Name: ______________________________________________

# Integration Exam Questions

## Topic Test and Revision

**Date:**

**Time:** 300

**Total marks available:** 257

**Total marks achieved:** ______

Calculator Allowed

**Mathvault.io**

## Questions

**Q1.**

Show that

$$\int_0^2 2x\sqrt{x+2}\,dx = \frac{32}{15}\left(2+\sqrt{2}\right)$$

let $u = x + 2$

$u_2 = 2+2 = 4$

$u_1 = 0+2 = 2$

$\frac{du}{dx} = 1$ , $du = dx$

$x = u - 2$

$$I = \int_2^4 2(u-2)\sqrt{u}\ du$$

$$I = \int_2^4 2u^{1/2}(u-2)\ du$$

$$I = \int_2^4 2u^{3/2} - 4u^{1/2}\ du$$

$$I = \left[\frac{2}{5}\times 2u^{5/2} - 4\times\frac{2}{3}u^{3/2}\right]_2^4$$

$$I = \left[\frac{4}{5}(\sqrt{4})^5 - \frac{8}{3}(\sqrt{4})^3\right] - \left[\frac{4}{5}(\sqrt{2})^5 - \frac{8}{3}(\sqrt{2})^3\right]$$

$$I = \left[\frac{128}{5} - \frac{64}{3}\right] - \left[\frac{16}{5}\sqrt{2} - \frac{16}{3}\sqrt{2}\right]$$

$$I = \frac{64}{15} - \left[\frac{-32}{15}\sqrt{2}\right]$$

$$I = \frac{64}{15} + \frac{32}{15}\sqrt{2}$$

$$I = \frac{32}{15}\left(2+\sqrt{2}\right)$$

**(7)**

**(Total for question = 7 marks)**

(**Q13 9MA0/01, June 2018**)

**Q2.**

Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} \mathrm{d}x$ is independent of $k$,

$$\frac{d}{dx}\left(\ln|3x-k|\right) = \frac{3}{3x-k}$$

$$\times \, 2/3 \qquad \times \frac{2}{3}$$

$$\frac{2}{3}\frac{d}{dx}\ln|3x-k| = \frac{2}{3x-k}$$

$$\frac{2}{3}\int d \ln|3x-k| = \int \frac{2}{3x-k}\,dx$$

$$\left[\frac{2}{3}\ln|3x-k|\right]_k^{3k} = \int_k^{3k} \frac{2}{3x-k}\,dx$$

$$\frac{2}{3}\ln|3(3k)-k| - \frac{2}{3}\ln|3k-k|$$

$$\frac{2}{3}\ln|8k| - \frac{2}{3}\ln|2k|$$

$$\frac{2}{3}\left[\ln 8k - \ln 2k\right] = \frac{2}{3}\ln\left|\frac{8k}{2k}\right| = \frac{2}{3}\ln 4$$

$\frac{2}{3}\ln 4$ does not include the constant $K$ hence $\int_K^{3k} \frac{2}{3x-k}\,dx$ is independent of $K$

(4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} \mathrm{d}x$ is inversely proportional to $k$.

$$I = 2\int_K^{2K} (2x-k)^{-2}\,dx$$

$$I = 2\left[\frac{1}{2}\times\frac{1}{-1}(2x-k)^{-1}\right]_K^{2K}$$

$$I = \left[-(2x-k)^{-1}\right]_k^{2k}$$

$$I = \left[\frac{-1}{2x-k}\right]_k^{2k}$$

$$I = \frac{-1}{2(2k)-k} - \left(\frac{-1}{2k-k}\right)$$

$$I = \frac{-1}{3k} + \frac{1}{k}$$

$$I = -\frac{1}{3k} + \frac{3}{3k}$$

$$I = \frac{2}{3k}$$

$$I = \frac{2}{3k}$$

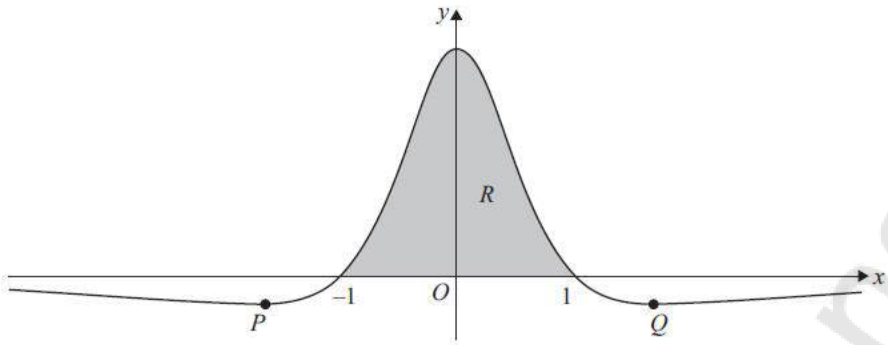
hence $\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to $K$

(3)

**(Total for question = 7 marks)**

(**Q07 9MA0/01, June 2018**)

**Q3.**




**In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

Figure 5 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1 - x^2}{(1 + x^2)^2}$$

The curve
- intersects the $x$-axis at −1 and 1
- has minimum turning points at $P$ and $Q$

as shown in Figure 5.

(a) Use calculus to find the exact coordinates of $P$.

$u = 1 - x^2 \qquad v = (1 + x^2)^2$

$u' = -2x \qquad v' = 2 \times 2x(1 + x^2)$

$v' = 4x(1 + x^2)$

$$f'(x) = \frac{-2x(1+x^2)^2 - 4x(1+x^2)(1-x^2)}{(1+x^2)^2}$$

P and Q occur at $f'(x) = 0$

$\therefore \quad -2x(1+x^2)^2 - 4x(1+x^2)(1-x^2) = 0$

$-2x(1+x^2)\left[(1+x^2) + 2(1-x^2)\right] = 0$

$-2x(1+x^2) = 0 \qquad (1+x^2) + 2(1-x^2) = 0$

$-2x = 0 \quad 1 + x^2 = 0 \qquad 1 + x^2 = -2(1 - x^2)$

$x = 0 \quad x^2 = -1 \qquad 1 + x^2 = -2 + 2x^2$

Maximum point hence not P or Q — No real roots — $x^2 = 3$

$x = \sqrt{3}$ and $x = -\sqrt{3}$

$$f(-\sqrt{3}) = \frac{1 - (-\sqrt{3})^2}{\left[1 + (-\sqrt{3})^2\right]^2} = \frac{1}{8}$$

$\therefore P\left(-\sqrt{3}, \frac{1}{8}\right)$

**(5)**

(b) Using the substitution $x = \tan\theta$ show that

$$\int_{-1}^{1} f(x)\,dx = \int_{\alpha}^{\beta} \cos 2\theta \, d\theta$$

where α and β are constants to be found.

$$f(x) = \frac{1 - x^2}{(1 + x^2)^2}$$

$1 = \tan\theta$ , $\theta = \tan^{-1}(1) = \pi/4$

$-1 = \tan\theta$ , $\theta = \tan^{-1}(-1) = -\pi/4$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 - \tan^2\theta}{(1 + \tan^2\theta)^2} \times \sec^2\theta \, d\theta$$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$x = \tan\theta$

$\frac{dx}{d\theta} = \sec^2\theta$

$dx = \sec^2\theta \times d\theta$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 - \tan^2\theta}{(\sec^2\theta)^2} \times \sec^2\theta \, d\theta$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 - \tan^2\theta}{\sec^2\theta} \, d\theta$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta} \, d\theta$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2\theta - \sin^2\theta \, d\theta$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2\theta \, d\theta$$



**(5)**

The finite region $R$, shown shaded in Figure 5, is bounded by the $x$-axis and the curve.

(b) Use algebraic integration to find the area of $R$.

$$R = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2\theta \, d\theta$$

$$\left[ \frac{\sin 2\theta}{2} \right]_{-\pi/4}^{\pi/4}$$

$$R = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin\left(-\frac{\pi}{2}\right)$$

$$R = \underline{\underline{1}}$$



**(3)**

**(Total for question = 13 marks)**

(**Q15 9MA0/02, June 2025**)

**Q4.**

The height above ground, $H$ metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

where $t$ is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1 \sin(0.25t)}$

$$\int \frac{1}{H}\, dH = \frac{1}{40}\int \cos(0.25t)\, dt$$

$$\ln H = \frac{1}{40} \times \sin(0.25t) \times \frac{1}{0.25} + c$$

$$\ln H = \frac{1}{10}\sin(0.25t) + c$$

At $H = 5$, $t = 0$

$$\ln 5 = \frac{1}{10}\sin(0.25 \times 0) + c$$

$$\ln 5 = c$$

$$\therefore \ln H = \frac{1}{10}\sin(0.25t) + \ln 5$$

$$e^{\ln H} = e^{\frac{1}{10}\sin(0.25t) + \ln 5}$$

$$H = \left(e^{\frac{1}{10}\sin(0.25t)}\right)\left(e^{\ln 5}\right)$$

$$H = 5e^{0.1\sin(0.25t)}$$

shown.



(5)

(b) State the maximum height of the passenger above the ground.

At $H_{max}$ , $\sin(0.25t) = 1$

$\therefore \quad H_{max} = 5e^{0.1}$

$H_{max} \approx 5.53 \text{ m} \quad (3 \text{ s.f.})$

**(1)**

The passenger reaches the maximum height, for the second time, *T* seconds after the start of the ride.

(c) Find the value of *T*.

$\sin(0.25t) = 1$

$0.25t = \sin^{-1}(1)$

$t = 4\sin^{-1}(1)$

$t = \underset{1st}{4\left(\frac{\pi}{2}\right)} \quad , \quad \underset{2nd}{4\left(\frac{5\pi}{2}\right)}$

$t = 2\pi \quad , \quad 10\pi$

$T = 10\pi \text{ s} \approx 31.4 \text{ seconds} \quad (3 \text{ s.f})$

**(2)**

**(Total for question = 8 marks)**

(**Q10 9MA0/01, June 2018**)

**Q5.**

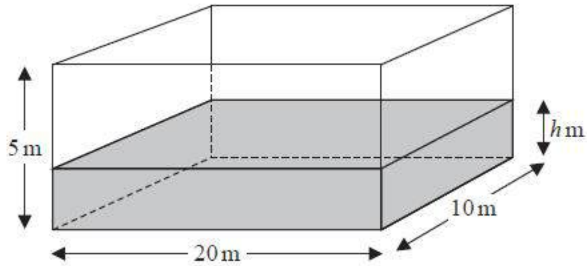



**Figure 1**

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was *h* m and the volume of water in the tank was *V* m$^3$

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of *V* is inversely proportional to the square root of *h*

(a) Show that

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

where $\lambda$ is a constant.

$V = 20 \times 10 \times h$

$V = 200h$

$\frac{dV}{dh} = 200 \quad , \quad \frac{dh}{dV} = \frac{1}{200}$

$\frac{dV}{dt} \propto \frac{1}{\sqrt{h}}$

$\therefore \frac{dV}{dt} = \frac{k}{\sqrt{h}}$

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

$\frac{dh}{dt} = \frac{k}{\sqrt{h}} \times \frac{1}{200}$

$\frac{dh}{dt} = \frac{0.005k}{\sqrt{h}}$

$\lambda = 0.005k \quad , \quad \frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$

**(3)**

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking $h$ with $t$, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where $A$ and $B$ are constants to be found.

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

$$\sqrt{h}\, dh = \lambda\, dt$$

$$\int h^{1/2}\, dh = \int \lambda\, dt$$

$$\frac{2}{3} h^{3/2} = \lambda t + c$$

At $t = 0$, $h = 1.44$

$$\frac{2}{3}(1.44)^{1.5} = \lambda(0) + c$$

$$1.152 = C$$

$$\therefore \frac{2}{3} h^{1.5} = \lambda t + 1.152$$

At $t = 8$, $h = 3.24$

$$\frac{2}{3}(3.24)^{1.5} = 8\lambda + 1.152$$

$$\lambda = \frac{1}{8}\left[\frac{2}{3}(3.24)^{1.5} - 1.152\right]$$

$$\lambda = 0.342$$

$$\therefore \frac{2}{3} h^{3/2} = 0.342t + 1.152$$

$$h^{3/2} = 0.513t + 1.728$$

**(5)**

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

Tank full at $h = 5$, $5^{1.5} = 0.513t + 1.728$

$$t = \frac{1}{0.513}\left(5^{1.5} - 1.728\right)$$

$$t \approx 18.4 \text{ minutes } (3 \text{ s.f.})$$

**(2)**

**(Total for question = 10 marks)**

(**Q11 9MA0/02, June 2023**)

**Q6.**

(a) Express $\frac{1}{V(25-V)}$ in partial fractions.

$$\frac{1}{V(25-V)} \equiv \frac{A}{V} + \frac{B}{25-V}$$

$$\frac{1}{V(25-V)} \equiv \frac{A(25-V) + B(V)}{V(25-V)}$$

$$1 \equiv 25A - AV + BV$$

$$1 \equiv (B-A)V + 25A$$

$25A = 1$, $A = \frac{1}{25}$

$B - A = 0$, $B = A$, $\therefore B = \frac{1}{25}$

$$\frac{1}{V(25-V)} \equiv \frac{1/25}{V} + \frac{1/25}{25-V}$$

**(2)**

The volume, $V$ microlitres, of a plant cell $t$ hours after the plant is watered is modelled by the differential equation

$$\frac{dV}{dt} = \frac{1}{10}V(25-V)$$

The plant cell has an initial volume of 20 microlitres.

(b) Find, according to the model, the time taken, in minutes, for the volume of the plant cell to reach 24 microlitres.

$$\frac{dV}{dt} = \frac{1}{10}V(25-V)$$

$$\int \frac{1}{V(25-V)}\, dV = \int \frac{1}{10}\, dt$$

$$\int \frac{1/25}{V} + \frac{1/25}{25-V}\, dV = \int 0.1\, dt$$

$$\frac{1}{25}\ln V - \frac{1}{25}\ln|25-V| = 0.1t + C$$

At $V = 20$, $t = 0$

$$\frac{1}{25}\ln 20 - \frac{1}{25}\ln 5 = C$$

$$\frac{1}{25}(\ln 20 - \ln 5) = C$$

$$\frac{1}{25}\ln\left|\frac{20}{5}\right| = C$$

$$\frac{1}{25}\ln 4 = C$$

Thus $\frac{1}{25}\ln V - \frac{1}{25}\ln|25-V| = 0.1t + \frac{1}{25}\ln 4$

At $V = 24$

$$\frac{1}{25}\ln 24 - \frac{1}{25}\ln 1 = 0.1t + \frac{1}{25}\ln 4$$

$$\ln 24 = 2.5t + \ln 4$$

$$\ln 24 - \ln 4 = 2.5t$$

$$\ln\frac{24}{4} = 2.5t$$

$$\ln 6 = 2.5t$$

$$t = \frac{\ln 6}{2.5} = 0.7167\ldots$$

$$t = \frac{\ln 6}{2.5} \times 60 \simeq \underline{\underline{43 \text{ minutes}}}$$

**(5)**

(c) Show that

$$V = \frac{A}{e^{-kt} + B}$$

where *A*, *B* and *k* are constants to be found.

$$\frac{1}{25}\ln v - \frac{1}{25}\ln|25 - v| = 0.1t + \frac{1}{25}\ln 4$$

$$\ln v - \ln|25 - v| = 2.5t + \ln 4$$

$$\ln\left|\frac{v}{25-v}\right| = 2.5t + \ln 4$$

$$e^{\ln\left|\frac{v}{25-v}\right|} = e^{2.5t + \ln 4}$$

$$\frac{v}{25-v} = \left(e^{2.5t}\right)\left(e^{\ln 4}\right)$$

$$\frac{v}{25-v} = \left(e^{2.5t}\right)(4)$$

$$v = 100e^{2.5t} - 4ve^{2.5t}$$

$$v + 4ve^{2.5t} = 100e^{2.5t}$$

$$v(1 + 4e^{2.5t}) = 100e^{2.5t}$$

$$v = \frac{100e^{2.5t}}{1 + 4e^{2.5t}} \begin{matrix} \div e^{2.5t} \\ \div e^{2.5t} \end{matrix} \Rightarrow V = \frac{100}{e^{-2.5t} + 4}$$

$A = 100$

$k = 2.5$

$B = 4$

**(3)**

The model predicts that there is an upper limit, *L* microlitres, on the volume of the plant cell.

(d) Find the value of *L*, giving a reason for your answer.

As $t \to \infty$

$e^{-2.5t} \to 0$

$e^{-2.5t} + 4 \to 4$

$V \to \frac{100}{4}$

$V \to 25$ $\quad \therefore \underline{\underline{L = 25}}$

**(2)**

**(Total for question = 12 marks)**

(**Q12 9MA0/02, June 2024**)

**Q7.**

$$f(x) = \frac{3kx - 18}{(x+4)(x-2)} \qquad \text{where } k \text{ is a positive constant}$$

(a) Express f($x$) in partial fractions in terms of $k$.

$$\frac{3kx - 18}{(x+4)(x-2)} \equiv \frac{A}{x+4} + \frac{B}{x-2}$$

$$3kx - 18 \equiv A(x-2) + B(x+4)$$

At $x = 2$,

$$6k - 18 = 6B$$

$$B = k - 3$$

At $x = -4$

$$-12k - 18 = -6A$$

$$A = 2k + 3$$

$$\therefore \quad f(x) = \frac{2k+3}{x+4} + \frac{k-3}{x-2}$$

**(3)**

(b) Hence find the exact value of $k$ for which

$$\int_{-3}^{1} f(x)\, dx = 21$$

$$\int_{-3}^{1} \frac{2k+3}{x+4} + \frac{k-3}{x-2}\, dx = 21$$

$$\Big[(2k+3)\ln|x+4| + (k-3)\ln|x-2|\Big]_{-3}^{1} = 21$$

$$\Big[(2k+3)\ln 5 + (k-3)\ln 1\Big] - \Big[(2k+3)\ln 1 + (k-3)\ln 5\Big] = 21$$

$$2k\ln 5 + 3\ln 5 - k\ln 5 + 3\ln 5 = 21$$

$$k\ln 5 + 6\ln 5 = 21$$

$$k\ln 5 = 21 - 6\ln 5$$

$$k = \frac{21}{\ln 5} - 6$$

**(4)**

**(Total for question = 7 marks)**

(**Q10 9MA0/02, June 2023**)

**Q8.**

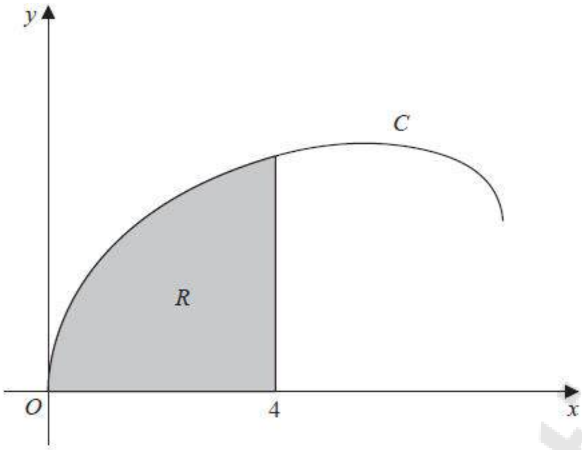



**Figure 6**

Figure 6 shows a sketch of the curve $C$ with parametric equations

$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \le t \le \frac{\pi}{2}$

The region $R$, shown shaded in Figure 6, is bounded by $C$, the $x$-axis and the line with equation $x = 4$

(a) Show that the area of $R$ is given by

$$\int_0^a (8 - 8\cos 4t + 48\sin^2 t\cos t)\,dt$$

where $a$ is a constant to be found.

$R = \int_0^4 y\,dx$

$x = 8\sin^2 t$

$\cos 2t = \cos^2 t - \sin^2 t$

$\sin^2 t + \cos^2 t = 1$

$\cos^2 t = 1 - \sin^2 t$

$\therefore \cos 2t = 1 - \sin^2 t - \sin^2 t$

$\cos 2t = 1 - 2\sin^2 t$

$2\sin^2 t = 1 - \cos 2t$

$8\sin^2 t = 4 - 4\cos 2t$

$\therefore x = 4 - 4\cos 2t$

$\frac{dx}{dt} = -4 \times 2 \times -\sin 2t$

$\frac{dx}{dt} = 8\sin 2t$

$dx = 8\sin 2t\,dt$

$4 = 8\sin^2 t$

$\frac{4}{8} = \sin^2 t$

$\frac{1}{2} = \sin^2 t$

$\frac{1}{\sqrt{2}} = \sin t$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = t$

$t = \frac{\pi}{4}$

$R = \int_0^{\pi/4} (2\sin 2t + 3\sin t) \times 8\sin 2t\,dt$

$R = \int_0^{\pi/4} 16\sin^2 2t + 24\sin t\sin 2t\,dt$

$\cos 2t = \cos^2 t - \sin^2 t$

$\cos 4t = \cos^2 2t - \sin^2 2t$

$\sin^2 2t = \cos^2 2t - \cos 4t$

$\sin^2 2t = 1 - \sin^2 2t - \cos 4t$

$2\sin^2 2t = 1 - \cos 4t$

$16\sin^2 2t = 8 - 8\cos 4t$

$R = \int_0^{\pi/4} 8 - 8\cos 4t + 24\sin t(2\sin t\cos t)\,dt$

$R = \int_0^{\pi/4} 8 - 8\cos 4t + 48\sin^2 t\cos t\,dt$

$a = \frac{\pi}{4}$

Shown

**(5)**

(b) Hence, using algebraic integration, find the exact area of *R*.

$$R = \int_0^{\pi/4} 8 - 8\cos 4t + 48\sin^2 t \cos t \, dt$$

$$\int_0^{\pi/4} 8 - 8\cos 4t \, dt = \left[ 8t - 2\sin 4t \right]_0^{\pi/4} = 2\pi$$

$$-48\int_0^{\pi/4} \sin^2 t \cos t \, dt$$

let $u = \sin t$ $\quad u_2 = \sin \pi/4 = \sqrt{2}/2$

$u_1 = \sin 0 = 0$

$$\frac{du}{dt} = \cos t$$

$$dt = \frac{du}{\cos t}$$

$$48\int_0^{\sqrt{2}/2} u^2 \cos t \times \frac{du}{\cos t}$$

$$48\int_0^{\sqrt{2}/2} u^2 \, du$$

$$48\left[ \frac{u^3}{3} \right]_0^{\sqrt{2}/2}$$

$$48 \times \frac{1}{3}\left[ \frac{\sqrt{2}}{2} \right]^3$$

$$48 \times \frac{\sqrt{2}}{12}$$

$$4\sqrt{2}$$

$$\therefore \quad R = \underline{\underline{2\pi + 4\sqrt{2}}}$$

**(4)**

**(Total for question = 9 marks)**

(**Q16 9MA0/01, June 2022**)

**Q9.**

The table below shows corresponding values of $x$ and $y$ for $y = e^{-x^2}$

The values of $y$ are given to 4 decimal places as appropriate.

| $x$ | 0 | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 |
|---|---|---|---|---|---|---|---|
| $y$ | 1 | 0.9394 | 0.7788 | 0.5698 | 0.3679 | 0.2096 | 0.1054 |

(a) Using the trapezium rule with all the values of $y$ in the table, find an estimate for

$$\int_0^{1.5} e^{-x^2}\,dx$$

giving your answer to 3 significant figures.

$h = 0.25 \quad b = 1.5 \quad a = 0$

$$\int_0^{1.5} e^{-x^2}\,dx \approx \frac{1}{2} \times \frac{1}{4}\left[(1 + 0.1054) + 2(0.9394 + 0.7788 + 0.5698 + 0.3679 + 0.2096)\right]$$

$$\int_0^{1.5} e^{-x^2}\,dx \approx \underline{\underline{0.855}} \quad (3\text{ s.f.})$$



**(3)**

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_{-1.5}^{1.5} e^{-x^2}\, dx$

(ii) $\int_{0}^{1.5} \left(e^{-x^2}+7\right) dx$

b i) $\int_{-1.5}^{1.5} e^{-x^2} dx \approx 2 \times 0.855 = \underline{\underline{1.71}}$ (3 s.f.)

ii) $\int_{0}^{1.5} e^{-x^2} dx + \int_{0}^{1.5} 7\, dx$

$0.855 + \left[7x\right]_{0}^{1.5}$

$0.855 + 7(1.5)$

$\underline{\underline{11.355}}$

(3)

**(Total for question = 6 marks)**

(**Q05 9MA0/02/M, June 2025**)

**Q10.**

(a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{dh}{4-\sqrt{h}} = -8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + k$$

where $k$ is a constant

$u = 4 - \sqrt{h}$

$u = 4 - h^{1/2}$

$\frac{du}{dh} = -\frac{1}{2}h^{-1/2}$

$\frac{du}{dh} = -\frac{1}{2\sqrt{h}}$

$dh = -2\sqrt{h}\ du$

$\sqrt{h} = 4 - u$

$\therefore\ dh = -2(4-u)\ du$

$\int \frac{-2(4-u)}{u}\ du$

$\int -\frac{8}{u} + \frac{2u}{u}\ du$

$-8\ln u + 2u + c$

$-8\ln|4-\sqrt{h}| + 2[4-\sqrt{h}] + c$

$-8\ln|4-\sqrt{h}| - 2\sqrt{h} + 8 + c$

let $-8+c = k$

$-8\ln|4-\sqrt{h}| - 2\sqrt{h} + k$

Shown.

**(6)**

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{dh}{dt} = \frac{t^{0.25}\left(4 - \sqrt{h}\right)}{20}$$

where $h$ is the height in metres and $t$ is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

At $h_{max}$, $\frac{dh}{dt} = 0$ $\therefore$ $4 - \sqrt{h} = 0$

$4 = \sqrt{h}$

$16 = h_{max}$

At start, $t = 0$, $h = 0$

range of tree height $\Rightarrow$ $0 \le h \le 16$

**(2)**

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

$\frac{dh}{dt} = \frac{t^{0.25}(4 - \sqrt{h})}{20}$

$\int \frac{1}{4 - \sqrt{h}}\, dh = \int \frac{t^{0.25}}{20}\, dt$

$-8 \ln|4 - \sqrt{h}| - 2\sqrt{h} = \frac{t^{1.25}}{25} + c$

At $t = 0$, $h = 1$ m

$-8 \ln 3 - 2 = c$

$\therefore$ $-8 \ln|4 - \sqrt{h}| - 2\sqrt{h} = \frac{t^{1.25}}{25} - 2 - 8 \ln 3$

$-8 \ln|4 - \sqrt{12}| - 2\sqrt{12} = \frac{t^{1.25}}{25} - 2 - 8 \ln 3$

$-200 \ln|4 - 2\sqrt{3}| - 50\sqrt{12} = t^{1.25} - 50 - 200 \ln 3$

$t^{1.25} = 200 \ln 3 + 50 - 200 \ln|4 - 2\sqrt{3}| - 50\sqrt{12}$

$t = \left(200 \ln 3 + 50 - 200 \ln|4 - 2\sqrt{3}| - 50\sqrt{12}\right)^{4/5}$

$t \simeq 75.2$ years. (3 s.f.)

**(7)**

**(Total for question = 15 marks)**

(**Q14 9MA0/02, June 2019**)

**Q11.**

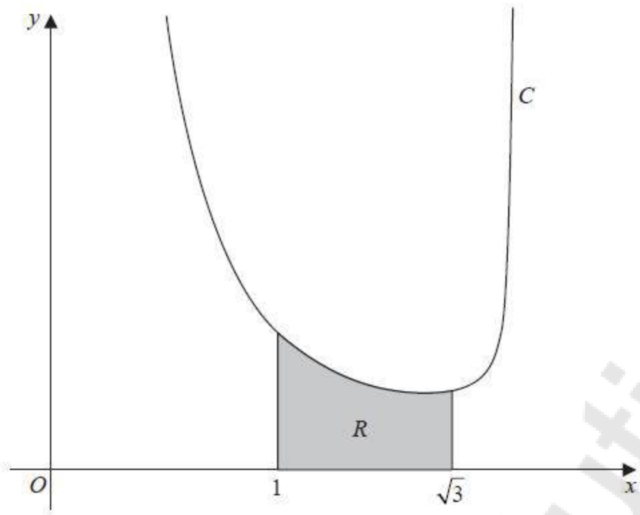



**Figure 5**

Figure 5 shows a sketch of the curve $C$ with equation

$$y = \frac{1}{x^2\sqrt{4-x^2}} \qquad 0 < x < 2$$

The region $R$, shown shaded in Figure 5, is bounded by $C$, the line with equation $x = 1$, the $x$-axis and the line with equation $x = \sqrt{3}$

(a) Use the substitution $x = 2\sin u$ to show that the area of $R$ is given by

$$\int_a^b k\operatorname{cosec}^2 u \, du$$

where $a$, $b$ and $k$ are constants to be found.

$x = 2\sin u$ $\qquad$ $\sqrt{3} = 2\sin u$

$\frac{dx}{du} = 2\cos u$ $\qquad$ $\frac{\sqrt{3}}{2} = \sin u, \quad u_2 = \frac{\pi}{3}$

$dx = 2\cos u \, du$ $\qquad$ $1 = 2\sin u$

$\frac{1}{2} = \sin u, \quad u_1 = \frac{\pi}{6}$

$x^2 = 4\sin^2 u$

$$\int_{\frac{\pi}{6}}^{\pi/3} \frac{1}{4\sin^2 u\sqrt{4 - 4\sin^2 u}} \times 2\cos u \, du$$

$$\int_{\pi/6}^{\pi/3} \frac{2\cos u}{4\sin^2 u\sqrt{4(1-\sin^2 u)}} \, du$$

$$\int_{\pi/6}^{\pi/3} \frac{2\cos u}{4\sin^2 u\sqrt{4\cos^2 u}} \, du$$

$$\int_{\pi/6}^{\pi/3} \frac{2\cos u}{4\sin^2 u \times 2\cos u} \, du$$

$K = 0.25$

$a = \frac{\pi}{6}$

$b = \frac{\pi}{3}$

**(4)**

$$\int_{\pi/6}^{\pi/3} 0.25\operatorname{cosec}^2 u \, du$$

Shown.

(b) Hence, using algebraic integration, find the exact area of $R$.

Give your answer in simplest form.

$$0.25 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{cosec}^2 u \, du$$

$$\int -k \,\text{cosec}^2 kx \, dx = \cot kx$$

$$\int \text{cosec}^2 x \, dx = -\cot x$$

$$0.25 \left[ -\cot u \right]_{\pi/6}^{\pi/3}$$

$$0.25 \left[ \left( -\frac{1}{\tan \frac{\pi}{3}} \right) - \left( -\frac{1}{\tan \frac{\pi}{6}} \right) \right]$$

$$0.25 \left[ -\frac{\sqrt{3}}{3} + \sqrt{3} \right]$$

$$-\frac{\sqrt{3}}{12} + \frac{\sqrt{3}}{4}$$

$$\underline{\underline{\frac{\sqrt{3}}{6}}}$$

**(3)**

**(Total for question = 7 marks)**

**(Q16 9MA0/01, June 2025)**

**Q12.**

**In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Show that

$$\int_0^2 \frac{x}{(2x+1)^3}\,dx = \frac{2}{25}$$

$$\int_0^2 \frac{x}{(2x+1)^3}\,dx$$

let $u = 2x+1$ $\quad u_2 = 5 \quad u_1 = 1$

$\frac{du}{dx} = 2$ , $dx = \frac{du}{2}$ $\quad x = \frac{u-1}{2}$

$$\int_1^5 \frac{u-1}{2} \times \frac{1}{u^3} \times \frac{du}{2}$$

$$\int_1^5 \frac{u-1}{4u^3}\,du$$

$$\int_1^5 \frac{u}{4u^3} - \frac{1}{4u^3}\,du$$

$$\int_1^5 0.25u^{-2} - 0.25u^{-3}\,du$$

$$\left[\frac{0.25u^{-1}}{-1} - \frac{0.25u^{-2}}{-2}\right]_1^5$$

$$\left[-\frac{0.25}{u} + \frac{0.125}{u^2}\right]_1^5$$

$$R = \left[\frac{-0.25}{5} + \frac{0.125}{5^2}\right] - \left[\frac{-0.25}{1} + \frac{0.125}{1^2}\right]$$

$$R = \frac{2}{25}$$

Shown.

**(Total for question = 5 marks)**

(**Q13 9MA0/01, June 2025**)

**Q13.**

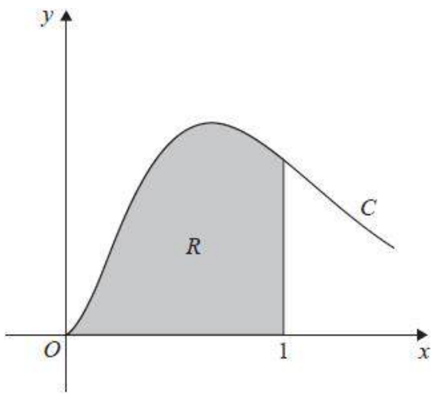



**Figure 5**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 5 shows a sketch of part of the curve *C* with equation

$$y = 8x^2e^{-3x} \qquad x \geqslant 0$$

The finite region *R*, shown shaded in Figure 5, is bounded by

- the curve *C*
- the line with equation $x = 1$
- the *x*-axis

Find the exact area of *R*, giving your answer in the form

$$A + Be^{-3}$$

where *A* and *B* are rational numbers to be found.

$R = \int_0^1 8x^2e^{-3x}\,dx$

$\int u\,dv = uv - \int v\,du$

$u = 8x^2 \qquad dv = e^{-3x}\,dx$

$\frac{du}{dx} = 16x \qquad \int dv = \int e^{-3x}\,dx$

$du = 16x\,dx \qquad v = \frac{e^{-3x}}{-3}$

$v = -\frac{1}{3}e^{-3x}$

$R = 8x^2\left(-\frac{1}{3}e^{-3x}\right) - \int -\frac{1}{3}e^{-3x} \times 16x\,dx$

$-\frac{8}{3}x^2e^{-3x} + \frac{16}{3}\int xe^{-3x}\,dx$

$u = x \qquad dv = e^{-3x}\,dx$

$\frac{du}{dx} = 1 \qquad \int dv = \int e^{-3x}\,dx$

$\therefore du = dx \qquad v = -\frac{1}{3}e^{-3x}$

$-\frac{8}{3}x^2e^{-3x} + \frac{16}{3}\left[-\frac{1}{3}xe^{-3x} - \int -\frac{1}{3}e^{-3x}\,dx\right]$

$-\frac{8}{3}x^2e^{-3x} + \frac{16}{3}\left[-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx\right]$

$-\frac{8}{3}x^2e^{-3x} + \frac{16}{3}\left[-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}\right]$

**(5)**

$\left[-\frac{8}{3}x^2e^{-3x} - \frac{16}{9}xe^{-3x} - \frac{16}{27}e^{-3x}\right]_0^1$

$\left[-\frac{8}{3}(1)^2e^{-3(1)} - \frac{16}{9}(1)e^{-3(1)} - \frac{16}{27}e^{-3(1)}\right] - \left[-\frac{16}{27}e^{-3(0)}\right]$

$-\frac{8}{3}e^{-3} - \frac{16}{9}e^{-3} - \frac{16}{27}e^{-3} + \frac{16}{27}$

$\frac{16}{27} - \frac{136}{27}e^{-3} \qquad \therefore A = \frac{16}{27} \qquad B = -\frac{136}{27}$

**(Total for question = 5 marks)**

**(Q11 9MA0/02, June 2024)**

**Q14.**

(a) Given that $a$ is a positive constant, use the substitution $x = a\sin^2\theta$ to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,dx = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,d\theta$$

$x = a\sin^2\theta$

$a = a\sin^2\theta$

$1 = \sin^2\theta$

$1 = \sin\theta$

$\theta_2 = \sin^{-1}(1) = \frac{\pi}{2}$

$0 = a\sin^2\theta$

$0 = \sin^2\theta$

$0 = \sin\theta$

$\theta_1 = \sin^{-1}(0) = 0$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - \sin^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1}{2} - \frac{\cos 2\theta}{2}$

$\therefore x = \frac{a}{2} - \frac{a\cos 2\theta}{2}$

$\frac{dx}{d\theta} = a\sin 2\theta$

$dx = a\sin 2\theta\,d\theta$

$\int_0^{\frac{\pi}{2}} (a\sin^2\theta)^{1/2}\sqrt{a - a\sin^2\theta} \times a\sin 2\theta\,d\theta$

$\int_0^{\frac{\pi}{2}} \sin\theta(\sqrt{a})(\sqrt{a(1-\sin^2\theta)}) \times a\sin 2\theta\,d\theta$

$\int_0^{\pi/2} \sin\theta(\sqrt{a})(\sqrt{a\cos^2\theta}) \times a\sin 2\theta\,d\theta$

$\int_0^{\frac{\pi}{2}} \sin\theta(\sqrt{a})(\sqrt{a})(\sqrt{\cos^2\theta}) \times a\sin 2\theta\,d\theta$

$\int_0^{\pi/2} a\sin\theta\cos\theta \times a\sin 2\theta\,d\theta$

$\int_0^{\pi/2} \frac{a}{2}\sin 2\theta \times a\sin 2\theta\,d\theta \rightarrow \frac{1}{2}a^2\int_0^{\pi/2}\sin^2 2\theta\,d\theta$

Shown.

**(4)**

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,dx = k\pi a^2$$

where $k$ is a constant to be found.

$I = \frac{1}{2}a^2\int_0^{\pi/2}\sin^2 2\theta\,d\theta$

$\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$

$\cos 4\theta = 1 - \sin^2 2\theta - \sin^2 2\theta$

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$\sin^2 2\theta = \frac{1}{2} - \frac{\cos 4\theta}{2}$

$I = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}} \frac{1}{2} - \frac{\cos 4\theta}{2}\,d\theta$

$I = \frac{1}{2}a^2\left[\frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_0^{\pi/2}$

$I = \frac{1}{2}a^2\left[\left(\frac{\pi}{4} - \frac{\sin 2\pi}{8}\right) - \left(\frac{0}{2} - \frac{\sin 0}{8}\right)\right]$

$I = \frac{1}{8}\pi a^2$

$\therefore k = \frac{1}{8}$

**(4)**

**(Total for question = 8 marks)**

(**Q13 9MA0/01, June 2024**)

**Q15.**

The curve $C$ with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where $p$ and $q$ are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

i) $x \neq 2 \quad \therefore \quad 2(2) - q = 0$

$4 - q = 0$

$\therefore q = 4$

ii)

$$y = \frac{p - 3x}{(2x-4)(x+3)}$$

$$\frac{1}{2} = \frac{p - 3(3)}{[2(3)-4][3+3]}$$

$$\frac{1}{2} = \frac{p-9}{12}$$

$6 = p - 9$

$\underline{\underline{p = 15}}$ shown.



(3)

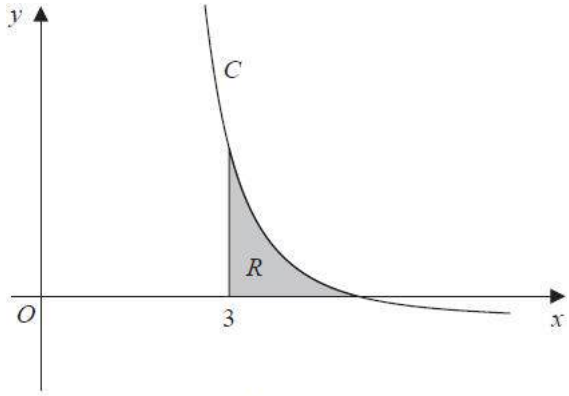



**Figure 4**

Figure 4 shows a sketch of part of the curve *C*. The region *R*, shown shaded in Figure 4, is bounded by the curve *C*, the *x*-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of *R* is $a\ln 2 + b\ln 3$, where *a* and *b* are rational constants to be found.

$$y = \frac{15 - 3x}{(2x-4)(x+3)}$$

At $y = 0$

$$0 = 15 - 3x$$

$$3x = 15$$

$$x = 5$$

$$R = \int_3^5 \frac{15 - 3x}{(2x-4)(x+3)}\,dx$$

$$\frac{15 - 3x}{(2x-4)(x+3)} = \frac{A}{2x-4} + \frac{B}{x+3}$$

$$15 - 3x = A(x+3) + B(2x-4)$$

At $x = -3$

$$24 = -10B$$

$$B = -\frac{12}{5}$$

At $x = 2$

$$9 = 5A$$

$$A = \frac{9}{5}$$

$$\frac{15 - 3x}{(2x-4)(x+3)} = \frac{9/5}{2x-4} - \frac{12/5}{x+3}$$

$$R = \int_3^5 \frac{9/5}{2x-4} - \frac{12/5}{x+3}\,dx$$

$$R = \left[\frac{9}{10}\ln|2x-4| - \frac{12}{5}\ln|x+3|\right]_3^5$$

$$R = \left(\frac{9}{10}\ln 6 - \frac{12}{5}\ln 8\right) - \left(\frac{9}{10}\ln 2 - \frac{12}{5}\ln 6\right)$$

$\ln 8 = \ln 2^3$
$\ln 8 = 3\ln 2$

$$R = \frac{9}{10}\ln 6 - \frac{36}{5}\ln 2 - \frac{9}{10}\ln 2 + \frac{12}{5}\ln 6$$

$$R = \frac{33}{10}\ln 6 - \frac{81}{10}\ln 2$$

$$R = \frac{33}{10}\ln(3\times 2) - \frac{81}{10}\ln 2$$

$$R = \frac{33}{10}\ln 3 + \frac{33}{10}\ln 2 - \frac{81}{10}\ln 2$$

$$R = -\frac{24}{5}\ln 2 + \frac{33}{10}\ln 3$$

$$a = -\frac{24}{5} \qquad b = \frac{33}{10}$$

**(8)**

**(Total for question = 11 marks)**

(**Q13 9MA0/01, June 2019**)

**Q16.**

The number of mice on an island is being monitored.

When monitoring began there were 4000 mice on the island.

In a simple model, the number of mice *x*, in thousands, is modelled by the equation

$$x = \frac{k(3t+2)}{4t+5}$$

where *k* is a constant and *t* is the number of years after monitoring began.

(a) Find a complete equation for this model.

$x = 4$ at $t = 0$

$$4 = \frac{2k}{5}$$

$$k = \frac{4 \times 5}{2}$$

$$k = 10$$

$$\therefore \; x = \frac{10(3t+2)}{4t+5}$$

**(2)**

(b) Hence calculate the long-term population of mice predicted by this model.

AS $t \to \infty$

$$x \to \frac{10000 \times 3}{4}$$

$x \to 7500$ mice is the long term population

**(1)**

In a second model, the number of mice $x$, in thousands, is modelled by the differential equation

$$4\frac{dx}{dt} = 6x - x^2$$

where $t$ is the number of years after monitoring began.

(c) Write $\frac{4}{6x - x^2}$ in partial fraction form.

$$\frac{4}{6x - x^2} = \frac{4}{x(6-x)}$$

$$\therefore \quad \frac{4}{x(6-x)} \equiv \frac{A}{x} + \frac{B}{6-x}$$

$$4 = A(6-x) + Bx$$

At $x = 6$

$4 = 6B$

$B = \frac{2}{3}$

At $x = 0$

$4 = 6A$

$A = \frac{2}{3}$

$$\therefore \quad \frac{4}{6x - x^2} = \frac{2/3}{x} + \frac{2/3}{6-x}$$

**(3)**

(d) Solve the differential equation with the given initial condition to show that

$$x = \frac{12e^{\frac{3}{2}t}}{1+2e^{\frac{3}{2}t}}$$

$$4\frac{dx}{dt} = 6x - x^2$$

$$\int \frac{4}{6x-x^2}\,dx = \int dt$$

$$\int \frac{2/3}{x} + \frac{2/3}{6-x}\,dx = \int dt$$

$$\frac{2}{3}\ln x - \frac{2}{3}\ln|6-x| = t + c$$

$$\frac{2}{3}\ln 4 - \frac{2}{3}\ln 2 = c$$

$$\frac{2}{3}(\ln 4 - \ln 2) = c$$

$$\frac{2}{3}\ln\frac{4}{2} = c$$

$$\frac{2}{3}\ln 2 = c$$

$$\therefore \frac{2}{3}\ln x - \frac{2}{3}\ln|6-x| = t + \frac{2}{3}\ln 2$$

$$\frac{2}{3}\ln\left|\frac{x}{6-x}\right| = t + \frac{2}{3}\ln 2$$

$$\ln\left|\frac{x}{6-x}\right| = \frac{3}{2}t + \ln 2$$

$$e^{\ln\left|\frac{x}{6-x}\right|} = e^{\frac{3}{2}t + \ln 2}$$

$$\frac{x}{6-x} = \left(e^{\frac{3}{2}t}\right)\left(e^{\ln 2}\right)$$

$$\frac{x}{6-x} = 2e^{\frac{3}{2}t}$$

$$x = 12e^{\frac{3}{2}t} - 2xe^{\frac{3}{2}t}$$

$$x + 2xe^{\frac{3}{2}t} = 12e^{\frac{3}{2}t}$$

$$x(1+2e^{\frac{3}{2}t}) = 12e^{\frac{3}{2}t}$$

$$x = \frac{12e^{\frac{3}{2}t}}{1+2e^{\frac{3}{2}t}}$$

Shown.

(5)

(e) Hence find the long-term population of mice predicted by this **second** model.

As $t \to \infty$

$e^{\frac{3}{2}t} \to \infty$

$x \to 6$

long term population = 6000 mice

(1)

**(Total for question = 12 marks)**

**(Q15 9MA0/02/M, June 2025)**

**Q17.**

Water flows at a constant rate into a large container.

There is a tap at the bottom of the container.

At time $t$ hours after the tap was opened

- the volume of water in the container is $V$ m$^3$
- water is flowing into the container at a constant rate of 0.45 m$^3$ per hour
- water is leaving the container through the tap at a rate of $0.3V$ m$^3$ per hour

(a) Show that

$$20\frac{dV}{dt} = 9 - 6V$$

$$\frac{dV}{dt} = 0.45 - 0.3V$$

$\times 20 \qquad \times 20 \qquad \times 20$

$$20\frac{dV}{dt} = 9 - 6V$$

**(2)**

Given that when the tap was opened, there was 0.25 $m^3$ of water in the container,

(b) solve the differential equation to show that

$$V = P - Qe^{-kt}$$

where $P$, $Q$ and $k$ are positive constants to be found.

$$20\frac{dv}{dt} = 9 - 6v$$

$$\int \frac{1}{9-6v}\,dv = \int \frac{1}{20}\,dt$$

$$-\frac{1}{6}\ln|9-6v| = \frac{t}{20} + c$$

$$-\frac{1}{6}\ln|9-6(0.25)| = c$$

$$-\frac{1}{6}\ln 7.5 = c$$

$$-\frac{1}{6}\ln|9-6v| = \frac{t}{20} - \frac{1}{6}\ln 7.5$$

$$\ln|9-6v| = -\frac{3}{10}t + \ln 7.5$$

$$e^{\ln|9-6v|} = e^{-0.3t+\ln 7.5}$$

$$9-6v = (e^{-0.3t})(e^{\ln 7.5})$$

$$9-6v = 7.5e^{-0.3t}$$

$$v = \frac{1}{6}(9 - 7.5e^{-0.3t})$$

$$V = \frac{3}{2} - \frac{5}{4}e^{-0.3t} \qquad P = \frac{3}{2} \quad Q = \frac{5}{4} \quad k = 0.3$$

**(5)**

Given that

- the capacity of the container is 2 $m^3$
- the tap remains open
- the water continues to flow into the tank at the same rate

(c) determine whether the container will ever become full, giving a reason for your answer.

As $t \to \infty$

$e^{-0.3t} \to 0$

$\therefore V \to \frac{3}{2}m^3$

$\frac{3}{2} < 2$ So the container will never become full.

**(2)**

**(Total for question = 9 marks)**

(**Q10 9MA0/02, June 2025**)

**Q18.**

**In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

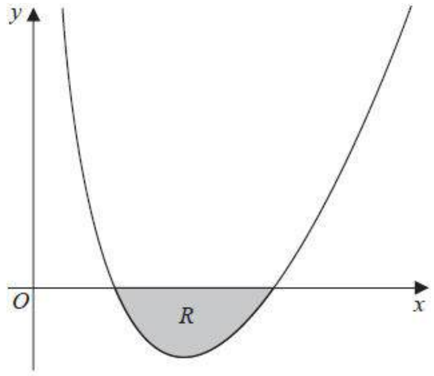



**Figure 3**

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region $R$, shown shaded in Figure 3, is bounded by the curve and the $x$-axis.

Find the exact area of $R$, writing your answer in the form $a\sqrt{2} + b$, where $a$ and $b$ are constants to be found.

$$y = \frac{x^2 - 6x + 8}{4x^{1/2}}$$

To find limits

At $y = 0$

$(x-2)(x-4) = 0$

$\therefore \quad x = 2 \quad x = 4$

$$y = \frac{x^2}{4x^{1/2}} - \frac{6x}{4x^{1/2}} + \frac{8}{4x^{1/2}}$$

$$y = 0.25x^{3/2} - 1.5x^{1/2} + 2x^{-1/2}$$

$$R = \int_2^4 0.25x^{3/2} - 1.5x^{1/2} + 2x^{-1/2}\, dx$$

$$R = \left[ \frac{0.25x^{5/2}}{5/2} - \frac{1.5x^{3/2}}{3/2} + \frac{2x^{1/2}}{1/2} \right]_2^4$$

$$R = \left[ 0.1(\sqrt{x})^5 - (\sqrt{x})^3 + 4\sqrt{x} \right]_2^4$$

$$R = \left[ 0.1(\sqrt{4})^5 - (\sqrt{4})^3 + 4\sqrt{4} \right] - \left[ 0.1(\sqrt{2})^5 - (\sqrt{2})^3 + 4\sqrt{2} \right]$$

$$R = \left[ \frac{16}{5} - 8 + 8 \right] - \left[ \frac{2}{5}\sqrt{2} - 2\sqrt{2} + 4\sqrt{2} \right]$$

$$R = \frac{16}{5} - \frac{12}{5}\sqrt{2} \qquad a = \frac{-12}{5} \qquad b = \frac{16}{5}$$

**(Total for question = 6 marks)**

(**Q08 9MA0/02, June 2022**)

**Q19.**

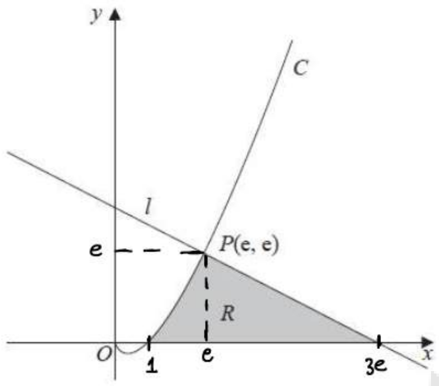



**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation $y = x \ln x$,  $x > 0$

The line $l$ is the normal to $C$ at the point $P(\mathrm{e}, \mathrm{e})$

The region $R$, shown shaded in Figure 2, is bounded by the curve $C$, the line $l$ and the $x$-axis.

Show that the exact area of $R$ is $A\mathrm{e}^2 + B$ where $A$ and $B$ are rational numbers to be found.

$y = x \ln x$

At $y = 0$

$0 = x \ln x$

$\therefore x = 0 \quad \ln x = 0$

$x = 1$

$\int_0^1 x \ln x \, dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln x \quad dv = x\,dx$

$\frac{du}{dx} = \frac{1}{x} \quad \int dv = \int x\,dx$

$\therefore du = \frac{dx}{x} \quad v = \frac{x^2}{2}$

$\int_1^e x \ln x\,dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{dx}{x}$

$\int_1^e x \ln x\,dx = \frac{x^2}{2} \ln x - \int \frac{x}{2}\,dx$

$\int_1^e x \ln x\,dx = \left[\frac{x^2}{2} \ln x - \frac{x^2}{4}\right]_1^e$

$\int_1^e x \ln x\,dx = \left(\frac{e^2}{2} \ln e - \frac{e^2}{4}\right) - \left(\frac{1^2}{2} \ln 1 - \frac{1^2}{4}\right)$

$\int_1^e x \ln x\,dx = \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2}{4} + \frac{1}{4}$

$y = x \ln x$

$y' = uv' + vu'$

$u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

$\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x$

$\frac{dy}{dx} = 1 + \ln x$

At $x = e$ $\quad m_T = 1 + \ln e$

$m_T = 1 + 1 = 2$

$m_N = -\frac{1}{2} \quad$ At $(e, e)$

Equation of Normal

$y - e = -\frac{1}{2}(x - e)$

At $y = 0$

$-e = -\frac{1}{2}(x - e)$

$-2e = -x + e$

$x = 2e + e$

$x = 3e$

Area of triangle

base $= 3e - e = 2e$

height $= e$

Area of $\Delta = \frac{2e \times e}{2}$

Area of $\Delta = e^2$

Area of $R = e^2 + \frac{e^2}{4} + \frac{1}{4}$

Area of $R = \frac{5}{4}e^2 + \frac{1}{4}$

**(10)**

**(Total for question = 10 marks)**

**(Q13 9MA0/02, June 2018)**

**Q20.**

A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.

(You should define the variables that you use.)

let t = time mint has been in mouth in minutes

let r = radius of mint after time, t in mm

$$-\frac{dr}{dt} = \frac{K}{r^2}$$

$$\int -r^2\,dr = \int K\,dt$$

$$-\frac{r^3}{3} = Kt + c$$

$$-\frac{(5)^3}{3} = c$$

$$c = \frac{-125}{3}$$

$$\therefore \frac{-r^3}{3} = Kt - \frac{125}{3}$$

$$\frac{-(3)^3}{3} = 4K - \frac{125}{3}$$

$$K = \frac{1}{4}\left[\frac{125}{3} - \frac{27}{3}\right]$$

$$K = \frac{49}{6}$$

$$\therefore \frac{-r^3}{3} = \frac{49}{6}t - \frac{125}{3}$$

**(5)**

(b) Hence find the total time taken for the mint to completely dissolve.

Give your answer in minutes and seconds to the nearest second.

Completely dissolved at $r = 0$

$$\therefore \frac{49}{6}t = \frac{125}{3}$$

$$t = \frac{125}{3} \times \frac{6}{49}$$

$t = 5.10204\ldots$ $t$ = 5 minutes 6 seconds

$0.10204\ldots \times 60 \approx 6$ seconds

**(2)**

(c) Suggest a limitation of the model.

The model does not consider the temperature of the mouth.

**(1)**

**(Total for question = 8 marks)**

(**Q10 9MA0/02, June 2018**)

**Q21.**

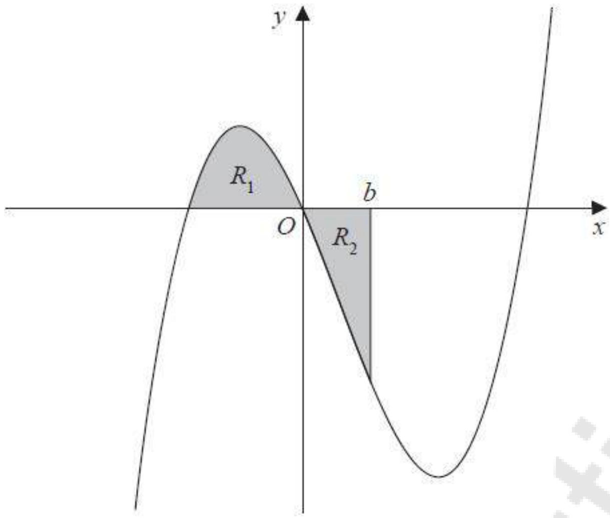



**Figure 2**

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region $R_1$ shown shaded in Figure 2 is bounded by the curve and the negative $x$-axis.

(a) Show that the exact area of $R_1$ is $\frac{20}{3}$

To find limits, $y = 0$

$0 = x(x+2)(x-4)$

$\therefore x = 0, \quad x = -2, \quad x = 4$

$R_1 = \int_{-2}^{0} x(x+2)(x-4)\,dx$

$R_1 = \int_{-2}^{0} x^3 - 2x^2 - 8x \, dx$

$R_1 = \left[ \frac{x^4}{4} - \frac{2}{3}x^3 - 4x^2 \right]_{-2}^{0}$

$R_1 = 0 - \left( \frac{(-2)^4}{4} - \frac{2}{3}(-2)^3 - 4(-2)^2 \right)$

$R_1 = 0 - \left( -\frac{20}{3} \right)$

$R_1 = \frac{20}{3}$

**(4)**

The region $R_2$ also shown shaded in Figure 2 is bounded by the curve, the positive $x$-axis and the line with equation $x = b$, where $b$ is a positive constant and $0 < b < 4$

Given that the area of $R_1$ is equal to the area of $R_2$

(b) verify that $b$ satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

$R_1 = R_2 = -\frac{20}{3}$

$-\frac{20}{3} = \int_0^b x^3 - 2x^2 - 8x \, dx$

$-\frac{20}{3} = \left[ \frac{x^4}{4} - \frac{2}{3}x^3 - 4x^2 \right]_0^b$

$-\frac{20}{3} = \frac{b^4}{4} - \frac{2}{3}b^3 - 4b^2$

$-80 = 3b^4 - 8b^3 - 48b^2$

$0 = 3b^4 - 8b^3 - 48b^2 + 80$

$(b+2)^2$ is a factor

$\therefore$ $b^2 + 4b + 4$ is a factor

$$\begin{array}{r|l} & 3b^2 - 20b + 20 \\ b^2 + 4b + 4 & 3b^4 - 8b^3 - 48b^2 + 0b + 80 \\ & -(3b^4 + 12b^3 + 12b^2) \\ & -20b^3 - 60b^2 + 0b \\ & -(-20b^3 - 80b^2 - 80b) \\ & 20b^2 + 80b + 80 \\ & 20b^2 + 80b + 80 \\ & 0 \quad 0 \quad 0 \end{array}$$

$0 = (b^2 + 4b^2 + 4)(3b^2 - 20b + 20)$

$0 = (b+2)^2 (3b^2 - 20b + 20)$

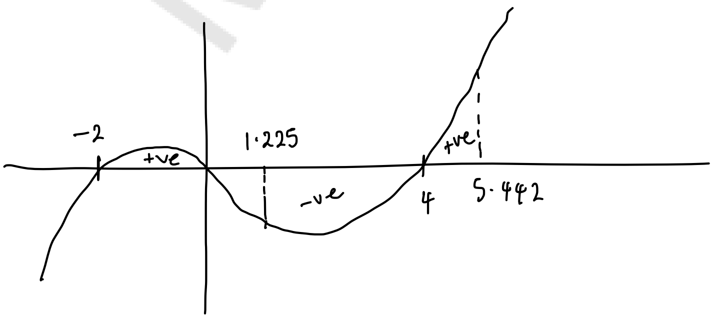
Shown.

**(4)**

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of $b$ is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

Area below $x$-axis is negative hence from 0 to 5.442 the net area which accounts for positive above the $x$-axis and negative below the $x$-axis results in $-\frac{20}{3}$. See sketch below.




**(2)**

**(Total for question = 10 marks)**

(**Q08 9MA0/01, June 2019**)

**Q22.**

(a) Express $\frac{3}{(2x-1)(x+1)}$ in partial fractions.

$$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$$

$$3 = A(x+1) + B(2x-1)$$

At $x = -1$ At $x = 1/2$

$3 = -3B$ $3 = 1.5A$

$B = -1$ $A = 2$

$$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$$



**(3)**

When chemical $A$ and chemical $B$ are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V$ m$^3$, $t$ hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)} \qquad V \geqslant 0 \quad t \geqslant k$$

where $k$ is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of 3 m$^3$ of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)}$$

$$\frac{dv}{dt} = \frac{3v}{(2t-1)(t+1)}$$

$$\int \frac{1}{v}\,dv = \int \frac{3}{(2t-1)(t+1)}\,dt$$

$$\int \frac{1}{v}\,dv = \int \frac{2}{2t-1} - \frac{1}{t+1}\,dt$$

$$\ln v = \ln|2t-1| - \ln|t+1| + c$$

$$\ln v = \ln\left|\frac{2t-1}{t+1}\right| + c$$

$$\ln 3 = \ln\left|\frac{3}{3}\right| + c$$

$$\ln 3 = \ln 1 + c$$

$$c = \ln 3$$

$$\ln v = \ln\left|\frac{2t-1}{t+1}\right| + \ln 3$$

$$\ln v = \ln\left|\frac{3(2t-1)}{t+1}\right|$$

$$e^{\ln v} = e^{\ln\left|\frac{3(2t-1)}{t+1}\right|}$$

$$v = \frac{3(2t-1)}{t+1}$$

Shown.

**(5)**

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

(c) (i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in $m^3$

i) At $V=0$, $0 = 2t-1$

$t = 0.5$ hours

Time delay is 30 minutes

ii) As $t \to \infty$

$V \to 6\,m^3$

The limit to total volume of oxygen produced is $6\,m^3$

**(2)**

**(Total for question = 10 marks)**

(**Q14 9MA0/02, June 2022**)

**Q23.**

(a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x$ as an integral.

$$\int_{2\cdot 1}^{6\cdot 3} \frac{2}{x}\, dx$$

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \ln k$$

where *k* is a constant to be found.

$$2\int_{2\cdot 1}^{6\cdot 3} \frac{1}{x}\, dx = \ln k$$

$$2\left[\ln x\right]_{2\cdot 1}^{6\cdot 3} = \ln k$$

$$2\left[\ln 6\cdot 3 - \ln 2\cdot 1\right] = \ln k$$

$$2\ln\left|\frac{6\cdot 3}{2\cdot 1}\right| = \ln k$$

$$2\ln 3 = \ln k$$

$$\ln 3^2 = \ln k$$

$$\ln 9 = \ln k$$

(2)

$$\therefore \underline{\underline{k = 9}}$$

**(Total for question = 3 marks)**

(**Q04 9MA0/01, June 2022**)

**Q24.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Find the first three terms, in ascending powers of $x$, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

$(3 + x)^{-2}$

$\left[3\left(1 + \frac{x}{3}\right)\right]^{-2}$

$3^{-2}\left(1 + \frac{x}{3}\right)^{-2}$

$\frac{1}{9}\left[1 + \frac{x}{3}\right]^{-2}$

$n = -2$

$X = \frac{x}{3}$

$\left(1 + \frac{x}{3}\right)^{-2} = 1 + (-2)\left(\frac{x}{3}\right) + (-2)(-3)\left(\frac{x}{3}\right)^2\left(\frac{1}{2}\right) + \dots$

$\left(1 + \frac{x}{3}\right)^{-2} = 1 - \frac{2}{3}x + \frac{x^2}{3} + \dots$

$(3 + x)^{-2} = \frac{1}{9}\left(1 + \frac{x}{3}\right)^{-2} = \frac{1}{9}\left(1 - \frac{2}{3}x + \frac{1}{3}x^2 + \dots\right)$

$(3 + x)^{-2} = \frac{1}{9} - \frac{2}{27}x + \frac{1}{27}x^2 + \dots$



**(4)**

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\,dx$$

giving your answer to 4 significant figures.

$$I = \int_{0\cdot2}^{0\cdot4} 6x(3+x)^{-2}\,dx$$

$$I = \int_{0\cdot2}^{0\cdot4} 6x\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27} + \cdots\right) dx$$

$$I = \int_{0\cdot2}^{0\cdot4} \left(\frac{2x}{3} - \frac{4x^2}{9} + \frac{2x^3}{9} + \cdots\right) dx$$

$$I = \left[\frac{1}{2}\times\frac{2}{3}x^2 - \frac{1}{3}\times\frac{4}{9}x^3 + \frac{1}{4}\times\frac{2}{9}x^4\right]_{0\cdot2}^{0\cdot4}$$

$$I = \left[\frac{x^2}{3} - \frac{4x^3}{27} + \frac{x^4}{18}\right]_{0\cdot2}^{0\cdot4}$$

$$I = \left[\frac{(0\cdot4)^2}{3} - \frac{4(0\cdot4)^3}{27} + \frac{(0\cdot4)^4}{18}\right] - \left[\frac{(0\cdot2)^2}{3} - \frac{4(0\cdot2)^3}{27} + \frac{(0\cdot2)^4}{18}\right]$$

$$I = \frac{223}{6750}$$

**(4)**

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\, dx$$

giving your answer in the form *a* ln *b* + *c*, where *a*, *b* and *c* are constants to be found.

$$I = \int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\, dx$$

let $u = 3+x \qquad x = u-3$

$\frac{du}{dx} = 1 \quad , \quad du = dx$

$u_2 = 3+0.4 = 3.4$

$u_1 = 3+0.2 = 3.2$

$$I = \int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\, du$$

$$I = \int_{3.2}^{3.4} \frac{6u}{u^2} - \frac{18}{u^2}\, du$$

$$I = \int_{3.2}^{3.4} \frac{6}{u} - 18u^{-2}\, du$$

$$I = \left[ 6\ln u + 18u^{-1} \right]_{3.2}^{3.4}$$

$$I = \left(6\ln 3.4 + \frac{18}{3.4}\right) - \left(6\ln 3.2 + \frac{18}{3.2}\right)$$

$$I = 6\ln\left|\frac{3.4}{3.2}\right| - \frac{45}{136}$$

$$I = 6\ln\left|\frac{17}{16}\right| - \frac{45}{136}$$

$a = 6 \qquad b = \frac{17}{16} \qquad c = \frac{-45}{136}$

**(5)**

**(Total for question = 13 marks)**

**(Q13 9MA0/02, June 2023)**

**Q25.**

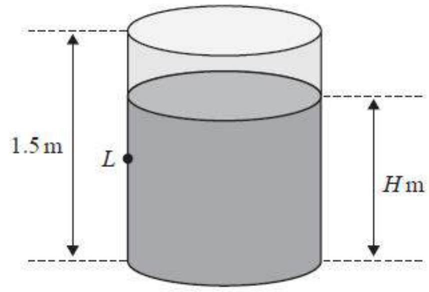



Diagram not drawn to scale.

**Figure 2**

Figure 2 shows a cylindrical tank of height 1.5 m.

Initially the tank is full of water.

The water starts to leak from a small hole, at a point *L*, in the side of the tank.

While the tank is leaking, the depth, *H* metres, of the water in the tank is modelled by the differential equation

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

where *t* hours is the time after the leak starts.

Using the model,

(a) show that

$$H = Ae^{-0.2t} + B$$

where *A* and *B* are constants to be found,

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

$$\int dH = -0.12\int e^{-0.2t}\,dt$$

$$H = -0.12 \times \frac{1}{-0.2} e^{-0.2t} + c$$

$$H = 0.6e^{-0.2t} + c$$

At $t = 0$, $H = 1.5$

$$1.5 = 0.6e^{-0.2(0)} + c$$

$$c = 1.5 - 0.6$$

$$c = 0.9$$

$$\therefore \quad H = 0.6e^{-0.2t} + 0.9$$

$A = 0.6 \quad B = 0.9$

**(3)**

(b) find the time taken for the depth of the water to decrease to 1.2 m. Give your answer in hours and minutes, to the nearest minute.

$$1.2 = 0.6e^{-0.2t} + 0.9$$

$$0.3 = 0.6e^{-0.2t}$$

$$0.5 = e^{-0.2t}$$

$$\ln 0.5 = \ln e^{-0.2t}$$

$$\ln 0.5 = -0.2t$$

$$t = \frac{\ln 0.5}{-0.2}$$

$$t = 3.4657\ldots$$

$$0.4657\ldots \times 60 \approx 28 \text{ minutes}$$

$$t \approx 3 \text{ hours } 28 \text{ minutes}$$

**(3)**

In the long term, the water level in the tank falls to the same height as the hole.

(c) Find, according to the model, the height of the hole from the bottom of the tank.

$$H = 0.6e^{-0.2t} + 0.9$$

$$t \to \infty$$

$$e^{-0.2t} \to 0$$

$$H \to 0.9$$

height of the hole is 0.9 m

**(2)**

**(Total for question = 8 marks)**

(**Q07 9MA0/01, June 2024**)

**Q26.**

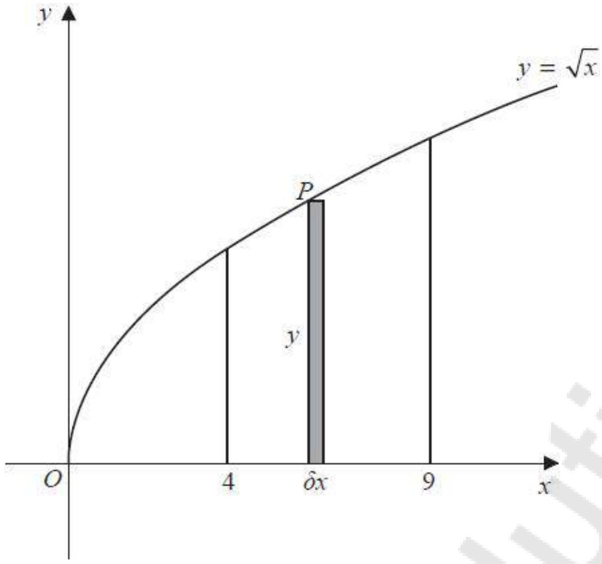



**Figure 3**

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point P($x$, $y$) lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width $\delta x$.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\, \delta x$$

$$I = \int_4^9 x^{1/2}\, dx$$

$$I = \left[ \frac{2}{3} x^{3/2} \right]_4^9$$

$$I = \frac{2}{3}(9)^{3/2} - \frac{2}{3}(4)^{3/2}$$

$$I = \frac{38}{3}$$



**(Total for question = 3 marks)**

(**Q05 9MA0/02, June 2019**)

**Q27.**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

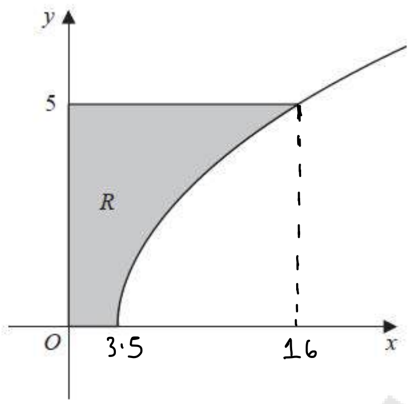



**Figure 3**

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{2x - 7}$$

The region *R*, shown shaded in Figure 3, is bounded by the curve, the *x*-axis, the *y*-axis and the line with equation *y* = 5

Find the exact area of *R*

At $y = 5$, $5 = \sqrt{2x-7}$

$25 = 2x - 7$

$2x = 32$

$x = 16$

Area of rectangle $= 16 \times 5 = 80$

At $y=0$, $0 = \sqrt{2x-7}$

$0 = 2x - 7$

$2x = 7$

$x = 3.5$

$I =$ Area between Curve and $x$-axis $= \int_{3.5}^{16} (2x-7)^{1/2}\, dx$

$I = \left[ \frac{2}{3} \times \frac{1}{2} \left[2x - 7\right]^{3/2} \right]_{3.5}^{16}$

$I = \left[ \frac{1}{3} (2x-7)^{3/2} \right]_{3.5}^{16}$

$I = \frac{1}{3}\left[ (2(16)-7)^{3/2} - (2(3.5)-7)^{3/2} \right]$

$I = \frac{125}{3}$

$R =$ Area of rectangle $- I$

$R = 80 - \frac{125}{3}$

$R = \frac{115}{3}$

**(Total for question = 6 marks)**

(**Q08 9MA0/02/M, June 2025**)

**Q28.**

Find

$$\int\left(x^4 - 6x^{\frac{1}{2}} - 3\right)\mathrm{d}x$$

giving the answer in simplest form.

$$\frac{x^5}{5} - 6\times\frac{2}{3}x^{3/2} - 3x + c$$

$$\frac{x^5}{5} - 4x^{3/2} - 3x + c$$



**(Total for question = 4 marks)**

(**Q02 9MA0/02, June 2025**)

**Q29.**

(a) Express $\lim_{\delta x \to 0} \sum_{x=1.44}^{2.89} \frac{2}{\sqrt{x}} \delta x$ as an integral.

$$\int_{1.44}^{2.89} \frac{2}{\sqrt{x}} \, dx$$

**(1)**

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=1.44}^{2.89} \frac{2}{\sqrt{x}} \delta x = k$$

where *k* is an integer to be found.

*(Solutions relying on calculator technology are not acceptable.)*

$$I = \int_{1.44}^{2.89} 2x^{-1/2} \, dx$$

$$I = \left[ 4x^{1/2} \right]_{1.44}^{2.89}$$

$$I = 4(2.89)^{1/2} - 4(1.44)^{1/2}$$

$$I = 2$$

$$\therefore \underline{\underline{k = 2}}$$

**(2)**

**(Total for question = 3 marks)**

(**Q05 9MA0/01, June 2025**)

**Q30.**

A balloon is being inflated.

In a simple model,

- the balloon is modelled as a sphere
- the rate of increase of the radius of the balloon is inversely proportional to the square root of the radius of the balloon

At time *t* seconds, the radius of the balloon is *r* cm.

(a) Write down a differential equation to model this situation.

$$\frac{dr}{dt} = \frac{K}{\sqrt{r}}$$

**(1)**

At the instant when $t = 10$

- the radius is 16 cm
- the radius is increasing at a rate of 0.9 cm $s^{-1}$

(b) Solve the differential equation to show that

$$r^{\frac{3}{2}} = 5.4t + 10$$

$$\frac{dr}{dt} = \frac{K}{\sqrt{r}}$$

$$0.9 = \frac{K}{\sqrt{16}}$$

$$K = 4 \times 0.9$$

$$K = 3.6$$

$$\sqrt{r}\,dr = K\,dt$$

$$\int r^{1/2}\,dr = \int K\,dt$$

$$\frac{2}{3}r^{3/2} = Kt + C$$

$$\frac{2}{3}r^{3/2} = 3.6t + C$$

$$\frac{2}{3}(16)^{3/2} = 3.6(10) + C$$

$$C = \frac{2}{3}(16)^{3/2} - 36$$

$$C = \frac{20}{3}$$

$$\therefore \frac{2}{3}r^{3/2} = 3.6t + \frac{20}{3}$$

**(5)**

$$r^{3/2} = 5.4t + 10$$

shown.

(c) Hence find the radius of the balloon when $t = 20$

Give your answer to the nearest millimetre.

$r = (5.4(20) + 10)^{2/3}$

$r = 24.05773139\ldots \text{ cm}$

$r \approx 24.1 \text{ cm}$



**(2)**

(d) Suggest a limitation of the model.

The balloon may burst as it cannot keep inflating forever.

**(1)**

**(Total for question = 9 marks)**

(**Q14 9MA0/01, June 2024**)

**Q31.**

Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,dx$$

writing each term in simplest form.

$$\int \frac{2x^{3/2}}{3} - \frac{5x^{1/2}}{3}\,dx$$

$$\frac{2}{5} \times \frac{2}{3}x^{5/2} - \frac{2}{3} \times \frac{5}{3}x^{3/2} + c$$

$$\frac{4}{15}x^{5/2} - \frac{10}{9}x^{3/2} + c$$



**(Total for question = 4 marks)**

(**Q01 9MA0/01, June 2023**)

**Q32.**

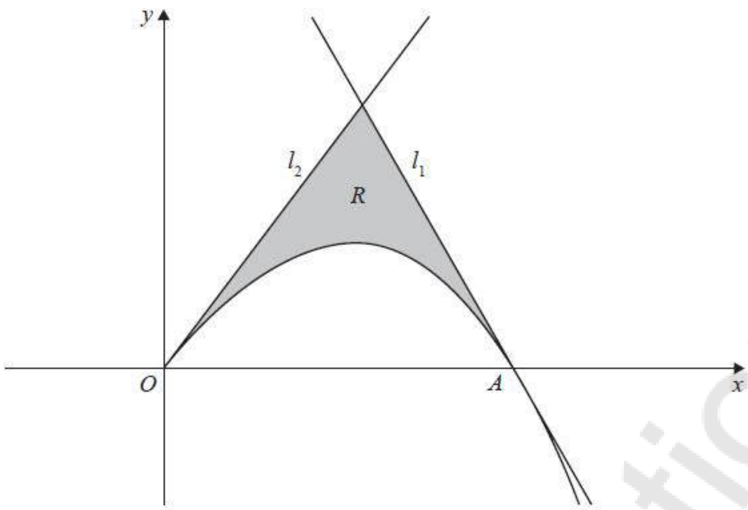



**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \qquad x \geqslant 0$$

The curve crosses the $x$-axis at the point $A$.

(a) Verify that the $x$ coordinate of $A$ is 4

At A, $y = 8(4) - (4)^{5/2}$

$y = 32 - 32$

$y = 0$ Thus, A is $(4, 0)$

**(1)**

The line $l_1$ is the tangent to the curve at $A$.

(b) Use calculus to show that an equation of line $l_1$ is

$$12x + y = 48$$

$y = 8x - x^{5/2}$

$\frac{dy}{dx} = 8 - \frac{5}{2}x^{3/2}$

$m_{l_1} = 8 - \frac{5}{2}(4)^{3/2} = -12$

$y - y_1 = m(x - x_1)$

$y - 0 = -12(x - 4)$

$y = -12x + 48$

$\underline{12x + y = 48}$

**(3)**

The line $l_2$ has equation $y = 8x$

The region $R$, shown shaded in Figure 3, is bounded by the curve, the line $l_1$ and the line $l_2$

(c) Use algebraic integration to find the exact area of $R$.

To find point where $l_1$ and $l_2$ meet.

$$12x + 8x = 48$$

$$20x = 48$$

$$x = 2.4$$

$$\therefore\ y = 8 \times 2.4 = 19.2$$

Area of triangle $= \dfrac{4 \times 19.2}{2} = 38.4$

Area under curve $= \displaystyle\int_0^4 8x - x^{5/2}\,dx$

$$\left[\frac{8x^2}{2} - \frac{x^{7/2}}{7/2}\right]_0^4$$

$$\left[4x^2 - \frac{2}{7}x^{7/2}\right]_0^4$$

$$\left[4(4)^2 - \frac{2}{7}(4)^{7/2}\right] - \left[4(0) - \frac{2}{7}(0)^{7/2}\right]$$

Area under curve $= \dfrac{192}{7}$

$$R = 38.4 - \frac{192}{7}$$

$$R = \frac{384}{35}$$

**(5)**

**(Total for question = 9 marks)**

**(Q10 9MA0/01, June 2024)**