MA0/01, June 2022)

Q9.

Question	Scheme	Marks	AOs
(a)	States or uses $h = 0.25$	B1	1.1a
	$\frac{...}{2}\{1+0.1054+2\times(0.9394+0.7788+0.5698+0.3679+0.2096)\}$	M1	1.1b
	$= 0.855$	A1	1.1b
		(3)	
(b)(i)	$\int_{-1.5}^{1.5} e^{-x^2}\, dx = 2\times(a) = 1.71$	B1ft	2.2a
(ii)	$\int_0^{1.5}\left(e^{-x^2}+7\right)dx = (a)+[7x]_0^{1.5} = (a)+7\times1.5$ $= 0.855+10.5$ $= 11.355$	M1 A1ft	3.1a 1.1b
		(3)	
			(6 marks)

Notes:

(a)

B1: States or uses $h = 0.25$

M1: A full attempt at the trapezium rule with their h.
All y terms must be present but condone copying slips.
Do not condone missing brackets unless implied by subsequent work.

A1: 0.855 only (not awrt)
(Note that the calculator answer is 0.856)

(b)(i) **Attempts to use the trapezium rule again score no marks.**

B1ft: For $2\times$ their answer to part (a). Correct method must be seen.

(b)(ii)

M1: Awarded for their answer to part (a) + 7×1.5 or their answer to part (a) + 10.5

A1ft: Correct working followed by awrt 11.4 but ft on their 0.855

(**Q05 9MA0/02/M, June 2025**)

Q10.

Question	Scheme	Marks	AOs
(a)	$\{u = 4-\sqrt{h} \Rightarrow\}\ \dfrac{du}{dh} = -\dfrac{1}{2}h^{-\frac{1}{2}}$ or $\dfrac{dh}{du} = -2(4-u)$ or $\dfrac{dh}{du} = -2\sqrt{h}$	B1	1.1b
	$\left\{\int \dfrac{dh}{4-\sqrt{h}} = \right\}\ \int \dfrac{-2(4-u)}{u}\,du$	M1	2.1
	$= \int\left(-\dfrac{8}{u}+2\right)du$	M1	1.1b
	$= -8\ln u + 2u\ \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln\lvert 4-\sqrt{h}\rvert + 2(4-\sqrt{h}) + c = -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} + k$ *	A1*	2.1
		(6)	
(b)	$\left\{\dfrac{dh}{dt} = \dfrac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow\right\}\ 4-\sqrt{h} = 0$	M1	3.4
	Deduces any of $0 < h < 16$, $0 \le h < 16$, $0 < h \le 16$, $0 \le h \le 16$, $h < 16$, $h \le 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\int \dfrac{1}{(4-\sqrt{h})}\,dh = \int \dfrac{1}{20}t^{0.25}\,dt$	B1	1.1b
	$-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \dfrac{1}{25}t^{1.25}\ \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t=0,\ h=1 \Rightarrow\}\ -8\ln(4-1) - 2\sqrt{(1)} = \dfrac{1}{25}(0)^{1.25} + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2 \Rightarrow -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \dfrac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\{h=12 \Rightarrow\}\ -8\ln\lvert 4-\sqrt{12}\rvert - 2\sqrt{12} = \dfrac{1}{25}t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$t^{1.25} = 221.2795202... \Rightarrow t = \sqrt[1.25]{221.2795...}$ or $t = (221.2795...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t =$ 75.2 (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	(7)	
(c) Way 2	$\int_1^{12} \dfrac{20}{(4-\sqrt{h})}\,dh = \int_0^T t^{0.25}\,dt$	B1	1.1b
	$\left[20(-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h})\right]_1^{12} = \left[\dfrac{4}{5}t^{1.25}\right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \dfrac{4}{5}T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202... \Rightarrow T = \sqrt[1.25]{221.2795...}$ or $T = (221.2795...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T =$ 75.2 (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	(7)	
			(15 marks)

Notes for Question	
(a)	
B1:	See scheme. Allow $du = -\frac{1}{2}h^{-\frac{1}{2}}dh$, $dh = -2(4-u)du$, $dh = -2\sqrt{h}\,du$ o.e.
M1:	Complete method for applying $u = 4-\sqrt{h}$ to $\int \frac{dh}{4-\sqrt{h}}$ to give an expression of the form $\int \frac{k(4-u)}{u} du$; $k \neq 0$
Note:	Condone the omission of an integral sign and/or du
M1:	Proceeds to obtain an integral of the form $\int \left(\frac{A}{u}+B\right)\{du\}$; $A, B \neq 0$
M1:	$\int \left(\frac{A}{u}+B\right)\{du\} \to D\ln u + Eu$; $A, B, D, E \neq 0$; with or without a constant of integration
A1:	$\int \left(-\frac{8}{u}+2\right)\{du\} \to -8\ln u + 2u$; with or without a constant of integration
A1*:	**dependent on all previous marks** Substitutes $u = 4-\sqrt{h}$ into their integrated result and completes the proof by obtaining the printed result $-8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + k$. Condone the use of brackets instead of the modulus sign.
Note:	They must combine $2(4)$ and their $+c$ correctly to give $+k$
Note:	Going from $-8\ln\left|4-\sqrt{h}\right| + 2(4-\sqrt{h}) + c$ to $-8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + k$, with no intermediate working or with no incorrect working is required for the final A1* mark.
Note:	Allow A1* for correctly reaching $-8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + c + 8$ and stating $k = c + 8$
Note:	Allow A1* for correctly reaching $-8\ln\left|4-\sqrt{h}\right| + 2(4-\sqrt{h}) + k = -8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + k$
	<u>Alternative (integration by parts) method for the 2nd M, 3rd M and 1st A mark</u> $\left\{\int \frac{-2(4-u)}{u} du = \int \frac{2u-8}{u} du\right\} = (2u-8)\ln u - \int 2\ln u\, du = (2u-8)\ln u - 2(u\ln u - u)\ \{+c\}$
2nd M1:	Proceeds to obtain an integral of the form $(Au+B)\ln u - \int A\ln u\{du\}$; $A, B \neq 0$
3rd M1:	Integrates to give $D\ln u + Eu$; $D, E \neq 0$; which can be simplified or un-simplified with or without a constant of integration.
Note:	Give 3rd M1 for $(2u-8)\ln u - 2(u\ln u - u)$ because it is an un-simplified form of $D\ln u + Eu$
1st A1:	Integrates to give $(2u-8)\ln u - 2(u\ln u - u)$ or $-8\ln u + 2u$ o.e. with or without a constant of integration.

(b)	
M1:	Uses the context of the model and has an understanding that the tree keeps growing until $\frac{dh}{dt} = 0 \Rightarrow 4-\sqrt{h} = 0$. Alternatively, they can write $\frac{dh}{dt} > 0 \Rightarrow 4-\sqrt{h} > 0$
Note:	Accept $h = 16$ or 16 used in their inequality statement for this mark.
A1:	See scheme
Note:	A correct answer can be given M1 A1 from any working.

Notes for Question	
(c)	**Way 1**
B1:	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Condone absence of integral signs.
M1:	Integrates $t^{0.25}$ to give $\lambda t^{1.25}$; $\lambda \neq 0$
A1:	Correct integration. E.g. $-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \frac{1}{25}t^{1.25}$ or $20(-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h}) = \frac{4}{5}t^{1.25}$ $-8\ln\lvert 4-\sqrt{h}\rvert + 2(4-\sqrt{h}) = \frac{1}{25}t^{1.25}$ or $20(-8\ln\lvert 4-\sqrt{h}\rvert + 2(4-\sqrt{h})) = \frac{4}{5}t^{1.25}$ with or without a constant of integration, e.g. k, c or A
Note:	There is no requirement for modulus signs.
M1:	Some evidence of ***applying*** both $t=0$ and $h=1$ to their model (which can be a changed equation) which contains a constant of integration, e.g. k, c or A
dM1:	**dependent on the previous M mark** Complete process of finding their constant of integration, followed by applying $h=12$ and their constant of integration to their changed equation
M1:	Rearranges their equation to make $t^{\text{their } 1.25} = \dots$ followed by a correct method to give $t = \dots$; $t > 0$
Note:	$t^{\text{their } 1.25} = \dots$ can be negative, but their '$t = \dots$' must be positive
Note:	"their 1.25" cannot be 0 or 1 for this mark
Note:	Do not give this mark if $t^{\text{their } 1.25} = \dots$ (usually $t^{0.25} = \dots$) is a result of substituting $t=12$ (or $t=11$) into the given $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20}$. **Note:** They will usually write $\frac{\mathrm{d}h}{\mathrm{d}t}$ as either 12 or 11.
A1:	awrt 75.2

(c)	**Way 2**
B1:	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working.
Note:	Integral signs and limits are not required for this mark.
M1:	Same as Way 1 (ignore limits)
A1:	Same as Way 1 (ignore limits)
M1:	Applies limits of 1 and 12 to their model (i.e. to their changed expression in h) and subtracts
dM1	**dependent on the previous M mark** Complete process of applying limits of 1 and 12 and 0 and T (or 't') appropriately to their changed equation
M1:	Same as Way 1
A1:	Same as Way 1

(**Q14 9MA0/02, June 2019**)

Q11.

Question	Scheme	Marks	AOs
(a)	$R=\int_{1}^{\sqrt{3}} \frac{1}{x^2\sqrt{4-x^2}}\,dx$		
	$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx=\int \frac{1}{4\sin^2 u\sqrt{4-4\sin^2 u}}2\cos u\,(du)$	M1	3.1a
	Uses $1-\sin^2 u=\cos^2 u \Rightarrow \sqrt{4-4\sin^2 u}=2\cos u$	dM1	1.1b
	$=\int \frac{1}{4\sin^2 u\times 2\cos u}2\cos u\,(du)=\int \frac{1}{4}\operatorname{cosec}^2 u\,du$	A1	2.1
	Deduces $a=\frac{\pi}{6}$ and $b=\frac{\pi}{3}$	B1	2.2a
		(4)	
(b)	$\int \frac{1}{4}\operatorname{cosec}^2 u\,(du)=-\frac{1}{4}\cot u\ \ (+c)$	B1ft	1.2
	$=\left[-\frac{1}{4}\cot u\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=-\frac{1}{4}\cot\frac{\pi}{3}+\frac{1}{4}\cot\frac{\pi}{6}$	M1	2.1
	$=\frac{1}{2\sqrt{3}}$ or $\frac{\sqrt{3}}{6}$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a)

M1: Proceeds to $\frac{1}{A\sin^2 u\sqrt{\pm C \pm C\sin^2 u}} \times B\cos u$ o.e. or may be implied by $\frac{\cos u}{D\sin^2 u\cos u}$ o.e in terms of u only $A, B, C, D \neq 0$

Requires $\mathrm{d}x \to ...\cos u\ (\mathrm{d}u)$ o.e. Condone the omission of the integral sign and du for this mark. Condone the $\pm B\cos u$ appearing after the du if present

It cannot be implied by $\frac{1}{D\sin^2 u}$ or $\frac{\operatorname{cosec}^2 u}{D}$ where $D \neq 0$

dM1: Attempts to use $\pm 1 \pm \sin^2 u = \pm\cos^2 u$ to change $\sqrt{\pm C \pm C\sin^2 u}$ to $p\cos u$ o.e. (Must be seen somewhere in their solution which may be part of side-workings)

May be seen as e.g. $\frac{1}{A\sin^2 u\sqrt{C\cos^2 u}} \times B\cos u$ or $\frac{1}{A\sin^2 u \times F\cos u} \times B\cos u$

It is dependent on the previous method mark.

A1: $\int \frac{1}{4}\operatorname{cosec}^2 u\,\mathrm{d}u$ Ignore any limits or the absence of them for this mark.

Requires the integral sign and du. Do not isw. Must be seen in (a)

B1: Deduces $a = \frac{\pi}{6}$ and $b = \frac{\pi}{3}$ (in radians) May be with their final integral in the correct positions or stated separately which value is a and which value is b. Must be seen in (a).

Alternative part (a) (Further Maths)

M1: $u = \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{M}{\sqrt{1 - Nx^2}}$ It cannot be implied by $\frac{1}{D\sin^2 u}$ or $\frac{\operatorname{cosec}^2 u}{D}$, $D \neq 0$

dM1: Uses $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{\sqrt{4 - x^2}}$ o.e , substitutes $x = 2\sin u$ and proceeds to $\frac{1}{4\sin^2 u}$ It is dependent on the previous method mark.

A1: $\int \frac{1}{4}\operatorname{cosec}^2 u\,\mathrm{d}u$ Ignore any limits or the absence of them for this mark.

Requires the integral sign and du. Do not isw. Must be seen in (a)

B1: Deduces $a = \frac{\pi}{6}$ and $b = \frac{\pi}{3}$ (must be in radians)

May be with their final integral in the correct positions or stated separately which value is a and which value is b. Must be seen in (a).

(b) Condone x instead of u provided the appropriate limits are substituted into the function

B1ft: $k\operatorname{cosec}^2 u \to -k\cot u$ (may be left in terms of $k \neq 0$)

e.g. $-2\operatorname{cosec}^2 u \to 2\cot u$ is B1 (look for the sign to change as well)

Note some candidates may prefer to change

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{4}\operatorname{cosec}^2 u\,(\mathrm{d}u) = -\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{1}{4}\operatorname{cosec}^2 u\,(\mathrm{d}u) = \frac{1}{4}\cot u \ (+c) \text{ which scores B1ft}$$

M1: Uses the limits $\frac{\pi}{6}$ and $\frac{\pi}{3}$ either way round (or condone use of $30°$ and $60°$) in an expression of the form $\pm q\cot u$ and subtracts (either way round).

Allow $q = 1$. May be implied by their final answer provided B1ft has been scored.

May write as e.g. $-\frac{1}{4}\frac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}} + \frac{1}{4}\frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$

Do not condone a sign slip here e.g. $-\frac{1}{4}\cot\frac{\pi}{3} - \frac{1}{4}\cot\frac{\pi}{6}$ is M0.

The expression is sufficient but if just a value is stated following integration to the required form then you may need to check this on your calculator.

If no algebraic integration is seen then M0

If only decimal values are seen then M0

A1: $\frac{1}{2\sqrt{3}}$ or $\frac{\sqrt{3}}{6}$ provided the previous two marks have been scored.

Note that incorrect integration e.g. $\int k\operatorname{cosec}^2 u\,\mathrm{d}u = k\cot u \to \frac{1}{2\sqrt{3}}$ scores B0ftM1A0

(Q16 9MA0/01, June 2025)

Q12.

Question	Scheme	Marks	AOs
	$\int \frac{x}{(2x+1)^3}\,dx = \frac{x(2x+1)^{-2}}{-4} + \int \frac{(2x+1)^{-2}}{4}(dx)$	M1	3.1a
	$= ... + \frac{(2x+1)^{-1}}{-8}$	dM1	1.1b
	$-\frac{x}{4(2x+1)^2} - \frac{1}{8(2x+1)}$	A1	1.1b
	e.g. $= \left(-\frac{2}{4(2\times2+1)^2} - \frac{1}{8(2\times2+1)}\right) - \left(-\frac{0}{4(2\times0+1)^2} - \frac{1}{8(2\times0+1)}\right)$	ddM1	1.1b
	e.g. $-\frac{2}{100} - \frac{1}{40} + \frac{1}{8} = \frac{2}{25}$ *	A1*	2.1
		(5)	
Alt I	$\int \frac{x}{(2x+1)^3}\,dx = \int \frac{u-1}{4u^3}(du)$ where $u = 2x+1$	M1	3.1a
	$= \int \frac{1}{4}u^{-2} - \frac{1}{4}u^{-3}\,du = ...$	dM1	1.1b
	$= -\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}$	A1	1.1b
	$\int_0^2 \frac{x}{(2x+1)^3}\,dx = \left[-\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}\right]_1^5 = \left(-\frac{1}{20} + \frac{1}{200}\right) - \left(-\frac{1}{4} + \frac{1}{8}\right)$	ddM1	1.1b
	e.g. $-\frac{1}{20} + \frac{1}{200} + \frac{1}{8} = \frac{2}{25}$ *	A1*	2.1
		(5)	
			(5 marks)

Notes:

M1: Obtains $\alpha x(2x+1)^{-2} \pm \beta\int(2x+1)^{-2}\ (dx)$ o.e. where $\alpha, \beta \neq 0$ but may be equal to each other (you do not need to be concerned about how they arrive at this)

dM1: Uses a correct method to integrate an expression of the form $\pm\beta\int(2x+1)^{-2}\ (dx) \to \pm\gamma(2x+1)^{-1}$, $\beta, \gamma \neq 0$

It is dependent on the previous method mark.

A1: $-\frac{x}{4(2x+1)^2} - \frac{1}{8(2x+1)}$ o.e. Allow this to be unsimplified

Watch out for the DI method

	D	I
+	x	$(2x+1)^{-3}$
–	1	$-\frac{(2x+1)^{-2}}{4}$
+	0	$\frac{(2x+1)^{-1}}{8}$

Giving correct integration e.g. $\int \frac{x}{(2x+1)^3}\,dx = -\frac{x(2x+1)^{-2}}{4} - \frac{(2x+1)^{-1}}{8}$

Score M1dM1 for obtaining $\pm px(2x+1)^{-2} \pm q(2x+1)^{-1}$, $p,q \neq 0$ and A1 for both correct.

ddM1: Substitutes 0 and 2 into an expression of the form $\pm px(2x+1)^{-2} \pm q(2x+1)^{-1}$, $p,q \neq 0$ or equivalent and subtracts either way round. Evidence of limits used cannot be the given answer. Condone slips but not directly evaluating the expression as 0 when x is 0. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the dx throughout

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Alternative method I – substitution using $u = 2x+1$

M1: Uses a suitable substitution e.g. $u = 2x+1$ and proceeds to $A\int \frac{u-1}{u^3}\,(du)$ o.e.

dM1: Splits into separate fractions and attempts to integrate $A\int u^{-2} - u^{-3}\,(du)$ Look for at least one correct index for one of the two terms i.e. $u^{-2} \to u^{-1}$ or $u^{-3} \to u^{-2}$. It is dependent on the previous method mark.

A1: $-\frac{1}{4}u^{-1} + \frac{1}{8}u^{-2}$ o.e.

ddM1: Substitutes correct limits (1 and 5 if in terms of u) into an expression of the correct form $...u^{-1} + ...u^{-2}$ (or may have substituted back in terms of x and substitutes in 0 and 2 into an expression of the correct form $\pm p(2x+1)^{-2} \pm q(2x+1)^{-1}$, $p,q \neq 0$ o.e and subtracts either way round – see above for ddM1). Evidence of limits used cannot be the given answer. Condone slips. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the du throughout

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Alternative method II – partial fractions

M1: Writes $\frac{x}{(2x+1)^3}$ as $\frac{0.5}{(2x+1)^2} - \frac{0.5}{(2x+1)^3}$ o.e. Allow $\pm\frac{M}{(2x+1)^2} \pm \frac{N}{(2x+1)^3}$ (where M and N are constants)

dM1: $\int \frac{x}{(2x+1)^3}(dx) = \int \frac{"0.5"}{(2x+1)^2}(dx) - \int \frac{"0.5"}{(2x+1)^3}(dx) = \pm...(2x+1)^{-1} \pm ...(2x+1)^{-2}$

A1: $-\frac{1}{4(2x+1)^1} + \frac{1}{8(2x+1)^2}$ o.e.

ddM1: Substitutes 0 and 2 into an expression of the correct form $\pm...(2x+1)^{-1} \pm ...(2x+1)^{-2}$ and subtracts either way round. Evidence of limits used cannot be the given answer. Condone slips but not directly evaluating the expression as 0 when x is 0. It is dependent on both of the previous method marks.

A1*: Shows **evidence of evaluating** after or when the limits are substituted in before proceeding to the given answer with **no errors seen including invisible brackets** and all previous marks scored. Condone missing/poor notation e.g. missing the dx throughout.

(**Q13 9MA0/01, June 2025**)

Q13.

Question	Scheme	Marks	AOs
	$\int 8x^2e^{-3x}\,dx = -\frac{8x^2}{3}e^{-3x} + \int \frac{16x}{3}e^{-3x}\,dx$	M1 A1	2.1 1.1b
	$= -\frac{8x^2}{3}e^{-3x} - \frac{16x}{9}e^{-3x} + \int \frac{16}{9}e^{-3x}\,dx$	dM1	1.1b
	$\left[-\frac{8x^2}{3}e^{-3x} - \frac{16x}{9}e^{-3x} - \frac{16}{27}e^{-3x}\right]_0^1$ $= -\frac{8}{3}e^{-3} - \frac{16}{9}e^{-3} - \frac{16}{27}e^{-3} - \left(-0 - 0 - \frac{16}{27}\right)$	M1	2.1
	$= \frac{16}{27} - \frac{136}{27}e^{-3}$	A1	1.1b
		(5)	
			(5 marks)

Notes
Mark positively in this question and do not penalise poor notation such as a missing "dx" or spurious integral signs, "+ c" etc. as long as the intention is clear.

M1: Obtains $\pm\alpha x^2 e^{-3x} \pm \beta \int x\, e^{-3x}\, dx$

(you do not need to be concerned about how they arrive at this)

A1: Correct expression **simplified or unsimplified**. E.g. allow $-\frac{8x^2}{3}e^{-3x} - \int -\frac{16x}{3}e^{-3x}\, dx$

Note that we condone the "8" missing for this mark so allow e.g. $-\frac{x^2}{3}e^{-3x} - \int -\frac{2x}{3}e^{-3x}\, dx$

Note that notation may be poor here but the intention clear e.g. if they obtain $-\frac{8x^2}{3}e^{-3x} + \left[\frac{16x}{3}e^{-3x}\right]$ and then attempt to integrate $\frac{16x}{3}e^{-3x}$ both marks can be implied.

dM1: Attempts parts again on $\pm\beta \int x\, e^{-3x}\, dx$ to obtain $\pm Axe^{-3x} \pm B\int e^{-3x}\, dx$

This may be seen in isolation and does not need to be seen as part of the complete integration. **Depends on the first method mark.**
Watch for the DI method (with or without the 8):

	D	I
+	$8x^2$	e^{-3x}
–	$16x$	$-\frac{1}{3}e^{-3x}$
+	16	$\frac{1}{9}e^{-3x}$
–	0	$-\frac{1}{27}e^{-3x}$

Giving the correct integration e.g. $\int 8x^2 e^{-3x}\, dx = -\frac{8x^2}{3}e^{-3x} - \frac{16x}{9}e^{-3x} - \frac{16}{27}e^{-3x}$

In such cases score M1dM1 for obtaining $\pm px^2e^{-3x} \pm qxe^{-3x} \pm re^{-3x}$, $p, q, r \neq 0$ and then A1 for the **correct** first 2 terms, with or without the factor of 8.
Note that for this approach M1A1dM0 is not possible.

M1: Substitutes the limits 1 and 0 into an expression of the form $\pm\alpha x^2 e^{-3x} \pm \beta x e^{-3x} \pm \gamma e^{-3x}$, $\alpha, \beta, \gamma \neq 0$ and subtracts the right way round.
Must see evidence of the use of **both** limits and subtraction and use of $e^0 = 1$.
Note that some candidates apply the limits as they go e.g. to the $\left[-\frac{8x^2}{3}e^{-3x}\right]$ which is acceptable but you will need to check carefully that overall they are satisfying the conditions above.
Condone not realising that the first 2 terms evaluate to 0 when substituting $x = 0$ e.g.
condone $-\frac{8}{3}e^{-3} - \frac{16}{9}e^{-3} - \frac{16}{27}e^{-3} - \left(-\frac{8}{3} - \frac{16}{9} - \frac{16}{27}\right)$ as we have evidence of $e^0 = 1$

Note that e.g. $-\frac{8}{3}e^{-3} - \frac{16}{9}e^{-3} - \frac{16}{27}e^{-3} - \left(-\frac{16}{27}e^0\right) = -\frac{136}{27}e^{-3} - \frac{16}{27}$ scores M0 as it suggests that $e^0 = -1$ not $+1$.

A1: Correct answer of $\frac{16}{27} - \frac{136}{27}e^{-3}$ but allow equivalent exact fractions and condone $\frac{16}{27} - \frac{136}{27e^3}$. Isw once the correct answer is seen.

Candidates who consistently misread $8x^2e^{-3x}$ as $8x^2e^{3x}$:

$$\int 8x^2e^{3x}\,dx = \frac{8x^2}{3}e^{3x} - \int \frac{16x}{3}e^{3x}\,dx$$

$$= \frac{8x^2}{3}e^{3x} - \frac{16x}{9}e^{3x} + \int \frac{16}{9}e^{3x}\,dx$$

$$\left[\frac{8x^2}{3}e^{3x} - \frac{16x}{9}e^{3x} + \frac{16}{27}e^{3x}\right]_0^1$$

$$= \frac{8}{3}e^3 - \frac{16}{9}e^3 + \frac{16}{27}e^3 - \left(\frac{16}{27}\right) = \frac{40}{27}e^3 - \frac{16}{27}$$

Scores a maximum of M1A0dM1M1A0

The main scheme can be applied similarly e.g.

M1: Attempts parts to obtain $\alpha x^2e^{3x} - \beta\int x\,e^{3x}\,dx$, $\alpha, \beta > 0$

A0: Not available

dM1: Attempts parts again on $\beta\int x\,e^{3x}\,dx$ to obtain $Cxe^{3x} - D\int e^{3x}\,dx$, $C, D > 0$

M1: Substitutes the limits 1 and 0 into an expression of the form $\pm\lambda x^2e^{3x} \pm \mu xe^{3x} \pm \gamma e^{3x}$, $\lambda, \mu, \gamma \neq 0$ and subtracts the right way round.
Must see evidence of the use of **both** limits and subtraction and use of $e^0 = 1$.

A0: Not available

But note, do **not** allow mixing of $3x$'s and $-3x$'s. If there are a mixture, apply the main scheme.

(**Q11 9MA0/02, June 2024**)

Q14.

Question	Scheme	Marks	AOs
(a)	$x = a\sin^2\theta \Rightarrow \dfrac{dx}{d\theta} = 2a\sin\theta\cos\theta$	B1	1.1b
	$\int x^{\frac{1}{2}}\sqrt{a-x}\,dx = \int \sqrt{a}\sin\theta\sqrt{a-a\sin^2\theta} \times 2a\sin\theta\cos\theta\{d\theta\}$	M1	2.1
	$= \int \sqrt{a}\sin\theta\sqrt{a}\cos\theta \times 2a\sin\theta\cos\theta\,d\theta = 2a^2\int \sin^2\theta\cos^2\theta\,d\theta$ $= 2a^2\int\left(\frac{1}{2}\sin 2\theta\right)^2 d\theta$	dM1	3.1a
	Replaces or considers limits $\{x = 0 \Rightarrow\}\theta = 0,\ \{x = a \Rightarrow\}\theta = \dfrac{\pi}{2}$ $= \dfrac{1}{2}a^2\int_0^{\frac{\pi}{2}} \sin^2 2\theta\,d\theta$ *	A1*	1.1b
		(4)	
(b)	$\ldots\int \sin^2 2\theta\,d\theta \to \ldots\int \dfrac{1-\cos 4\theta}{2}\,d\theta$	M1	1.1b
	$\to \ldots\left(\dfrac{\theta}{2} - \dfrac{1}{8}\sin 4\theta\right)$	dM1	2.1
	$\mu\int \sin^2 2\theta\,d\theta \to \dfrac{\mu}{2}\left(\theta - \dfrac{1}{4}\sin 4\theta\right)$	A1	1.1b
	$\left\{\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,dx\right\} = \dfrac{a^2}{4}\left(\dfrac{\pi}{2} - 0 - 0\right) = \dfrac{1}{8}\pi a^2$	A1	1.1b
		(4)	
			(8 marks)

Notes
(a)
B1: $\dfrac{dx}{d\theta} = 2a\sin\theta\cos\theta$ or $dx = 2a\sin\theta\cos\theta\,d\theta$ o.e. seen or implied by their substitution.
Note that writing $x = a\sin^2\theta = \dfrac{a}{2}(1-\cos 2\theta) \to \dfrac{dx}{d\theta} = a\sin 2\theta$ is correct. Condone use of $\dfrac{dx}{da}$
M1: Attempts to substitute, fully replacing $x^{\frac{1}{2}}$ and $\sqrt{a-x}$ with θ's and dx with their $dx = \ldots$
Look for $x^{\frac{1}{2}}\sqrt{a-x}\,dx \to f(\theta)g(\theta)h(\theta)$ where
• $f(\theta)$ is an attempt at $\sqrt{a\sin^2\theta}$ e.g. allow $a\sin\theta$ but just $a\sin^2\theta$ is not condoned
• $g(\theta)$ is an attempt at $\sqrt{a-a\sin^2\theta}$ but not $\sqrt{a} - \sqrt{a\sin^2\theta}$ unless $\sqrt{a-a\sin^2\theta}$ is attempted first
• $h(\theta) =$ their dx or their $\dfrac{dx}{d\theta}$ or $\dfrac{1}{\text{their } \frac{dx}{d\theta}}$ (in terms of θ only but condone da seen)
Condone slips provided the intention is clear, e.g. $x^{\frac{1}{2}} \to \sqrt{a}\sin^2\theta$ but x must be eliminated. There is no need to replace the limits and condone the absence of $d\theta$ and/or the integral sign for this mark.

dM1: Attempts to use $\sin 2\theta = 2\sin\theta\cos\theta$ to convert an integral of the form $\int \sin^2\theta\cos^2\theta\,d\theta$ or e.g. the form $\int \sin\theta\cos\theta\sin 2\theta\,d\theta$ to $\int ...\sin^2 2\theta\,d\theta$

There is no need to replace the limits and condone the absence of $d\theta$ and/or the integral sign for this mark.

A1*: Replaces or considers limits $\{x=0\Rightarrow\}\theta=0,\ \{x=a\Rightarrow\}\theta=\frac{\pi}{2}$ at some stage before the given answer and proceeds with no errors to the given answer. The replaced limits may appear with their integral symbol and do not have to be justified and do not have to appear on every line. Condone infrequent slips in notation, e.g. $\sin\theta^2$ in a line as long as it is not consistently poor. You must see the integral sign with the correct limits and the $d\theta$ together in the given answer.

(b)

M1: Adopts an appropriate strategy by using the double angle identity to obtain an integrable form $...\int \sin^2 2\theta\,d\theta \to ...\int \frac{\pm1\pm\cos 4\theta}{2}\,d\theta$ which may be seen as $\lambda\int \sin^2 2\theta\,d\theta \to \frac{\lambda}{2}\int \pm1\pm\cos 4\theta\,d\theta$ with the $\frac{1}{2}$ absorbed into their coefficient of the integral.

dM1: Integrates into the form $\pm p\theta \pm q\sin 4\theta$

A1: Correct integration of $\mu\int \sin^2 2\theta\,d\theta$ to $\frac{\mu}{2}\left(\theta-\frac{1}{4}\sin 4\theta\right)$. Here μ may be 1.

Condone lack of limits here.

A1: Applies limits to the correct integral and proceeds to $\frac{1}{8}\pi a^2$ following correct work.

There is no need to see 0 substituted in and condone any omission of integral signs and/or $d\theta$

Note that $\frac{a^2}{4}\left[\theta+\frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}}=\frac{1}{8}\pi a^2$ is incorrect and scores M1dM1A0A0

Use of $\sin^2 2\theta = \frac{\pm1\pm\cos k\theta}{2}$ with $k\neq 4$ scores M0dM0A0A0 but may lead to $\frac{1}{8}\pi a^2$

Condone use of x in place of θ e.g. $\frac{a^2}{4}\left[x-\frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{2}}=\frac{1}{8}\pi a^2$

See overleaf for some alternative approaches.

Alternative (a) working backwards:

$$\frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta$$

$\frac{\mathrm{d}x}{\mathrm{d}\theta}=2a\sin\theta\cos\theta$ score B1 (as in main scheme)

$$=\frac{1}{2}a^2\int_0^{\frac{\pi}{2}}(2\sin\theta\cos\theta)(2\sin\theta\cos\theta)\,\mathrm{d}\theta$$

$$=a\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\,\mathrm{d}x$$

Score M1 here for using the double angle identity and replacing $\ldots\sin\theta\cos\theta\,\mathrm{d}\theta$ with $\mathrm{d}x$

$$=a\int_0^{a}\sqrt{\frac{x}{a}}\sqrt{1-\frac{x}{a}}\,\mathrm{d}x$$

Score dM1 here for a full attempt to replace all trig leading to everything in terms of x only

Must come from the form $\int \ldots\sin\theta\cos\theta\,\mathrm{d}x$

A1 fully correct with limits replaced / considered before the final line and the final line fully correct with limits, integral sign and dx as per the main scheme.

Alternative (b) via IBP Way 1:

$$\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta$$

$\left\{\begin{matrix} u=\sin^2 2\theta & v'=1 \\ u'=4\sin 2\theta\cos 2\theta & v=\theta \end{matrix}\right\}$

$$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-4\int_0^{\frac{\pi}{2}}\theta\sin 2\theta\cos 2\theta\,\mathrm{d}\theta$$

$$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\int_0^{\frac{\pi}{2}}\theta\sin 4\theta\,\mathrm{d}\theta$$

$\left\{\begin{matrix} u=\theta & v'=\sin 4\theta \\ u'=1 & v=-\frac{\cos 4\theta}{4} \end{matrix}\right\}$

$$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\left(\left[-\frac{\theta\cos 4\theta}{4}\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}-\frac{\cos 4\theta}{4}\mathrm{d}\theta\right)$$

Score M1 here.

$$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\left[-\frac{\theta\cos 4\theta}{4}+\frac{\sin 4\theta}{16}\right]_0^{\frac{\pi}{2}}$$

Score dM1 here, A1 if correct (ignoring limits).

$$=\left[\theta\sin^2 2\theta+\frac{\theta\cos 4\theta}{2}-\frac{\sin 4\theta}{8}\right]_0^{\frac{\pi}{2}}$$

$$=\left(0+\frac{\pi}{4}-0\right)-(0+0-0)=\frac{\pi}{4}\;\rightarrow\;\left\{\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x=\right\}=\frac{1}{8}\pi a^2$$

score A1* (with no errors).

Alternative (b) via IBP Way 2:

$$\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta \qquad \left\{ \begin{matrix} u = \sin 2\theta & v' = \sin 2\theta \\ u' = 2\cos 2\theta & v = -\dfrac{\cos 2\theta}{2} \end{matrix} \right\}$$

$$= \left[-\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos^2 2\theta \, d\theta$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta + \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta = \left[-\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos^2 2\theta \, d\theta + \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta$$

$$\Rightarrow 2\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta = \left[-\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} 1 \, d\theta$$ Score M1 here.

$$\Rightarrow 2\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta = \left[-\frac{\sin 2\theta \cos 2\theta}{2} + \theta \right]_0^{\frac{\pi}{2}}$$ Score dM1 here, A1 if correct including the 2 (ignoring limits).

$$\Rightarrow \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta = \frac{1}{2}\left(0 + \frac{\pi}{2}\right) - \frac{1}{2}(0+0) = \frac{\pi}{4}$$

$$\rightarrow \left\{ \int_0^a x^{\frac{1}{2}} \sqrt{a-x} \, dx = \right\} = \frac{1}{8}\pi a^2$$ score A1* (with no errors).

(**Q13 9MA0/01, June 2024**)

Q15.

Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x - q = 0$ when $x = 2$ oe Hence $q = 4$ *	B1*	2.4
	(ii) Substitutes $\left(3, \frac{1}{2}\right)$ into $y = \dfrac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\dfrac{1}{2} = \dfrac{p-9}{(2)\times(6)} \Rightarrow p - 9 = 6 \Rightarrow p = 15$ *	A1*	2.1
		(3)	

(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	$\text{I}=\int\frac{15-3x}{(2x-4)(x+3)}\text{d}x=m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	$\text{I}=\int\frac{15-3x}{(2x-4)(x+3)}\text{d}x=0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5\frac{15-3x}{(2x-4)(x+3)}\text{d}x$ Or $[\ldots\ldots\ldots\ldots]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ or $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
(11marks)			

(a)

B1*: Is able to link $2x-q=0$ and $x=2$ to explain why $q=4$

Eg "The asymptote $x=2$ is where $2x-q=0$ so $4-q=0 \Rightarrow q=4$"

"The curve is not defined when $2\times 2-q=0 \Rightarrow q=4$"

There **must be some words** explaining why $q = 4$ and in most cases, you should see a reference to either "the asymptote $x=2$", "the curve is not defined at $x=2$", 'the denominator is 0 at $x=2$"

M1: Substitutes $\left(3, \frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves

Alternatively substitutes $\left(3, \frac{1}{2}\right)$ into $y=\frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2}=\frac{6}{(2)\times(6)}$ oe

A1*: Full proof showing all necessary steps $\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15$

In the alternative there would have to be some recognition that these are equal eg ✓ hence $p=15$

(b)

M1: Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of x.

M1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B

A1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$, or for example $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$, $\frac{9}{(10x-20)}-\frac{12}{(5x+15)}$ oe

Must be written in PF form, not just for correct A and B

M1: Area $R = \int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(2x-4)+n\ln(x+3)$

OR $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(x-2)+n\ln(x+3)$

Note that $\int \frac{l}{(x-2)}\,dx \to l\ln(kx-2k)$ and $\int \frac{m}{(x+3)}\,dx \to m\ln(nx+3n)$

A1ft: $= \int \frac{15-3x}{(2x-4)(x+3)}\,dx = 0.9\ln(2x-4)-2.4\ln(x+3)$ oe. FT on their A and B

B1: Deduces that the limits for the integral are 3 and 5. It cannot just be awarded from 5 being marked on Figure 4. So award for sight of $\int_3^5 \frac{15-3x}{(2x-4)(x+3)}(dx)$ or $[\ldots\ldots\ldots\ldots]_3^5$ having performed an integral which may be incorrect

dM1: Uses correct ln work seen at least once eg $\ln 6=\ln 2+\ln 3$, $\ln 8 = 3\ln 2$ or $m\ln 6k - m\ln 2k = m\ln 3$

This is an attempt to get either of the above ln's in terms of ln2 and/or ln3

It is dependent upon the correct limits and having achieved $m\ln(2x-4)+n\ln(x+3)$ oe

A1: $=3.3\ln 3-4.8\ln 2$ oe

Q16.

Question	Scheme	Marks	AOs
(a)	Substitutes $t=0, x=4 \Rightarrow 4=\dfrac{k\times 2}{5} \Rightarrow k=...$	M1	3.1b
	$x=\dfrac{10(3t+2)}{4t+5}$	A1	3.3
		(2)	
(b)	7500	B1ft	3.4
		(1)	
(c)	$\dfrac{4}{6x-x^2}=\dfrac{A}{x}+\dfrac{B}{6-x} \Rightarrow 4=A(6-x)+Bx \Rightarrow A=...,B=...$	M1	1.1b
	$A=\dfrac{2}{3}$ or $B=\dfrac{2}{3}$	A1	1.1b
	$\dfrac{4}{6x-x^2}=\dfrac{2}{3x}+\dfrac{2}{3(6-x)}$ o.e. e.g. $\dfrac{2}{3x}+\dfrac{2}{18-3x}$	A1	1.1b
		(3)	

(a)

M1: Substitutes $x=4$ and $t=0$ into the given equation and solves to find a value for k.

A1: Correct **equation** $x=\dfrac{10(3t+2)}{4t+5}$ oe e.g. $x=\dfrac{30t+20}{4t+5}$ but not just for $k=10$

(b)

B1ft: 7500 oe e.g. 7.5 thousand. Follow through their k so allow for $0.75k\times 1000$

(c)

M1: Correct method of partial fractions leading to values for A and B

A1: Correct A or B

A1: $\dfrac{4}{6x-x^2}=\dfrac{2}{3x}+\dfrac{2}{3(6-x)}$ Correct **partial fractions** not just correct values but award if correct fractions are seen subsequently.

Question	Scheme	Marks	AOs
(d)	$\displaystyle\int \frac{4}{6x-x^2}\,dx=\int dt \Rightarrow \frac{2}{3}\ln x-\frac{2}{3}\ln(6-x)=t(+c)$	M1 A1ft	3.1a 1.1b
	$t=0, x=4 \Rightarrow c=\left(\dfrac{2}{3}\ln 2\right)$	M1	3.4
	$\dfrac{2}{3}\ln x-\dfrac{2}{3}\ln(6-x)=t+\dfrac{2}{3}\ln 2 \Rightarrow \ln\dfrac{x}{2(6-x)}=\dfrac{3}{2}t$ $\Rightarrow x+2xe^{\frac{3}{2}t}=12e^{\frac{3}{2}t}$	M1	2.1
	$x=\dfrac{12e^{\frac{3}{2}t}}{1+2e^{\frac{3}{2}t}}$ *	A1*	1.1b
		(5)	
(e)	6000	B1	3.4
		(1)	
		(12 marks)	

(d)

M1: Correct attempt at integration using lns. Look for $\alpha\ln x \pm \beta\ln(6-x) = t(+c)$

A1ft: Correct integration ft on their values for A and B. No requirement for + c here.

Note that $\frac{2}{3}\ln 3x - \frac{2}{3}\ln 3(6-x) = t(+c)$ is also correct.

dM1: Correct method to find "c" using $t = 0$ and $x = 4$

dM1: **Depends on the <u>first</u> method mark only.**

For the key steps in making x the subject of the equation.

- Uses correct ln work and proceeds to a form $\ln f(x) = at$
- Uses e as the inverse function to ln and cross multiplies (o.e) to get both x terms on the same side of the equation
- Requires a constant of integration that may or may not have been evaluated.

A1*: Correct processing leading to the given answer $x = \dfrac{12e^{\frac{3}{2}t}}{1+2e^{\frac{3}{2}t}}$

Note that 2nd and 3rd method marks may be awarded in the reverse order e.g.

$$\frac{2}{3}\ln x - \frac{2}{3}\ln(6-x) = t + c \Rightarrow \frac{2}{3}\ln\frac{x}{6-x} = t + c \Rightarrow \ln\frac{x}{6-x} = \frac{3t}{2} + k$$

$$\Rightarrow \frac{x}{6-x} = Ae^{\frac{3t}{2}} \Rightarrow x = 6Ae^{\frac{3t}{2}} - xAe^{\frac{3t}{2}} \Rightarrow x = \frac{6Ae^{\frac{3t}{2}}}{1+Ae^{\frac{3t}{2}}}$$

$$4 = \frac{6A}{1+A} \Rightarrow A = 2 \Rightarrow x = \frac{12e^{\frac{3t}{2}}}{1+2e^{\frac{3t}{2}}}*$$

(e)

B1: 6000 oe e.g. 6 thousand

(Q15 9MA0/02/M, June 2025)

Q17.

Question	Scheme		Marks	AOs
(a)	$\frac{dV}{dt} = 0.45$ or $\frac{dV}{dt} = \pm 0.3V$		M1	3.1b
	$\frac{dV}{dt} = 0.45 - \frac{3}{10}V$ $20\frac{dV}{dt} = 9 - 6V$ *		A1*	2.1
			(2)	
(b)	e.g. $\frac{1}{9-6V}\frac{dV}{dt} = \frac{1}{20} \to \int \frac{1}{9-6V}dV = \int \frac{1}{20}dt$		B1	1.1b
	$\frac{1}{9-6V} \to \ldots \ln\|9-6V\|$		M1	1.1b
	$-\frac{1}{6}\ln\|9-6V\| = \frac{t}{20} \quad (+c)$		A1	1.1b
	$-\frac{1}{6}\ln\|9-6V\| = \frac{t}{20} + c$ $9 - 6V = Ae^{-\frac{3t}{10}}$ $t = 0,\ V = 0.25 \Rightarrow A = (7.5)$	$-\frac{1}{6}\ln\|9-6V\| = \frac{t}{20} + c$ $t = 0,\ V = 0.25 \Rightarrow c = \left(-\frac{1}{6}\ln 7.5\right)$	dM1	3.1a
	$9 - 6V = 7.5e^{-\frac{3t}{10}}$ $V = \frac{3}{2} - \frac{5}{4}e^{-\frac{3t}{10}}$	$\frac{1}{6}\ln 7.5 - \frac{1}{6}\ln\|9-6V\| = \frac{t}{20}$ $\ln\frac{7.5}{9-6V} = 0.3t$ $9 - 6V = 7.5e^{-0.3t}$ $V = \frac{3}{2} - \frac{5}{4}e^{-0.3t}$	A1	2.1
			(5)	

Notes	
(a)	**Marks for part (a) may not be scored in part (b)**
M1:	Either $\frac{dV}{dt}=0.45$ or $\frac{dV}{dt}=\pm 0.3V$ o.e. e.g. $\frac{dV}{dt}=\frac{9}{20}$ or $\frac{dV}{dt}=\pm\frac{3}{10}V$ seen or implied by e.g. $\frac{dV}{dt}=0.45-\frac{3}{10}V$ (but not implied by just stating the given answer). Condone use of $\dot{V}$ It may be seen as part of their $\frac{dV}{dt}$ e.g. $\frac{dV}{dt}=0.45+V+0.3V$ scores M1A0* Condone e.g. change in volume = (inflow – outflow =) 0.45 – 0.3V for this mark.
A1*:	Achieves $20\frac{dV}{dt}=9-6V$ with no errors, following $\frac{dV}{dt}=0.45-\frac{3}{10}V$ o.e. (including the $\frac{dV}{dt}$ or $\dot{V}$ but note that it must be $\frac{dV}{dt}$ in the final line and not $\dot{V}$). change in volume = $0.45-0.3V \rightarrow 20\frac{dV}{dt}=9-6V$ scores M1A0*. Ignore any units used in their working for both marks.
(b)	
B1:	Separates the variables correctly, e.g., $\int\frac{1}{9-6V}dV=\int\frac{1}{20}dt$ or $\int\frac{20}{9-6V}dV=\int\{1\}dt$ o.e. The integral symbol and/or dV and/or dt may be implied if they go on to integrate **both** sides to the correct form $...\ln\lvert\alpha(9-6V)\rvert=...t \quad (+c)$ with or without the modulus brackets.
M1:	Attempts to integrate the reciprocal term $\frac{\beta}{9-6V}\rightarrow...\ln\lvert 9-6V\rvert$ or $\rightarrow...\ln\lvert\alpha(6V-9)\rvert$ for some constant β (and α if used). Condone e.g. $\frac{20}{9-6V}\rightarrow...\ln 9-6V$ or $\rightarrow...\ln 6V-9$
A1:	Correct integration for both sides. They do not need the + c for this mark. Note scoring this mark implies the earlier B1 (unless it is a verification attempt – see SC). Note that e.g. $-\frac{1}{6}\ln\lvert 3-2V\rvert=\frac{t}{20} \quad (+c)$ or $-\frac{10}{3}\ln\lvert 2V-3\rvert=t \quad (+c)$ are also correct. $-\frac{1}{6}\ln(9-6V)=\frac{t}{20} \quad (+c)$ is also correct. Condone log being used in place of ln.
dM1:	Requires constant of integration now. Substitutes (or states) $t=0$ and $V=0.25$ and finds a value for c, which may be "A" = e^c if they rearrange first to eliminate ln terms. Dependent on the previous method mark. Do not be concerned about their processing to find c or "A" = e^c and does not need to be exact.

A1:	Achieves the required form e.g. $V=\frac{3}{2}-\frac{5}{4}\mathrm{e}^{-\frac{3t}{10}}$ with no errors and clear working. Allow equivalent fractions or decimals e.g. $V=1.5-1.25\mathrm{e}^{-\frac{6}{20}t}$
SC:	**Attempts by verification may score maximum B0M1A1dM1A0 – see below.**
Alt:	**Use of an integrating factor – see below.**
(b)	**Special Case: Attempts by verification may score maximum B0M1A1dM1A0**
B0:	This mark may not be scored via this approach.
M1:	Differentiates $V=P-Q\mathrm{e}^{-kt}$ to the form $\frac{\mathrm{d}V}{\mathrm{d}t}=\alpha\mathrm{e}^{-kt}$ where α is a constant (note it should be $\frac{\mathrm{d}V}{\mathrm{d}t}=Qk\mathrm{e}^{-kt}$) **and** substitutes both this and $V=P-Q\mathrm{e}^{-kt}$ into $20\frac{\mathrm{d}V}{\mathrm{d}t}=9-6V$ **and** deduces a value for P **or** k by comparing coefficients.
A1:	Correct values for both P **and** k.
dM1:	Substitutes (or states) $t=0$ and $V=0.25$ and finds a value for Q. Requires a value for P to have been found using the above approach.
A0:	This mark may not be scored via this approach.
	Alternative: Using Integrating Factor (Further Maths)
B1:	Deduces the correct integrating factor for the equation, $\mathrm{e}^{0.3t}$ This should come from $\frac{\mathrm{d}V}{\mathrm{d}t}+0.3V=0.45\Rightarrow \text{I.F.}=\mathrm{e}^{\int 0.3\mathrm{d}t}=\mathrm{e}^{0.3t}$ May be implied by sight of $\frac{\mathrm{d}\left(V\mathrm{e}^{0.3t}\right)}{\mathrm{d}t}=\ldots$
M1:	Fully multiplies through by their integrating factor and integrates both sides. Score for $V\mathrm{e}^{kt}=\int\ldots\mathrm{e}^{kt}\mathrm{d}t=\ldots\mathrm{e}^{kt}$ Condone missing $\mathrm{d}t$
A1:	Correct integration $V\mathrm{e}^{0.3t}=\int 0.45\mathrm{e}^{0.3t}\mathrm{d}t=\frac{3}{2}\mathrm{e}^{0.3t}(+c)$
dM1:	As main scheme.
A1:	As main scheme.

Question	Scheme	Marks	AOs
(c)	**Examples:** (1) $\frac{dV}{dt}=0 \Rightarrow V=(1.5)$ (or e.g. max V is 1.5) (2) As $t\to\infty$, $e^{-"0.3"t}\to 0$ (or $V\to$ "1.5") (3) Flow in = flow out at max V so $0.3V=0.45\Rightarrow V=1.5$ (4) As $e^{-"0.3"t}>0$, $V<"1.5"$ (5) When $V>1.5$, $\frac{dV}{dt}<0$ (6) $V=2\Rightarrow\frac{dV}{dt}=-0.15$ or compares $\frac{dV_{out}}{dt}$ $(=0.6)$ against $\frac{dV_{in}}{dt}$ $(=0.45)$ at $V=2$ (7) $V=2\Rightarrow"1.5"-"1.25"e^{-"0.3"t}=2\Rightarrow e^{-"0.3"t}<0$ (8) $"1.5"-"1.25"e^{-"0.3"t}=2\Rightarrow\ln(-0.4)$ is undefined (condone e.g. gives a maths error)	M1	3.2a
	• The (upper) limit for V is 1.5 (m^3) so no (the container will not become full) (first 4 bullets) • If $V=2$ (or $V>1.5$), it would be emptying so no (it can never be full) (bullets 5, 6) • No, as the equation cannot be solved (or is not true/doesn't work) when $V=2$ (bullets 7, 8)	A1ft	2.4
		(2)	
			(9 marks)

Notes
(c) M1: See main scheme. If using the answer to part (b) it must be of the form $V=P-Qe^{-kt}$ but there is no limitation on the values of their P, Q or k. Substitution of a large value for t may score this mark but it is unlikely to be recovered to score the A1 unless they reference e.g. V_{max} being "1.5". Reference to an (upper) limit of "1.5" or their P can imply the method mark. If setting $V=2$ in their equation they must reach either ln(–ve) **or** solve the equation to reach a value for t to score this mark. A1ft: Must conclude "no" or equivalent e.g. "the container will not become full". Makes a correct interpretation for their method (see bullets 1-8) with a clear conclusion e.g. "no". To score this mark through ft, their V must be of the form $V=P-Qe^{-kt}$ with $k>0$, $Q>0$ **and** $0<P<2$ if used (but note that they can still use the answer to part (a) to score both marks via bullets 1, 3, 5 or 6). Allow "it" in place of the "container"/"tank". Just stating "the equation cannot be solved when $V=2$" without any evidence is M0A0. There must be no incorrect working if solving their equation or contradictory statements such as "t cannot be negative" but condone notational errors provided the intention is clear.

(Q10 9MA0/02, June 2025)

Q18.

Question	Scheme	Marks	AOs
	$y=\frac{(x-2)(x-4)}{4\sqrt{x}}=\frac{x^2-6x+8}{4\sqrt{x}}=\frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\int \frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\mathrm{d}x=\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$	M1	2.2a
	$\left(\frac{32}{10}-8+8\right)-\left(\frac{2}{5}\sqrt{2}-2\sqrt{2}+4\sqrt{2}\right)=\frac{16}{5}-\frac{12}{5}\sqrt{2}$ Area R $=\frac{12}{5}\sqrt{2}-\frac{16}{5}$ $\left(\text{or } \frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
		(6)	
			(6 marks)
Notes:			

M1: Correct attempt to write $\frac{(x-2)(x-4)}{4\sqrt{x}}$ as a sum of terms with **indices**.

Look for at least two different terms with the correct index e.g. two of $x^{\frac{3}{2}}, x^{\frac{1}{2}}, x^{-\frac{1}{2}}$ which have come from the correct places.

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

A1: $\frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ which can be left unsimplified e.g. $\frac{1}{4}x^{2-\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$

or as e.g. $\frac{1}{4}\left(x^{\frac{3}{2}}-6x^{\frac{1}{2}}+8x^{-\frac{1}{2}}\right)$

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

dM1: Integrates $x^n \to x^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $x^{\frac{3}{2}}\to x^{\frac{5}{2}}$, $x^{\frac{1}{2}}\to x^{\frac{3}{2}}$, $x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$

It is dependent upon the first M so at least two terms must have had a correct index.

A1: $\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$. Allow unsimplified e.g. $\frac{1}{4}\times\frac{2}{5}x^{\frac{3}{2}+1}-\frac{1}{2}\times\frac{2}{3}x^{\frac{1}{2}+1}-\frac{2}{3}x^{\frac{1}{2}+1}+2\times2x^{\frac{1}{2}}$

or as e.g. $\frac{1}{4}\left(\frac{2}{5}x^{\frac{5}{2}}-4x^{\frac{3}{2}}+16x^{\frac{1}{2}}\right)(+c)$.

M1: Substitutes the limits 4 and 2 to their $\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$ and subtracts either way round.

There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{10}\times4^{\frac{5}{2}}-4^{\frac{3}{2}}+4\times4^{\frac{1}{2}}-\frac{1}{10}\times2^{\frac{5}{2}}-2^{\frac{3}{2}}+4\times2^{\frac{1}{2}}$

This is an independent mark but the limits must be applied to an expression that is not y so they may even have differentiated.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw

Integration by parts:

$\int \frac{(x-2)(x-4)}{4\sqrt{x}}\,dx = \int \frac{1}{4}(x-2)(x-4)x^{-\frac{1}{2}}\,dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx$	M1 A1	1.1b 1.1b
$\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\,dx$ $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ Or e.g. $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	dM1 A1	3.1a 1.1b
Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	M1	2.2a
$0 - \frac{16}{3} + \frac{128}{15} - \left(0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}\right)$ Area $R = \frac{12}{5}\sqrt{2} - \frac{16}{5}$ $\left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies integration by parts and reaches the form $\alpha(x-2)(x-4)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ $\alpha, p \neq 0$

oe e.g. $\alpha\left(x^2 - 6x + 8\right)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ $\alpha, p \neq 0$

A1: Correct first application of parts in any form

dM1: Attempts their $\int (px+q)x^{\frac{1}{2}}\,dx$ by expanding and integrating or may attempt parts again.

E.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \int \left(2x^{\frac{3}{2}} - 6x^{\frac{1}{2}}\right)dx = \ldots$ or e.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}}(2x-6) - \frac{4}{3}\int x^{\frac{3}{2}}\,dx$

If they expand then at least one term requires $x^n \to x^{n+1}$ or if parts is attempted again, the structure must be correct.

A1: Fully correct integration in any form

M1: Substitutes the limits 4 and 2 to their $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ and subtracts either way round. There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $0 - \frac{16}{3} + \frac{128}{15} - 0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}$

This is an independent mark but the limits must be applied to a "changed" function.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents

Attempts at integration by parts "the other way round" should be sent to review.

Integration by substitution example:

$u=\sqrt{x}\left(x=u^2\right)\Rightarrow\int\frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ $=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}2u\mathrm{d}u$	M1 A1	1.1b 1.1b
$=\frac{1}{2}\int\left(u^4-6u^2+8\right)\mathrm{d}u=\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$	dM1 A1	3.1a 1.1b
Deduces limits of integral are $\sqrt{2}$ and 2 and applies to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$	M1	2.2a
$\frac{1}{2}\left(\frac{32}{5}-16+16-\left(\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)\right)$ Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\left(\text{or }\frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies the substitution e.g. $u=\sqrt{x}$ and attempts $k\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$

A1: Fully correct integral in terms of u in any form e.g. $\frac{1}{2}\int\left(u^2-2\right)\left(u^2-4\right)\mathrm{d}u$

dM1: Expands the bracket and integrates $u^n\to u^{n+1}$ for at least 2 correct indices
i.e. at least 2 of $u^4\to u^5$, $u^2\to u^3$, $k\to ku$

A1: $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$. Allow unsimplified.

M1: Substitutes the limits 2 and $\sqrt{2}$ to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$ and subtracts either way round.

There is no requirement to evaluate but 2 and $\sqrt{2}$ must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{2}\left(\frac{32}{5}-16+16-\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)$

Alternatively reverses the substitution and applies the limits 4 and 2 with the same conditions.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw.

There may be other substitutions seen and the same marking principles apply.

(Q08 9MA0/02, June 2022)

Q19.

Question	Scheme	Marks	AOs
	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ **or** $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\}\ \frac{1}{2}x^2\ln x - \int \frac{1}{x}\cdot\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\ \{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question	
M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } g'(x))$, where $g(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\mathrm{e}, \mathrm{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \mathrm{e} = m_N(x - \mathrm{e})$ to find $x = ...$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\mathrm{e}$, allowing un-simplified values for x such as $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$
Note:	Allow $x =$ awrt 8.15
M1:	Scored for either • Area under curve $= \int_1^{\mathrm{e}} x\ln x \, dx = [\ ... \ ]_1^{\mathrm{e}} = ...$, with limits of e and 1 and some attempt to substitute these and subtract • **or** Area under line $= \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2 \ln x - \int B\left(\frac{x^2}{x}\right)\{dx\}$; $A \neq 0, B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$
Note:	$\text{Area}(R_2)$ can also be found by integrating the line l between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\mathrm{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\mathrm{e}\right) dx = [\ ... \ ]_{\mathrm{e}}^{\text{their } x_A} = ...$
Note:	**Calculator approach with no algebra, differentiation or integration seen:** • Finding l cuts through the x-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the x-axis between $x = 1$ and $x = \mathrm{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972... + 7.3890... = 9.4862...$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.

(Q13 9MA0/02, June 2018)

Q20.

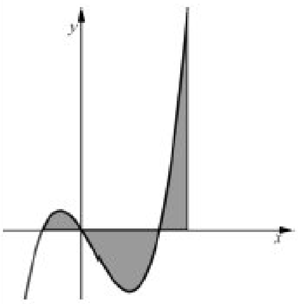
Question	Scheme		Marks	AOs
(a)	$\frac{dr}{dt} \propto \pm\frac{1}{r^2}$ or $\frac{dr}{dt} = \pm\frac{k}{r^2}$ (for k or a numerical k)		M1	3.3
	$\int r^2\,dr = \int \pm k\,dt \Rightarrow \ldots$ (for k or a numerical k)		M1	2.1
	$\frac{1}{3}r^3 = \pm kt\ \{+c\}$		A1	1.1b
	$t=0, r=5$ and $t=4, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth	$t=0, r=5$ and $t=240, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	M1 A1	3.1a 1.1b
			(5)	
(b)	$r=0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow\ 0 = -49t + 250\ \Rightarrow t = \ldots$		M1	3.4
	time = 5 minutes 6 seconds		A1	1.1b
			(2)	
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth		B1	3.5b
			(1)	
				(8 marks)

Notes for Question	
(a)	
M1:	Translates the description of the model into mathematics. See scheme.
M1:	Separates the variables of their differential equation which is in the form $\frac{dr}{dt} = f(r)$ and some attempt at integration. (e.g. attempts to integrate at least one side). e.g. $\int r^2 \, dr = \int \pm k \, dt$ and some attempt at integration. Condone the lack of integral signs
Note:	You can imply the M1 mark for $r^2 dr = -k\,dt \Rightarrow \frac{1}{3}r^3 = -kt$
Note:	A numerical value of k (e.g. $k = \pm 1$) is allowed for the first two M marks
A1:	Correct integration to give $\frac{1}{3}r^3 = \pm kt$ with or without a constant of integration, c
M1:	For a complete process of using the boundary conditions to find both their unknown constants and finds an equation linking r and t So applies either • $t = 0, r = 5$ and $t = 4, r = 3$, or • $t = 0, r = 5$ and $t = 240, r = 3$, *on their integrated equation* to find their constants k and c *and* obtains an equation linking r and t
A1:	Correct equation, with variables r and t fully defined including correct reference to units. • $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth • $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$, {or an equivalent equation,} where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth
Note:	Allow correct equations such as • in minutes, $r = \sqrt[3]{\frac{250 - 49t}{2}}$, $r^3 = -\frac{49}{2}t + 125$ or $t = \frac{250 - 2r^3}{49}$ • in seconds, $r = \sqrt[3]{\frac{15000 - 49t}{120}}$, $r^3 = -\frac{49}{120}t + 125$ or $t = \frac{15000 - 120r^3}{49}$
Note:	t defined as "the time from the start" is not sufficient for the final A1

(b)	
M1:	Sets $r = 0$ in their part (a) equation which links r with t and rearranges to make $t = \ldots$
A1:	5 minutes 6 seconds cao (**Note:** 306 seconds with no reference to 5 minutes 6 seconds is A0)
Note:	Give M0 if their equation would solve to give a negative time or a negative time is found
Note:	You can mark part (a) and part (b) together
(c)	
B1:	See scheme
Note:	Do not accept by itself • mint may not dissolve at a constant rate • rate of decrease of mint must be constant • $0 \le t < \frac{250}{49}$, $r \ge 0$; without any written explanation • reference to a mint having $r > 5$

(**Q10 9MA0/02, June 2018**)

Q21.

Question	Scheme	Marks	AOs
(a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \, dx \to \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y \, dx$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2(3b^2 - 20b + 20) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x-axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to $x^3 - 2x^2 - 8x$ (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \to x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function.

The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2} = \frac{20}{3}$ oe for this mark

A1*: For a rigorous argument leading to area of $R_1 = \frac{20}{3}$ For this to be awarded the integration must be correct and the limits must be the correct way around and embedded or calculated values must be seen.

Eg. Look for $-\left(4+\frac{16}{3}-16\right)$ or $-\left(\frac{1}{4}(-2)^4-\frac{2}{3}(-2)^3-4(-2)^2\right)$ oe before you see the $\frac{20}{3}$

Note: It is possible to do this integration by parts.

(b)

M1: For setting their $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=\pm\frac{20}{3}$ or $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{b}=0$

A1: Deduces that $3b^4-8b^3-48b^2+80=0$. Terms may be in a different order but expect integer coefficients.

It must have followed $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=-\frac{20}{3}$ oe.

Do not award this mark for $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2+\frac{20}{3}=0$ unless they attempt the second part of this question by expansion and then divide the resulting expanded expression by 12

M1: Attempts to factorise $3b^4-8b^3-48b^2\pm80=(b+2)(b+2)(3b^2...b...20)$ via repeated division or inspection. FYI $3b^4-8b^3-48b^2+80=(b+2)(3b^3-14b^2-20b+40)$ Allow an attempt via inspection $3b^4-8b^3-48b^2\pm80=(b^2+4b+4)(3b^2...b...20)$ but do not allow candidates to just write out $3b^4-8b^3-48b^2\pm80=(b+2)^2(3b^2-20b+20)$ which is really just copying out the given answer.

Alternatively attempts to expand $(b+2)^2(3b^2-20b+20)$ achieving terms of a quartic expression

A1*: Correctly reaches $(b+2)^2(3b^2-20b+20)=0$ with no errors and must have $=0$

In the alternative obtains both equations in the same form **and states that they are same**. Allow ✓ QED etc here.

(c) Please watch for candidates who answer this on Figure 2 which is fine

B1: Sketches the curve and a vertical line to the right of 4 ($x = 5.442$ may not be labelled.)

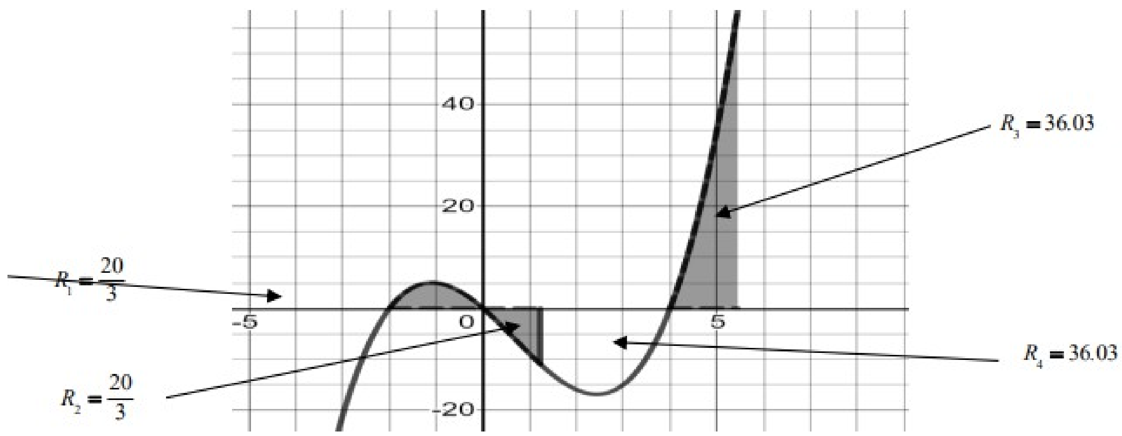
B1: Explains that (between $x=-2$ and $x=5.442$) the area above the x-axis = area below the x-axis with appropriate areas shaded or labelled.

Alternatively states that the area between 1.225 and 4 is the same as the area between 4 and 5.442

Another correct statement is that the net area between 0 and 5.442 is $-\frac{20}{3}$

Look carefully at what is written. There are many correct statements/ deductions.

Eg. " (area between 0 and 4) - (area between 4 and 5.442) = 20/3". Diagram below for your information.



(**Q08 9MA0/01, June 2019**)

Q22.

Question	Scheme	Marks	AOs
(a)	$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow A = ..., B = ...$	M1	1.1b
	Either $A = 2$ or $B = -1$	A1	1.1b
	$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$	A1	1.1b
		(3)	
(b)	$\int \frac{1}{V} \mathrm{d}V = \int \frac{3}{(2t-1)(t+1)} \mathrm{d}t$	B1	1.1a
	$\int \frac{2}{2t-1} - \frac{1}{t+1} \mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$	M1	3.1a
	$\ln V = \ln(2t-1) - \ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = (\ln 3)$	M1	3.4
	$\ln V = \ln(2t-1) - \ln(t+1) + \ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	

	(b) Alternative separation of variables:		
	$\int \frac{1}{3V} \mathrm{d}V = \int \frac{1}{(2t-1)(t+1)} \mathrm{d}t$	B1	1.1a
	$\frac{1}{3}\int \frac{2}{2t-1} - \frac{1}{t+1} \mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$	M1	3.1a
	$\frac{1}{3}\ln 3V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = \left(\frac{1}{3}\ln 3\right)$	M1	3.4
	$\frac{1}{3}\ln V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1) + \frac{1}{3}\ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	
(c)	(i) 30 (minutes)	B1	3.2a
	(ii) 6 (m^3)	B1	3.4
		(2)	
			(10 marks)
Notes:			

(a)

M1: Correct method of partial fractions leading to values for their A and B

E.g. substitution: $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(x+1) + B(2x-1) \Rightarrow A = ..., B = ...$

Or compare coefficients $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = x(A+2B) + A - B \Rightarrow A = ..., B = ...$

Note that $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(2x-1) + B(x+1) \Rightarrow A = ..., B = ...$ scores M0

A1: Correct value for "A" or "B"

A1: Correct partial fractions not just values for "A" and "B". $\frac{2}{2x-1} - \frac{1}{x+1}$ or e.g. $\frac{2}{2x-1} + \frac{-1}{x+1}$

Must be seen as **fractions** but if not stated here, allow if the correct fractions appear later.

(b)

B1: Separates variables $\int \frac{1}{V} \mathrm{d}V = \int \frac{3}{(2t-1)(t+1)} \mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1) + ...\ln(t+1)$ where … are constants.

Condone missing brackets around the $(2t-1)$ and/or the $(t+1)$ for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for $+c$ here.

The brackets around the $(2t-1)$ and/or the $(t+1)$ must be seen or implied for this mark

M1: Attempts to find "c" or e.g. "$\ln k$" using $t = 2,\ V = 3$ following an attempt at integration.

Condone poor algebra as long as $t = 2,\ V = 3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

A1*: Correct processing leading to the given answer $V = \frac{3(2t-1)}{(t+1)}$

Alternative:

B1: Separates variables $\int \frac{1}{3V}\,dV = \int \frac{1}{(2t-1)(t+1)}\,dt$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the $(2t-1)$ and/or the $(t+1)$ for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for $+c$ here.

The brackets around the $(2t-1)$ and/or the $(t+1)$ must be seen or implied for this mark

M1: Attempts to find "c" or e.g. "ln k" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

A1*: Correct processing leading to the given answer $V=\frac{3(2t-1)}{(t+1)}$

(Note the working may look like this:

$$\frac{1}{3}\ln 3V=\frac{1}{3}\ln(2t-1)-\frac{1}{3}\ln(t+1)+c,\ \frac{1}{3}\ln 9=\frac{1}{3}\ln(3)-\frac{1}{3}\ln 3+c,\ c=\frac{1}{3}\ln 9$$

$$\ln 3V=\ln\frac{9(2t-1)}{(t+1)}\Rightarrow 3V=\frac{9(2t-1)}{(t+1)}\Rightarrow V=\frac{3(2t-1)}{(t+1)}*)$$

Note that B0M1A1M1A1 is not possible in (b) as the B1 must be implied if all the other marks have been awarded.

Note also that some candidates may use different variables in (b) e.g.

$\frac{dy}{dx}=\frac{3y}{(2x-1)(x+1)}\Rightarrow\int\frac{1}{y}\,dy=\int\frac{3}{(2x-1)(x+1)}\,dx$ etc. In such cases you should award marks for equivalent work but they must revert to the given variables at the end to score the final mark.

Also if e.g. a "t" becomes an "x" within their working but is recovered allow full marks.

(c)

B1: Deduces 30 minutes. Units not required so just look for 30 but allow equivalents e.g. ½ an hour.

If units are given they must be correct so do not allow e.g. 30 hours.

B1: Deduces 6 m^3. Units not required so just look for 6. Condone $V<6$ or $V\le 6$

If units are given they must be correct so do not allow e.g. 6 m.

(Q14 9MA0/02, June 2022)

Q23.

Question	Scheme	Marks	AOs
(a)	$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \int_{2.1}^{6.3} \frac{2}{x}\,dx$	B1	1.2
		(1)	
(b)	$= [2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1$	M1	1.1b
	$= \ln 9$ CSO	A1	1.1b
		(2)	
			(3 marks)
Notes:			

Mark (a) and (b) as one

(a)

B1: States that $\int_{2.1}^{6.3} \frac{2}{x}\,dx$ or equivalent such as $2\int_{2.1}^{6.3} x^{-1}\,dx$ but must include the limits and the dx.

Condone $dx \leftrightarrow \delta x$ as it is very difficult to tell one from another sometimes

(b)

M1: Know that $\int \frac{1}{x}\,dx = \ln x$ and attempts to apply the limits (either way around)

Condone $\int \frac{2}{x}\,dx = p\ln x$ (including $p = 1$) or $\int \frac{2}{x}\,dx = p\ln qx$ as long as the limits are applied.

Also be aware that $\int \frac{2}{x}\,dx = \ln x^2$, $\int \frac{2}{x}\,dx = 2\ln|x| + c$ and $\int \frac{2}{x}\,dx = 2\ln cx$ o.e. are also correct

$[p\ln x]_{2.1}^{6.3} = p\ln 6.3 - p\ln 2.1$ is sufficient evidence to award this mark

A1: CSO $\ln 9$. Also answer $= \ln 3^2$ so $k = 9$ is fine. Condone $\ln|9|$

The method mark must have been awarded. Do not accept answers such as $\ln\frac{39.69}{4.41}$

Note that solutions appearing from "rounded" decimal work when taking lns should not score the final mark. It is a "show that" question

E.g. $[2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1 = 2.197 = \ln k \Rightarrow k = e^{2.197} = 8.998 = 9$

(Q04 9MA0/01, June 2022)

Q24.

Question	Scheme	Marks	AOs
(a)	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=3^{-2}(1+...x+...x^2)$	M1	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	M1	1.1b
	$\left(1+\frac{x}{3}\right)^{-2}=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	A1	1.1b
	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$	A1	2.1
		(4)	

(a)

M1: Attempts a binomial expansion by taking out a factor of 3^{-2} or $\frac{1}{3^2}$ or $\frac{1}{9}$ and achieves at least the first 3 terms in their expansion. May be seen separately e.g. $\frac{1}{9}$ and $(1+...x+...x^2)$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)(kx)$ or $\frac{(-2)(-2-1)}{2!}(kx)^2$ where $k \neq 1$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $\left(1+\frac{x}{3}\right)^{-2}$ e.g. $=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2-...$ or $1-\frac{2x}{3}+\frac{x^2}{3}-...$ Do not condone missing brackets unless they are implied by subsequent work.

Condone $\left(-\frac{x}{3}\right)^2$ for $\left(\frac{x}{3}\right)^2$

Also allow this mark for 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

<u>Direct expansion, if seen, should be marked as follows:</u>

$$\left((3+x)^{-2}=3^{-2}-2\times3^{-3}\times x+\frac{-2(-2-1)}{2!}\times3^{-4}\times x^2\right)$$

M1: For $(3+x)^{-2}=3^{-2}+3^{-3}\times\alpha x+3^{-4}\times\beta x^2$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)\times3^{-3}x$ or $\frac{(-2)(-2-1)}{2!}\times3^{-4}x^2$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $(3+x)^{-2}$ e.g. $3^{-2}-2\times3^{-3}\times x+\frac{-2(-2-1)}{2!}\times3^{-4}\times x^2$

Also award for at least 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

Note that M0M1A1A0 is a possible mark trait in either method

Note regarding a possible misread in parts (b) and (c)

Some candidates are misreading $\int \frac{6x}{(3+x)^2}\,dx$ in parts (b) and (c) as $\int \frac{6}{(3+x)^2}\,dx$

If parts (b) and (c) are consistently attempted with $\int \frac{6}{(3+x)^2}\,dx$ then we will allow the M marks in (b) **only**. **M1** for $x^n \to x^{n+1}$ applied to their expansion in part (a) or 6× (their expansion in part (a)) and **dM1** for substituting in 0.4 and 0.2 and subtracting either way round (may be implied).

No marks are available in part (c)

MARK PARTS (b) and (c) TOGETHER

(b)	$\int 6x \times "\left(\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}\right)"\,dx = \int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx = ...$	**M1**	1.1b
	$\int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx = \frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}$ oe	**A1**	1.1b
	$\left["\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}"\right]_{0.2}^{0.4} = \left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right)-\left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right)$	**dM1**	3.1a
	$=$ awrt 0.03304 or $\frac{223}{6750}$	**A1**	1.1b
		(4)	

(b)

M1: Attempts to multiply their expansion from part (a) by $6x$ or just x and attempts to integrate. Condone copying slips and slips in expanding. Look for $x^n \to x^{n+1}$ at least once having multiplied by $6x$ or x.
Ignore e.g. spurious integral signs.

A1: Correct integration, simplified or unsimplified.

$$\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18} \text{ oe e.g. } \frac{1}{9}\left(3x^2-\frac{4x^3}{3}+\frac{x^4}{2}\right),\ 6\left(\frac{x^2}{18}-\frac{2x^3}{81}+\frac{x^4}{108}\right)$$

If they have extra terms they can be ignored.
Ignore e.g. spurious integral signs.

dM1: An overall problem-solving mark for

- using part (a) by integrating $6x \times$ their binomial expansion and
- substituting in 0.4 and 0.2 and subtracting either way round (may be implied)

For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the limits 0.4 and 0.2.

This could be e.g. $\left[\mathrm{f}(x)\right]_{0.2}^{0.4} = ...$ provided the first M was scored. If the integration was correct, evidence can be taken from answer of awrt 0.0330 if limits are not seen elsewhere. **Depends on the first M mark.**

A1: awrt 0.03304 (NB allow the exact value which is $\frac{223}{6750} = 0.033037037...$).

Isw following a correct answer.
Note answers which use additional terms in the expansion to give a different approximation score A0
Also note that the actual value is 0.032865…

Some may use integration by parts in (b) and the following scheme should be applied.

Integration by parts in (b):

Either by taking $u = 6x$ and $\frac{dv}{dx} = "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = 6x \times \left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - 6\int\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right)dx$$

$$= 6x\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - \left(\frac{1}{3}x^2 - \frac{2x^3}{27} + \frac{6x^4}{324}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = kx \times f(x) - k\int f(x)dx = kx \times f(x) - kg(x)$$

Where f(x) is an attempt to integrate their expansion from (a) with $x^n \to x^{n+1}$ at least once **and** g(x) is an attempt to integrate their f(x) with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

Or by taking $u = "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"$ and $\frac{dv}{dx} = 6x$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int 3x^2 \times \left(-\frac{2}{27} + \frac{2x}{27}\right)dx$$

$$= 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int\left(\frac{6x^3}{27} - \frac{6x^2}{27}\right)dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \left(\frac{6x^4}{108} - \frac{6x^3}{81}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = kx^2 \times f(x) - k\int x^2 g(x)dx = kx^2 \times f(x) - kh(x)$$

Where f(x) is their expansion from (a) and g(x) is an attempt to differentiate their f(x) with $x^n \to x^{n-1}$ at least once **and** h(x) is an attempt to integrate their $x^2 g(x)$ with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

(c)	Overall problem-solving mark (see notes)	M1	3.1a
	$u = 3 + x \Rightarrow \int_{3.2}^{3.4} f(u)\,du \Rightarrow \int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,du = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2}\,du \Rightarrow ...\ln u + ...u^{-1}$	M1	1.1b
	$\int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,du = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2}\,du \Rightarrow 6\ln u + 18u^{-1}$	A1	1.1b
	$\left[6\ln u + 18u^{-1}\right]_{3.2}^{3.4} = \left(6\ln 3.4 + \frac{18}{3.4}\right) - \left(6\ln 3.2 + \frac{18}{3.2}\right) = ...$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1
		(5)	
(c) Alt 1	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2}\,dx = \frac{...x}{3+x} \pm ...\int(3+x)^{-1}dx = \frac{...x}{3+x} \pm ...\ln(3+x)$ oe	M1	1.1b
	$= 6\ln(3+x) - \frac{6x}{3+x}$ oe	A1	1.1b
	$\left(6\ln(3+0.4) - \frac{6(0.4)}{3+0.4}\right) - \left(6\ln(3+0.2) - \frac{6(0.2)}{3+0.2}\right) = ...$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1

(c) Alt 2	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2}\,dx = \int\left(\frac{...}{(3+x)}+\frac{...}{(3+x)^2}\right)dx = ...\ln(3+x)+\frac{...}{3+x}$ oe	M1	1.1b
	$=6\ln(3+x)+\frac{18}{3+x}$ oe	A1	1.1b
	$\left(6\ln(3+0.4)+\frac{18}{3+0.4}\right)-\left(6\ln(3+0.2)+\frac{18}{3+0.2}\right)=...$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right)-\frac{45}{136}$ oe	A1	2.1
			(13 marks)

Notes

(c) There are various methods which can be used

M1: An overall problem-solving mark for <u>all of</u>

- using an appropriate integration technique e.g. substitution, by parts or partial fractions – note that this may not be correct but mark positively if they have tried one of these approaches
- integrates one of their terms to a natural logarithm, e.g., $\frac{a}{3+x}\to b\ln(3+x)$ or $\frac{a}{u}\to b\ln u$
- substitutes in correct limits and subtracts either way round

M1: Integrates to achieve an expression of the required form for their chosen method

- substitution: $u=x+3\to\pm\frac{a}{u}\pm b\ln u$ or e.g. $u=(x+3)^2\to\pm\frac{a}{\sqrt{u}}\pm b\ln u$
- parts: $\pm a\ln(3+x)\pm\frac{bx}{3+x}$ condone missing brackets e.g. $...\ln x+3$ for $...\ln(3+x)$
- partial fractions: $\pm a\ln(3+x)\pm\frac{b}{3+x}$ condone missing brackets e.g. $...\ln 3+x$ for $...\ln(3+x)$

A1: Correct integration for their method e.g.

- substitution: $u=x+3\to 6\ln u+18u^{-1}$ or e.g. $u=(x+3)^2\to 3\ln u+\frac{18}{\sqrt{u}}$
- parts: $6\ln(3+x)-\frac{6x}{3+x}$
- partial fractions: $6\ln(3+x)+\frac{18}{3+x}$ oe e.g. $3\ln(9+6x+x^2)+\frac{18}{3+x}$

Note that the above terms may appear "separated" but must be correct with the correct signs. (ignore any reference to a constant of integration)

Do not condone missing brackets e.g. $6\ln x+3$ for $6\ln(3+x)$ unless they are implied by later work.

ddM1: Substitutes in the correct limits for their integral and subtracts either way round to find a value

Depends on both previous method marks.

For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the appropriate limits.

This could be e.g. $\left[f(x)\right]_{0.2}^{0.4}=...$ provided both previous M marks were scored.

Note that for substitution they may revert back to $3+x$ and so should be using 0.4 and 0.2

A1: A full and rigorous argument leading to $6\ln\left(\frac{17}{16}\right)-\frac{45}{136}$ or exact equivalent e.g. $3\ln\left(\frac{289}{256}\right)-\frac{45}{136}$ or e.g. $-6\ln\left(\frac{16}{17}\right)-\frac{45}{136}$

The brackets are not required around the $\frac{17}{16}$ and allow exact equivalents e.g. allow 1.0625 or $1\frac{1}{16}$ but not e.g. $\frac{3.4}{3.2}$. The $\frac{45}{136}$ must be exact or an exact equivalent. Also allow e.g. $6\ln\left|\frac{17}{16}\right|-\frac{45}{136}$

Ignore spurious integral signs that may appear as part of their solution.

(Q13 9MA0/02, June 2023)

Q25.

Question	Scheme	Marks	AOs
(a)	$\{H=\}0.6\mathrm{e}^{-0.2t}\{+c\}$	M1	1.1b
	$t=0, H=1.5 \Rightarrow 1.5 = "0.6"+c$ $\Rightarrow c=0.9$	dM1	3.4
	$\Rightarrow H=0.6\mathrm{e}^{-0.2t}+0.9$	A1	2.1
		(3)	
(b)	$1.2=0.6\mathrm{e}^{-0.2t}+0.9 \Rightarrow 0.6\mathrm{e}^{-0.2t}=0.3$	M1	3.4
	$\mathrm{e}^{-0.2t}=\frac{1}{2}$ $\Rightarrow t=-5\ln\left(\frac{1}{2}\right)$	dM1	1.1b
	$\{t=\}$3 hours 28 minutes	A1	3.2a
		(3)	
(c)	{As t gets large $H\to$} 0.9	M1	3.1b
	0.9 m or 90 cm	A1ft	2.2b
		(2)	
			(8 marks)

Notes

(a)

M1: Attempts integration to achieve $\{H=\}k\,\mathrm{e}^{-0.2t}\{+c\}$ with k a numerical constant $\neq -0.12$
Note that we will condone $k=(-0.2)(-0.12)$ $\{=0.024\}$
If they divide by -0.12 first before integrating they need $aH=be^{-0.2t}\{+c\}$ with a and b numerical and $b\neq 1$. Condone a spurious integral symbol remaining after integration.

dM1: Uses $t=0, H=1.5$ and a model of the form $H=k\,\mathrm{e}^{-0.2t}+c$ (or $aH=be^{-0.2t}+c$) to find the value of the constant c. They cannot just "make up" a value for k.
Do not be concerned with their processing to find c but they cannot just state B (or c) is 0.
For reference if they divide by -0.12 first they should reach $-\frac{25}{3}H=-5\mathrm{e}^{-0.2t}-7.5$ o.e.

A1: Correct complete equation in the required form: $H=0.6\mathrm{e}^{-0.2t}+0.9$ with the $H=$ present.
May be awarded if seen at the start of (b) but not in (c). Condone $-\frac{1}{5}$ in place of -0.2
Finding correct values for A and B is insufficient for this mark.
Allow exact equivalents but they must be in the required form, e.g. $H=\frac{6}{10}\mathrm{e}^{-0.2t}+\frac{9}{10}$
A minimally acceptable answer is $\{H=\}0.6\mathrm{e}^{-0.2t}+c \to H=0.6\mathrm{e}^{-0.2t}+0.9$ score M1dM1A1.
Note: sight of differentiating the given form to e.g. $\frac{\mathrm{d}H}{\mathrm{d}t}=-0.2A\mathrm{e}^{-0.2t}$ in their working without clear evidence of integration of the original differential equation should be marked using the special case below.

SC: For candidates starting with the given answer $H=A\mathrm{e}^{-0.2t}+B$ it is possible to use $\frac{\mathrm{d}H}{\mathrm{d}t}=-0.2A\mathrm{e}^{-0.2t}=-0.12\mathrm{e}^{-0.2t}$ to deduce that $A=0.6$. This can be awarded SC M1dM0A0
If they go on to find B as in the main scheme then this can be awarded SC M1dM1A0
Answer with no working scores 110.

Note:	If the special case is applied they may go on to achieve the rest of the marks in (b) and (c).
(b)	Note: A and B must be numbers but may be "made up" if they did not have an answer to (a).
M1:	Uses $H = 1.2$ in a model of the form $H = Ae^{-0.2t} + B,\ B \neq 0$ and rearranges to make $Ae^{\pm 0.2t}$ or $e^{\pm 0.2t}$ the subject. Condone slips in rearranging, e.g. dividing the LHS by 0.9 instead of subtracting 0.9. Rearranging first before substituting is acceptable but they must get to $Ae^{\pm 0.2t}$ or $e^{\pm 0.2t}$ as the subject.
dM1:	Correct use of ln to make t the subject. Requires $A > 0,\ 0 < B < 1.2$ **and** $e^{\pm 0.2t} = \lambda > 0$ If they had a negative value for A in part (a) they cannot just make it positive at this stage. Any of $5\ln 2$ or $-5\ln\frac{1}{2}$ or awrt 3.46 or awrt 3.47 following a correct equation will imply M1dM1. If they do not show their method for an incorrect $H = Ae^{-0.2t} + B$ with $A > 0,\ 0 < B < 1.2$ you may need to check their value for $t > 0$ as it may imply M1dM1.
A1:	Correct time in hours and minutes $\{t =\}$ 3 hours 28 minutes, but condone e.g. 3h 28m Must come from correct values of A and B in (a).
Note:	If their $B = 0$ then they should end up with $t = -3.46\ldots$ however, they did not score the first M1. They cannot "recover" this by making it positive and finding $t = 3$ hours 28 minutes.
(c)	Note that 0.9 or 0.9m must come from a correct value of B in (a) to score any marks.
M1:	Identifies the requirement to establish the limit as t tends to infinity. It can be implied by stating that $H = Ae^{-0.2t} + B \to B$ or $\left(\lim_{t\to\infty}\left[0.6e^{-0.2t} + 0.9\right]\right) = 0.9$ Stating "B" on its own will score this mark. Substituting a large value is M0 unless it leads to their value for B at which point the A1 is available as well.
A1ft:	Correct height including units. Follow through on their value of B where $0 < B < 1.2$ Correct ft height including units implies M1A1, while e.g. 0.9 (no units) would imply M1A0. Evidence of an incorrect method such as $1.5 - 0.6e^{-0.2(0)} = 0.9$ m scores M0A0. Misreading as $\frac{dH}{dt} = -0.12e^{0.2t}$ can score a maximum (a) 110 (b) 100 (c) 10.

(**Q07 9MA0/01, June 2024**)

Q26.

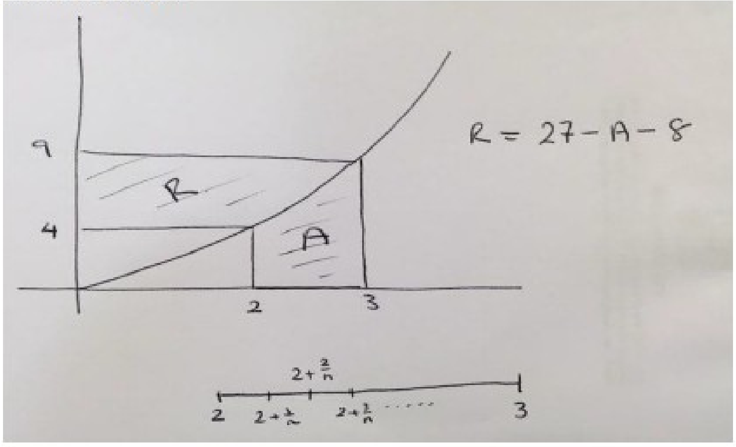
Question	Scheme	Marks	AOs
	States $\left\{\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\ \delta x \text{ is}\right\}$ $\int_4^9 \sqrt{x}\,dx$	B1	1.2
	$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9$	M1	1.1b
	$= \frac{2}{3}\times 9^{\frac{3}{2}} - \frac{2}{3}\times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

Notes for Question 5

B1:	States $\int_4^9 \sqrt{x}\,dx$ with or without the 'dx'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}}\right]_4^9$ or for $\lambda\times 9^{\frac{3}{2}} - \lambda\times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\,dx - \int_1^3 \sqrt{x}\,dx$ or for $\int_3^9 \sqrt{x}\,dx$ without reference to a correct $\int_4^9 \sqrt{x}\,dx$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\,dx = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3}x^{\frac{3}{2}} + c\right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\,dx$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Notes for Question Continued

Alt The following method is correct:



$$\text{Area }(A) = \lim_{n\to\infty}\sum_{i=1}^{n}(x_i - x_{i-1})\mathrm{f}(x_i) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(2+\frac{i}{n}\right)^2$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{4i}{n}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{1}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{4n}{n} + \frac{4}{n^2}\left(\frac{1}{2}n(n+1)\right) + \frac{1}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{4n^2+4n}{2n^2} + \frac{2n^3+3n^2+n}{6n^3}\right]$$

$$= \lim_{n\to\infty}\left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$$

$$= 4 + 2 + \frac{1}{3} = \frac{19}{3}$$

So, $\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x = \text{Area}(R) = (3\times 9) - (2\times 4) - \frac{19}{3}$

$$= \frac{38}{3} \quad \text{or} \quad 12\frac{2}{3} \quad \text{or} \quad \text{awrt } 12.7$$

(Q05 9MA0/02, June 2019)

Q27.

Question	Scheme	Marks	AOs
	$\int \sqrt{2x-7}\,dx = \frac{(2x-7)^{\frac{3}{2}}}{3}$	M1 A1	1.1b 1.1b
	Deduces **one** of; top limit is 16, bottom limit is $\frac{7}{2}$ or area rectangle = 80	B1	2.2a
	Deduces **all three** of ; top limit is 16, bottom limit is $\frac{7}{2}$ and area rectangle = 80	B1	2.2a
	Full strategy to find area R $= 80 - \left[\frac{(2x-7)^{\frac{3}{2}}}{3}\right]_{\frac{7}{2}}^{16}$	M1	3.1a
	Area $R = 80 - \frac{125}{3} = \frac{115}{3}$	A1	1.1b
		(6)	
			(6 marks)

Notes:

M1: Attempts to integrate $\sqrt{2x-7}$ to achieve $k(2x-7)^{\frac{3}{2}}$

May be implied if a substitution is used e.g. $u = 2x-7 \Rightarrow \int \sqrt{2x-7}\,dx = ku^{\frac{3}{2}}$

A1: $\int \sqrt{2x-7}\,dx = \frac{(2x-7)^{\frac{3}{2}}}{3}$ or e.g. $\frac{1}{3}u^{\frac{3}{2}}$ where $u = 2x-7$ which can be left unsimplified

B1: Deduces correct top limit, bottom limit or area of rectangle

B1: Deduces correct top limit, bottom limit and area of rectangle

M1: Fully correct strategy to find the area of R e.g. a correct unsimplified expression for area. Follow through their integration but limits etc. must be correct.

A1: $\frac{115}{3}$ or exact equivalent. Allow $38.\dot{3}$ or $38\frac{1}{3}$ but not 38.3

Alt method (not on specification but may be seen):

Attempts $\int_0^5 x\,dy = \int_0^5 \frac{y^2+7}{2}\,dy$ scores M1 A1 B1

$$= \left[\frac{y^3}{6} + \frac{7}{2}y\right]_0^5 = \frac{125}{6} + \frac{35}{2} = \frac{115}{3} \quad \text{B1 M1 A1}$$

Q28.

Question	Scheme	Marks	AOs
	$x^4 \to \ldots x^5$ or $x^{\frac{1}{2}} \to \ldots x^{\frac{3}{2}}$ or $-3 \to \ldots x$	M1	1.1a
	Any of $\frac{1}{5}x^5$ or $-\frac{6}{\left(\frac{3}{2}\right)}x^{\frac{3}{2}}$ or $-3x$	A1	1.1b
	Any two of $\frac{1}{5}x^5$ or $-\frac{6}{\left(\frac{3}{2}\right)}x^{\frac{3}{2}}$ or $-3x$	A1	1.1b
	$\frac{1}{5}x^5 - 4x^{\frac{3}{2}} - 3x + c$	A1	1.1b
		(4)	
		(4 marks)	

Notes

M1: For increasing any power by one. Score for $x^n \to x^{n+1}$ in any term, including, $-3 \to \ldots x$ where … is a constant, but not for $+\,c$. Allow the indices to be unprocessed, e.g., x^{4+1}

A1: One correct term which may be unsimplified and indices may be unprocessed.
Condone e.g. $-3x^1$ or $-\frac{6}{\left(\frac{1}{2}+1\right)}x^{\frac{1}{2}+1}$ for this mark. Not scored for $+\,c$

A1: Two correct terms which may be unsimplified but indices must be processed.
Condone $-3x^1$ for this mark. Not scored for $+\,c$

A1: cao Requires all terms simplified and $+\,c$
Ignore the LHS i.e. ignore what they call their integral.
Allow $0.2x^5$ for $\frac{1}{5}x^5$ and $-4\sqrt{x^3}$ or $-4\sqrt{x}^3$ or $-4x\sqrt{x}$ or $-4x^{1.5}$ for $-4x^{\frac{3}{2}}$
Do not allow $-3x^1$ for this mark.
Condone spurious integral signs e.g. $\int \frac{1}{5}x^5 - 4x^{\frac{3}{2}} - 3x + c$ or dx left in their answer
ISW after a correct expression seen e.g. if they multiply through by 5 or e.g. try to solve = 0
Do not allow e.g. $\frac{1}{5}x^5 + -4x^{\frac{3}{2}} + -3x + c$

(**Q02 9MA0/02, June 2025**)

Q29.

Question	Scheme	Marks	AOs
(a)	$\lim_{\delta x \to 0} \sum_{x=1.44}^{2.89} \frac{2}{\sqrt{x}} \delta x = \int_{1.44}^{2.89} \frac{2}{\sqrt{x}} \mathrm{d}x$	B1	1.2
		(1)	
(b)	$= \left[4\sqrt{x}\right]_{1.44}^{2.89} = 4\times 1.7 - 4\times 1.2$	M1	1.1b
	$= 2$	A1cso	1.1b
		(2)	
			(3 marks)

Notes:

Mark (a) and (b) together

(a)

B1: States that $\int_{1.44}^{2.89} \frac{2}{\sqrt{x}} \mathrm{d}x$ or equivalent such as $2\int_{1.44}^{2.89} x^{-\frac{1}{2}} \mathrm{d}x$ or $2\int_{1.44}^{2.89} x^{-0.5} \mathrm{d}x$ but must include the limits and the dx. Condone $\mathrm{d}x \leftrightarrow \delta x$ as it is very difficult to tell one from another sometimes.

(b)

M1: Uses $\int \frac{1}{\sqrt{x}} \mathrm{d}x = a\sqrt{x}$ or $ax^{\frac{1}{2}}$ (allow a to be 1) **and** applies the given limits to their $ax^{\frac{1}{2}}$ subtracting either way round. (Condone with the constant of integration included)

You do not need to be concerned by fractions within fractions as this is still of the required form e.g. $\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}$. Only condone transcription errors of 2.89 or 1.44 when substituting the limits into the expression.

This mark can be scored for

e.g. $\left[4\sqrt{x}\right]_{1.44}^{2.89} = 4\times\sqrt{2.89} - 4\times\sqrt{1.44}$ or e.g. $\frac{2x^{\frac{1}{2}}}{\frac{1}{2}} \to \frac{2(2.89)^{\frac{1}{2}} - 2(1.44)^{\frac{1}{2}}}{\frac{1}{2}}$

May already be partially evaluated so allow e.g. $\frac{34}{5} - \frac{24}{5}$ o.e. provided it is not just 2.

A1: 2 cso

The method mark must have been awarded. Do not withhold this mark for poor notation or e.g. a missing dx in their solution.

Q30.

Question	Scheme	Marks	AOs
(a)	$\frac{dr}{dt}=\frac{k}{\sqrt{r}}$	B1	3.3
		(1)	
(b)	$0.9=\frac{k}{\sqrt{16}}\Rightarrow k=3.6$	B1	3.4
	$\int\sqrt{r}\ dr=\int"3.6"dt\Rightarrow...$	M1	2.1
	$\frac{2}{3}r^{\frac{3}{2}}="3.6"t\quad\{+c\}$	A1	1.1b
	$t=10,r=16\Rightarrow\frac{2}{3}\times16^{\frac{3}{2}}=3.6\times10+c\Rightarrow c=...$	dM1	3.4
	$r^{\frac{3}{2}}=5.4t+10$ *	A1*	1.1b
		(5)	
(c)	$t=20\Rightarrow r=(5.4(20)+10)^{\frac{2}{3}}=...$	M1	3.4
	$r=24.1$ cm	A1	1.1b
		(2)	
(d)	(The model will not hold indefinitely as) the balloon may burst	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)

B1: Correctly sets up the model. $\frac{dr}{dt}\propto\frac{1}{\sqrt{r}}$ scores B0 unless e.g. $\frac{dr}{dt}=\pm\frac{k}{\sqrt{r}}$ is seen but condone $\frac{dr}{dt}=\pm\frac{k}{\sqrt{r}}$ being seen at the start of (b). They may use any letter except t or r in place of k.

You may see $\frac{dr}{dt}=\pm\frac{1}{k\sqrt{r}}$ which is acceptable provided it is clear that it is not the k^{th} root.

(b) **Note:** candidates using $\frac{dr}{dt}=\pm\frac{1}{\sqrt{r}}$ in (b) can score maximum B0M1A1dM0A0

B1: $k=3.6$ coming from $\frac{dr}{dt}=\frac{k}{\sqrt{r}}$ (or $k=\frac{5}{18}$ coming from $\frac{dr}{dt}=\frac{1}{k\sqrt{r}}$) and from use of $r=16$ and $\frac{dr}{dt}=0.9$ but note that this may occur later in their working, which is perfectly fine provided it is from acceptable work. Note e.g. $k=-3.6$ coming from $\frac{dr}{dt}=-\frac{k}{\sqrt{r}}$ is correct.

Note, however, that an attempt to find k from comparing coefficients between $r^{\frac{3}{2}}=1.5kt+c$ and $r^{\frac{3}{2}}=5.4t+10$ is not acceptable (see special case).

They can also find k by differentiating **their** $r=f(t)$ and substituting $t=10$, $r=16$ and $\frac{dr}{dt}=0.9$ **if** they have also used $t=10$, $r=16$ in **their** $r=f(t)$. This sets up simultaneous equations where c can be eliminated. Use Review if you are unsure if their approach is acceptable.

M1:	Separates the variables for their differential equation **correctly** and attempts to integrate both sides. Must be a differential equation of the form $\frac{\mathrm{d}r}{\mathrm{d}t}=\mathrm{f}(r)$ for some function f(r) independent of t. Evidence of $r^n \to r^{n+1}$, or e.g. $\frac{1}{r}\to \ln r$ is sufficient for their attempt to integrate in r, but k must be integrated to kt o.e. e.g. $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{k}{r^2}\Rightarrow r^2\frac{\mathrm{d}r}{\mathrm{d}t}=k\Rightarrow \lambda r^3 = kt\{+c\}$ would score this mark. Note that they may divide by k (or 3.6) prior to integrating. Here, 1 must be integrated to t.
A1:	Correct integration for their k. Allow this mark if they have not found k, so allow e.g. $\frac{2}{3}r^{\frac{3}{2}} = kt\{+c\}$ with/without the constant of integration but the $\frac{2}{3}$ must be evident in some way. Condone spurious notation for this intermediate mark e.g. integral signs left in after integrating.
dM1:	Uses t = 10, r = 16 in their equation to find the constant of integration. This mark is dependent on the first method mark. Must have already found a value for k using a valid strategy and the constant of integration must be present. Those that found k from comparing coefficients between $r^{\frac{3}{2}}=\frac{3}{2}kt+c$ and $r^{\frac{3}{2}}=5.4t+10$ may not score this mark.
A1*:	Correct equation from correct working. May be seen at the start of (c). Must follow A1 earlier so do check if this has been obtained fortuitously.
SC:	It is possible to compare coefficients (following integration) between $r^{\frac{3}{2}}=\frac{3}{2}kt+c$ and $r^{\frac{3}{2}}=5.4t+10$ to deduce the value of k as 3.6 (or just write their coefficient as 5.4). The maximum that can be scored this way or by a similar invalid approach is B0M1A1dM0A0

(c)	
M1:	Substitutes t = 20 into the given equation and uses correct processing to find the value of r e.g. substitutes $t=20$ into $\frac{2}{3}r^{\frac{3}{2}}=72+\frac{20}{3}\Rightarrow r=\left(\frac{3}{2}\left(72+\frac{20}{3}\right)\right)^{\frac{2}{3}}=...$ Their work *should* lead to the correct answer so the index work must be correct e.g. $\sqrt{118^3}$ is M0. $\sqrt[3]{118^2}$ or $118^{\frac{2}{3}}$ are acceptable as values, i.e., the bracket must be evaluated. May be implied by awrt 24 (cm) following $r^{\frac{3}{2}}=118$ or by awrt 24.1 (cm). Ignore units for M1.
A1:	cao 241mm or 241 or 24.1cm but do not accept 24.1 or e.g. $\sqrt[3]{118^2}$ cm Correct answer with units implies both marks.
(d)	
B1:	Examples of acceptable answers (which must relate to the **model in context**): • (The model will not hold indefinitely as) the balloon may burst/pop • The balloon is unlikely to be (perfectly) spherical (condone circular) • The model predicts the balloon will increase in size without limit (which is unrealistic) Note $t\to\infty\Rightarrow r\to\infty$ is unrealistic / impossible scores B0 unless they reference e.g. the radius. Condone the presence of additional remarks such as "the balloon may not inflate at the same rate" or "the radius of the balloon might not start at 0" that have already been addressed in the model, but these answers alone score B0.

(Q14 9MA0/01, June 2024)

Q31.

Question	Scheme	Marks	AOs
	$x^{\frac{1}{2}}(2x-5)=...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ or $\dfrac{x^{\frac{1}{2}}(2x-5)}{3}=\dfrac{...x^{\frac{3}{2}}+...x^{\frac{1}{2}}}{3}$	M1	1.1b
	$\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}$	A1	1.1b
	$\displaystyle\int \dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}\,dx=...x^{\frac{5}{2}}\pm...x^{\frac{3}{2}}\ (+c)$	dM1	1.1b
	$\dfrac{4}{15}x^{\frac{5}{2}}-\dfrac{10}{9}x^{\frac{3}{2}}+c$	A1	1.1b
		(4)	
			(4 marks)

Notes

M1: Attempts to multiply out the brackets of the numerator and either writes the expression (or just the numerator) as a sum of terms with **indices**. Award for either one correct index of $...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ which comes from a correct method. Condone appearing as terms on separate lines for this mark. The correct index may be implied later when e.g. $\sqrt{x}\to x^{\frac{3}{2}}$ or $\left(\sqrt{x}\right)^3\to x^{\frac{5}{2}}$ after they have integrated.

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: $\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}$ or equivalent e.g. $\dfrac{1}{3}\left(2x^{\frac{3}{2}}-5x^{\frac{1}{2}}\right)$. Condone $\dfrac{2x^{\frac{3}{2}}-5x^{\frac{1}{2}}}{3}$ May be implied by further work.

The correct index may be implied later when e.g. $\sqrt{x}\to x^{\frac{3}{2}}$ or $\left(\sqrt{x}\right)^3\to x^{\frac{5}{2}}$ after they have integrated.

Ignore incorrect integration notation around the terms. Ignore any presence or absence of dx.

Be aware that a factor of $\frac{1}{3}$ may be taken outside of the integral so you may need to look at further work to award the first A mark if work on the two terms is done separately or in a list.

May be unsimplified and the two terms may appear in a list which is fine.

Coefficients must be exact.

dM1: Increases the power by one on an x^n term where n is a fraction. The index does not need to be processed.

e.g. $...x^{\frac{3}{2}+1}$ or $...x^{\frac{1}{2}+1}$ It is dependent on the previous method mark so at least one of the terms must have had a correct index.

Note that integrating the numerator and denominator e.g. $\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}\to\dfrac{...x^{\frac{5}{2}}}{3x}-\dfrac{...x^{\frac{3}{2}}}{3x}$ is dM0.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ and including the constant or simplified exact equivalent such as

$\frac{4}{15}\sqrt{x^5}-\frac{10}{9}\sqrt{x^3}+c$ or $\frac{4}{15}x^{2.5}-\frac{10}{9}x^{1.5}+c$ or $\frac{1}{45}\left(12x^{\frac{5}{2}}-50x^{\frac{3}{2}}\right)+c$ or $\frac{x^{\frac{3}{2}}}{45}(12x-50)+c$. Fractions must be in their lowest terms and indices processed.

Do not accept e.g. $0.267x^{\frac{5}{2}}-1.11x^{\frac{3}{2}}+c$ but allow $0.2\dot{6}x^{\frac{5}{2}}-1.\dot{1}x^{\frac{3}{2}}+c$

Isw once a correct answer is seen but withhold this mark if there is spurious notation around their final answer e.g. $\int \frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c\ \mathrm{d}x$ is M1A1dM1A0

Alternative method using integration by parts example

M1: e.g. $\int x^{\frac{1}{2}}(2x-5)\,\mathrm{d}x = ...x^{\frac{3}{2}}(2x-5)-\int ...x^{\frac{3}{2}}\,\mathrm{d}x$ (applies integration by parts correctly to typically achieve this form – the $(2x-5)$ may also be split up as well – send to review if unsure how to mark)

This may also be done the other way round e.g. $\int x^{\frac{1}{2}}(2x-5)\,\mathrm{d}x = ...x^{\frac{1}{2}}(x^2-5x)-\int ...x^{\frac{3}{2}} \pm ...x^{\frac{1}{2}}\,\mathrm{d}x$

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: A correct intermediate stage applying integration by parts with correct coefficients.

e.g. $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}}\left(\frac{2x-5}{3}\right)-\int \frac{4}{9}x^{\frac{3}{2}}\,\mathrm{d}x$ (or unsimplified equivalent).

Coefficients must be exact. (See main scheme notes above) The other way round this could appear as

e.g. $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,\mathrm{d}x = \frac{1}{3}x^{\frac{1}{2}}(x^2-5x)-\frac{1}{6}\int x^{\frac{3}{2}}-5x^{\frac{1}{2}}\,\mathrm{d}x$. Condone a missing dx. May be implied.

dM1: Increases the power by one on an x^n term where n is a fraction e.g. $\int ...x^{\frac{3}{2}}\,\mathrm{d}x \rightarrow ...x^{\frac{5}{2}}$ The index does not need to be processed. It is dependent on the previous method mark.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ or exact simplified equivalent. (See main scheme notes above)

Alternative method using the substitution method

M1: e.g. let $u = x^{\frac{1}{2}} \Rightarrow \int ...u^4+...u^2\,\mathrm{d}u$ (uses a substitution to express the integral in terms of another variable. Allow slips with the coefficients, but the indices should be correct for their substitution)

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: e.g. $\int \frac{4u^4}{3}-\frac{10u^2}{3}\,\mathrm{d}u$ or unsimplified equivalent. Coefficients must be exact. See main scheme notes above). May be implied by further work. Condone a missing dx.

dM1: $\int ...u^4+...u^2\,\mathrm{d}u \rightarrow ...u^5+...u^3$ (increases the power by one on at least one of their indices – does not need to be processed. It is dependent on the previous method mark.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ or exact simplified equivalent. (See main scheme notes above)

There may be alternative substitutions, but the same marking principles apply.

Q32.

Question	Scheme	Marks	AOs
(a)	$8(4)-4^{\frac{5}{2}}=32-32=0$	B1	1.1b
		(1)	
(b)	$8-\frac{5}{2}x^{\frac{3}{2}}$	B1	1.1b
	$x=4\Rightarrow\left\{\frac{dy}{dx}=\right\}8-\frac{5}{2}\times 8=-12$ $\Rightarrow y\{-0\}="-12"(x-4)$	M1	1.1b
	$12x+y=48$ *	A1*	1.1b
		(3)	
(c)	Attempts to find one of the coordinates of the point of intersection $y=8x,\ 12x+y=48\Rightarrow y=19.2$ (or $x=2.4$)	M1	1.1b
	Triangle area is $\frac{1}{2}\times 4\times "19.2"\left(=38.4 \text{ or } \frac{192}{5}\right)$ or $\int_0^{"2.4"} 8x\ dx+\int_{"2.4"}^{4} "(48-12x)"\ dx$	dM1	3.1a
	$\int\left(8x-x^{\frac{5}{2}}\right)dx=4x^2-\frac{2}{7}x^{\frac{7}{2}}$	B1	1.1b
	$A=38.4-\left["4x^2-\frac{2}{7}x^{\frac{7}{2}}"\right]_0^4=38.4-64+\frac{256}{7}$	ddM1	3.1a
	$=\frac{384}{35}$	A1	1.1b
		(5)	
			(9 marks)

Notes
(a)
B1: Substitutes $x=4$ into the equation of the curve and verifies that $y=0$. Accept "$8(4)-4^{\frac{5}{2}}=0$"   Alternatively, sets $8x-x^{\frac{5}{2}}=0$ and solves with correct processing to achieve $x=4$.   As a minimum accept e.g. $8x-x^{\frac{5}{2}}=0\ \Rightarrow\ x^{\frac{3}{2}}=8\ \Rightarrow\ \{x=\}\ 4$ which may follow factorisation.
(b)
B1: Correct differentiation. The $\frac{dy}{dx}=$ need not be present.
M1: Correct method for finding the equation of the tangent at $A(4, 0)$.   Requires substitution of $x=4$ into their $\frac{dy}{dx}$ and an attempt at the equation of the line using this gradient. If using $y-y_1=m(x-x_1)$ then condone the omission of the -0.   If $y=mx+c$ is used they must proceed as far as $c=\ldots$ Accept $\frac{dy}{dx}=-12$ or $m=-12$ without explicit substitution of $x=4$ provided $8-\frac{5}{2}x^{\frac{3}{2}}$ is seen.

A1*: Correct work leading to the given equation having scored B1M1.
Condone $y+12x=48$ and apply isw once seen.
Do not condone $12x+y-48=0$ (unless a correct equation = 48 is seen).

(c) **Note:** Condone poor notation such as missing d*x* or spurious $\int$ symbols throughout.

M1: Attempts to find either the *x* or *y* coordinate of the intersection of line l_1 and line l_2
You may need to check the diagram or limits to their integrals.

dM1: Correct method for the area of the triangle. e.g. Triangle area is $\frac{1}{2}\times 4\times "19.2"\left(=38.4 \text{ or } \frac{192}{5}\right)$

If integration is attempted then condone slips in their rearrangement of $12x+y=48$ to $y=48-12x$ and note that their integrals do need not to be evaluated, so for example

look for $\int_0^{"2.4"} 8x\,dx+\int_{"2.4"}^{4} "(48-12x)"\,dx \qquad \left\{=\frac{576}{25}+\frac{384}{25}=23.04+15.36\right\}$

B1: Correct integration of **curve** ignoring limits, i.e. $4x^2-\frac{2}{7}x^{\frac{7}{2}}$ but condone e.g. $\frac{8x^{1+1}}{2}-\frac{x^{\frac{5}{2}+1}}{\frac{7}{2}}$

ddM1: Fully correct strategy including substitution which would lead to an **exact** area.
Does not need to reach a value. Dependent on both previous M marks.
Implied by $38.4-\frac{192}{7}$ or a correct final answer $\frac{384}{35}$
Note that the decimal approximation that might be seen is 10.97142857 and implies ddM0 unless there is evidence of substitution (which need not be evaluated).

A1: Correct exact value. Either $\frac{384}{35}$ or $10\frac{34}{35}$

Alternative using lines – curve:

M1: Attempts to find either the *x* or *y* coordinate of the intersection of line l_1 and line l_2
You may need to check the diagram or limits to their integrals.

dM1: Correct method for at least one part (0 to "2.4" **or** "2.4" to 4) of the area of *R* including limits.
Condone slips in their rearrangement of $12x+y=48$ to $y=48-12x$ and note that their integrals do need not to be evaluated, so for example

look for $\int_0^{"2.4"} 8x-\left(8x-x^{\frac{5}{2}}\right)dx$ or $\int_{"2.4"}^{4} "(48-12x)"-\left(8x-x^{\frac{5}{2}}\right)dx$ (or a sum of both)

B1: Correct integration of **both** regions ignoring limits. May be completed as a sum or separately.
Condone e.g. $\frac{x^{\frac{5}{2}+1}}{\frac{7}{2}}$ in place of $\frac{2}{7}x^{\frac{7}{2}}$ Note that each integral may have been simplified.

$\int_{...}^{...} x^{\frac{5}{2}}\,dx$ and {+} $\int_{...}^{...} 48-20x+x^{\frac{5}{2}}\,dx \rightarrow \left[\frac{2}{7}x^{\frac{7}{2}}\right]_{...}^{...}$ and {+} $\left[48x-10x^2+\frac{2}{7}x^{\frac{7}{2}}\right]_{...}^{...}$

ddM1: Fully correct strategy including substitution which would lead to an **exact** area.
Does not need to reach a value. Dependent on both previous M marks.
This approach requires:

- substitution of 0, their 2.4 and 4 in the correct places
- the $\frac{2}{7}(2.4)^{\frac{7}{2}}-\frac{2}{7}(2.4)^{\frac{7}{2}}$ to be cancelled (may be implied by a correct final answer $\frac{384}{35}$)

Note that the decimal approximation that might be seen is 10.97142857 and implies ddM0 unless there is evidence that the $\frac{2}{7}(2.4)^{\frac{7}{2}}-\frac{2}{7}(2.4)^{\frac{7}{2}}$ has been cancelled e.g. ~~6.118...~~ − ~~6.118...~~

A1: Correct exact value. Either $\frac{384}{35}$ or $10\frac{34}{35}$

(Q10 9MA0/01, June 2024)