Name: ____

Sequence and Series Exam Questions Mark Scheme

Date:

Time:

Total marks available: 231

Total marks achieved: ______

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Uses the common ratio $\frac{5+2\sin\theta}{12\cos\theta}=\frac{6\tan\theta}{5+2\sin\theta}$ o.e.	M1	3.1a
	Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\times12\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(a) Alt	**(a) Alternative example:**		
	Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ $\Rightarrow 12\cos\theta\left(\frac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$	M1	3.1a
	Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(b)	$4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\frac{1}{2}\left(,\frac{25}{2}\right)$	M1	1.1b
	$\theta=\frac{5\pi}{6}$	A1	1.2
		(2)	

Question	Scheme	Marks	AOs
(c)	Attempts a value for either a or r e.g. $a=12\cos\theta=12\times-\frac{\sqrt{3}}{2}$ **or** $r=\frac{5+2\sin\theta}{12\cos\theta}=\frac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$	M1	3.1a
	$"a"=-6\sqrt{3}$ **and** $"r"=-\frac{1}{\sqrt{3}}$ o.e.	A1	1.1b
	Uses $S_\infty=\frac{a}{1-r}=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$	dM1	2.1
	Rationalises denominator $S_\infty=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\frac{-18}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$	ddM1	1.1b
	$(S_\infty=)9(1-\sqrt{3})$	A1	2.1
		(5)	
			(10 marks)
Notes:			

(a)

M1: For the key step in using the ratio of $\frac{a_2}{a_1}=\frac{a_3}{a_2}$

dM1: Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Alternative:

M1: Expresses the 2nd and 3rd terms in terms of the first term and the common ratio and eliminates "r"

dM1: Multiplies up and uses $\tan\theta \times \cos\theta = \sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Other approaches may be seen in (a) and can be marked in a similar way e.g. M1 for correctly obtaining an equation in θ using the GP, M1 for applying $\tan\theta \times \cos\theta = \sin\theta$ or equivalent and eliminating fractions, A1 as above
Example: $u_2 = \frac{u_1 \times u_3}{u_2} \Rightarrow 5 + 2\sin\theta = \frac{12\cos\theta \times 6\tan\theta}{5 + 2\sin\theta}$ **M1**
$\Rightarrow (5 + 2\sin\theta)^2 = 72\sin\theta$ **dM1**
$25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$ **A1**
$\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0$ *

(b)

M1: Attempts to solve $4\sin^2\theta - 52\sin\theta + 25 = 0$. Must be clear they have found $\sin\theta$ and not e.g. just x from $4x^2 - 52x + 25 = 0$. Working does not need to be seen but see general guidance for solving a 3TQ if necessary. Note that the $\frac{25}{2}$ does not need to be seen.

A1: $\theta = \frac{5\pi}{6}$ and no other values unless they are rejected or the $\frac{5\pi}{6}$ clearly selected here and not in (c)

A minimum requirement in (b) is e.g. $\sin\theta = \frac{1}{2}$, $\theta = \frac{5\pi}{6}$

Do **not** allow 150° for $\frac{5\pi}{6}$

(c) Allow full marks in (c) if e.g. $\theta = \frac{\pi}{6}$ is their answer to (b) but $\theta = \frac{5\pi}{6}$ is used here.

or if e.g. $\theta = \frac{5\pi}{6}$ is their answer to (b) but $\theta = \frac{\pi}{6}$ is used here allow the M marks only.

M1: For attempting a value (exact or decimal) for either a or r using **their** θ

E.g. $a = 12\cos\theta = \left(12 \times -\frac{\sqrt{3}}{2}\right)$ or $r = \frac{5 + 2\sin\theta}{12\cos\theta} = \left(\frac{5 + 2 \times \frac{1}{2}}{12 \times -\frac{\sqrt{3}}{2}}\right)$ oe e.g. $r = \frac{6\tan\theta}{5 + 2\sin\theta} = \left(\frac{6 \times -\frac{1}{\sqrt{3}}}{5 + 2 \times \frac{1}{2}}\right)$

A1: Finds both $a = -6\sqrt{3}$ and $r = -\frac{1}{\sqrt{3}}$ which can be left unsimplified but $\sin\theta = \frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}$ and $\tan\theta = -\frac{\sqrt{3}}{3}$ (if required) must have been used.

dM1: Uses both values of "a" and "r" with the equation $S_\infty = \frac{a}{1-r} = \frac{-6\sqrt{3}}{1 + \frac{1}{\sqrt{3}}}$ to create an expression involving surds where a and r have come from appropriate work and $|r| < 1$

Depends on the first method mark.

ddM1: Rationalises denominator. The denominator must be of the form $p \pm q\sqrt{3}$ oe e.g. $p + \frac{q}{\sqrt{3}}$

Depends on both previous method marks.

Note that stating e.g. $\frac{k}{p + q\sqrt{3}} \times \frac{p - q\sqrt{3}}{p - q\sqrt{3}}$ or $\frac{k}{p + \frac{q}{\sqrt{3}}} \times \frac{p - \frac{q}{\sqrt{3}}}{p - \frac{q}{\sqrt{3}}}$ is sufficient.

A1: Obtains $(S_\infty =)9(1-\sqrt{3})$

Note that full marks are available in (c) for the use of $\theta = 150°$

Note also that marks may be implied in (c) by e.g.

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\theta}{1-\frac{5+2\sin\theta}{12\cos\theta}} = \frac{144\cos^2\theta}{12\cos\theta - 5 - 2\sin\theta} = \frac{144\cos^2\frac{5\pi}{6}}{12\cos\frac{5\pi}{6} - 5 - 2\sin\frac{5\pi}{6}}$$

$$= \frac{108}{-6-6\sqrt{3}} = \frac{108}{-6-6\sqrt{3}} \times \frac{-6+6\sqrt{3}}{-6+6\sqrt{3}} = \frac{-648+648\sqrt{3}}{-72} = 9(1-\sqrt{3})$$

Scores M1A1 implied dM1 ddM1 A1

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} \quad \text{or e.g.} \quad S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}}$$

And nothing else

scores M1A0dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} = 9(1-\sqrt{3})$$

Scores M1A1dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}} = 9(1+\sqrt{3})$$

Scores M1A0dM1ddM0A0

$S_\infty = 9(1-\sqrt{3})$ with no working scores no marks

(**Q15 9MA0/02, June 2022**)

Q2.

Question	Scheme	Marks	AOs
(a)(i)	$a_1=3,\ a_2=5,\ a_3=3\ ...$	B1	1.1b
(ii)	2	B1	1.1b
		(2)	
(b)	$\sum_{n=1}^{85} a_n = 42\times(3+5)+3$ o.e.	M1	3.1a
	$=339$	A1	1.1b
		(2)	
			(4 marks)
Notes:			

(a)(i) Mark (a)(i) and (a)(ii) together.

B1: States the values of at least $a_2=5$ and $a_3=3$. This is sufficient but if more terms are given they must be correct. There is no need to see e.g. $a_2=...,\ a_3=...$ just look for values.

Allow an algebraic approach e.g. $a_{n+1}=8-a_n,\ a_{n+2}=8-(8-a_n)=a_n$

A conclusion is **not** needed.

(a)(ii)

B1: States that the order of the periodic sequence is 2

Allow "second order", "it repeats every 2 numbers" or equivalent statements that convey the idea of the period being 2.

Note that ±2 is B0

(b)

M1: Attempts a <u>correct</u> method to find $\sum_{n=1}^{85} a_n$

For example $\sum_{n=1}^{85} a_n = 42\times(3+5)+3$, $\sum_{n=1}^{85} a_n = \frac{84}{2}\times 3+42\times 5+3$ or $\sum_{n=1}^{85} a_n = 43\times(3+5)-5$

or $\sum_{n=1}^{85} a_n = 43\times 3+42\times 5$ or $\sum_{n=1}^{85} a_n = \frac{85}{2}\times 8-1$

There may be other methods e.g. "Chunking": 5×(3 + 5) = 40, 40 × 8 = 320, 320 + 3×3 +2×5 = 339

A1: 339. Correct answer only scores both marks.

Attempts to use an AP formula score M0

(**Q03 9MA0/02, June 2022**)

Q3.

Question	Scheme	Marks	AOs
(a)	$16+(21-1)\times d=24\Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\}\Rightarrow S_{500}=\frac{1}{2}\times500\{2\times16+499\times\text{"}0.4\text{"}\}$	M1	1.1b
	$=57900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times\text{"}0.4\text{"}=215.6\Rightarrow S_{500}=\frac{1}{2}\times500\{16+\text{"}215.6\text{"}\}$	M1	1.1b
	$=57900$	A1	1.1b
			(4 marks)

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using $a = 16$, and $n = 21$ and the 24. The method may be implied by their working.
If the AP term formula is quoted it must be correct, so use of e.g. $u_n = a + nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20}, \frac{4}{10}, \frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with $a = 16$, $n = 500$ and their numerical d from part (a)
If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their l

A1: Correct value

Note that some candidates are showing implied use of $u_n = a + nd$ by showing the following:

$$\text{(a) } d=\frac{24-16}{21}=\frac{8}{21} \quad \text{(b) } S_{500}=\frac{1}{2}\times500\left\{2\times16+499\times\frac{8}{21}\right\}=55523.80952...$$

This scores (a) M0A0 (b) M1A0

(**Q01 9MA0/02, Oct 2021**)

Q4.

Question	Scheme	Marks	AOs
(a)	$(u_{12} =)400+11\times -10 = 290$* or e.g. $(u_{12} =)400-110 = 290$* or e.g. $(u_{12} =)400+(12-1)\times -10 = 290$* or e.g. $(u_{12} =)410+12\times -10 = 290$*	B1*	1.1b
		(1)	
	Alternative 1:		
	$400+(n-1)\times -10 = 290$ $\Rightarrow 400-10n+10 = 290 \Rightarrow 10n = 120 \Rightarrow n = 12$*	B1*	1.1b
	Alternative 2:		
	$290 = 400+(12-1)d \Rightarrow 11d = -110 \Rightarrow d = -10$*	B1*	1.1b
(b)	$8100 = \frac{1}{2}N(2\times 400+(N-1)\times -10)$ or e.g. $8100 = \frac{1}{2}N(400+400+(N-1)\times -10)$	M1	1.1b
	$8100 = \frac{1}{2}N(2\times 400+(N-1)\times -10)$ $\Rightarrow 16200 = 800N-10N^2+10N$ or e.g. $\Rightarrow 8100 = 400N-5N^2+5N$ $\Rightarrow N^2-81N+1620 = 0$*	A1*	2.1
		(2)	
(c)	$N^2-81N+1620 = 0 \Rightarrow (N-45)(N-36) = 0 \Rightarrow N = 45, 36$	M1	1.1b
	$(N=)$ 36	A1	2.3
		(2)	
			(5 marks)

Notes

(a)

B1*: Correct working to obtain 290. Must be a correct calculation so do not condone missing brackets unless they are recovered. E.g. $(u_{12} =)400+12-1\times -10 = 290$ scores B0 unless followed by $= 400+11\times -10 = 290$. Condone $(u_{12} =)400+(12-1)-10 = 290$

The "£" symbol is not required but the "290" must appear.

Alternative 1:

B1*: Correct working using the 290 to obtain $n = 12$.
There must be at least one intermediate line after setting up the equation and must be correct work so do not condone missing brackets unless they are recovered (as above).
A conclusion is **not** required with this approach as long as 12 is correctly obtained.

Alternative 2:

B1*: Correct working using the 290 and 400 to obtain $d = -10$.
There must be at least one intermediate line after setting up the equation and must be correct work so do not condone missing brackets unless they are recovered (as above).
A conclusion is **not** required with this approach as long as −10 is correctly obtained.

Allow candidates to list terms and show the 12th term is 290 e.g.
400, 390, 380, 370, 360, 350, 340, 330, 320, 310, 300, 290
Must list all 12 terms which must be correct and end with 290
Condone if missing 400, 390, 380 as these are given in the question.

(b) **Mark (b) and (c) together**

M1:	Uses a <u>correct</u> sum formula in terms of N or n with a = 400 and d = −10 or +10 and sets = 8100. Condone e.g. > 8100 and allow A1 if this is recovered to become "=" before the final line. Condone $8100=\frac{1}{2}N\left(2\times400+\left(N-1\right)-10\right)$ if recovered or not.
A1*:	Fully correct proof with sufficient working shown and no unrecovered errors. Do not condone e.g. missing brackets or e.g. a missing N/n unless recovered before the final given answer. Condone the use of n instead of N for **both** marks. Condone terms in a different order as long as they are correct. Condone $0=N^2-81N+1620$* Sufficient working requires all brackets to be removed to obtain an unsimplified expanded quadratic before proceeding to the given answer including the "=0".
Alternative (further maths method): Series summation approach:	
	$$\sum_{r=1}^{N}\left(410-10r\right)=8100\Rightarrow 410N-10\times\frac{1}{2}N\left(N+1\right)$$ $$\Rightarrow 410N-5N^2-5N=8100\Rightarrow N^2-81N+1620=0*$$
M1:	Attempt to sum an appropriate series with first term 400. Condone use of +10 as in the main scheme.
A1*:	As main scheme.
(c)	
M1:	Solves the **given** quadratic equation by any correct method including a calculator to obtain at least one value for N. See general guidance for solving a 3-term quadratic. If values are <u>just written down</u> and only one value is given it must be 45 or 36. If both values are <u>just written down</u> they must both be correct.
A1:	Realises that the smaller value is required and so selects (N =) 36. Ignore any units if given. The "N =" is not required, just look for the correct value. It must be clear that this value has been selected. This may be indicated by e.g. underlining the 36 or the omission of the 45. If the 45 is not rejected score A0. N = 36 with no working scores M1A1

(**Q02 9MA0/02, June 2024**)

Q5.

Question	Scheme	Marks	AOs
(a)	Attempts to solve $10-6k=2k-10 \Rightarrow k=...$	M1	3.1a
	$(k=)\ \dfrac{5}{2}$ o.e.	A1	1.1b
		(2)	
(b)	Deduces the value of "d" $= -5$	B1 ft	2.2a
	$S_{50}=\dfrac{50}{2}\left(2\times"15"+49\times"-5"\right)$	M1	1.1b
	$= -5375$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a) **Condone using other letters for *a* and *d* (e.g. may use *r* for *d*)**

M1: Attempts a valid method to solve the problem

- Uses the common difference to form a correct equation and attempts to solve to find a value for k

  e.g. Attempts to solve $10-6k=2k-10$ o.e such as $2k-6k=2(10-6k)$

- Uses 10 as the mean of $2k$ and $6k$: $\dfrac{2k+6k}{2}=10 \Rightarrow k=...$
- Sets up correct equations $a=6k$, $a+d=10$ and $a+2d=2k$, or may be seen as $6k+d=10$ and $10+d=2k$, **and** proceeds to find k.
- Uses the summation formula $S_3=\dfrac{3}{2}(6k+2k)=6k+10+2k$ **and** proceeds to find k

In each attempt the initial equation (or simultaneous equations) must be correct but do not be concerned by the mechanics of the rearrangement to find k. May be implied by $k=\dfrac{5}{2}$

A1: $(k=)\ \dfrac{5}{2}$ o.e.

(b) **Work seen in (a) can only be scored if seen or used in (b)**

B1ft: Common difference $=-5$ **or** ft on their value for k (even if k has been found from an incorrect method) e.g. $10-6\times"\dfrac{5}{2}"$ or e.g. $2\times"\dfrac{5}{2}"-10$ if only a numerical value is seen.

May be implied or seen in a term or summation formula.

Note that some candidates may work in terms of k throughout so only allow B1ft to be scored when they substitute in their numerical value for k, following $10-6k$ o.e. correctly embedded in a correct formula. They may make arithmetical slips before they substitute in their numerical value for k which can be condoned.

M1:	Attempts to use a **correct formula**. The expression is sufficient to score this mark but they must be using a correct value for a and $\pm d$ (or ft on their value for k for a and d) which are correctly placed in the formula. e.g. $\left(S_{50}=\right)\frac{50}{2}\left(2\times"6k"+49\times"\pm d"\right)$. Alternatively, they may find the 50th term $u_{50}="15"+(50-1)\times"-5"=-230$ and use $\left(S_{50}=\right)\frac{50}{2}\left("15"+"-230"\right)$. If working in terms of k they must substitute in their value for k e.g. $\left(S_{50}=\right)\frac{50}{2}\left(2\times 6k+49\times(10-6k)\right)=-7050k+12250=-7050\times"\frac{5}{2}"+12250$ Do not withhold this mark for omission of brackets around (-5) e.g. $\frac{50}{2}\left(2\times 15+\left(49\right)-5\right)$ scores M1
A1:	-5375 cao

(**Q03 9MA0/01, June 2025**)

Q6.

Question	Scheme	Marks	AOs
(i)	States that $S = a+(a+d)+\ldots\ldots\ldots\ldots+(a+(n-1)d)$	B1	1.1a
	$S = a + \quad (a+d)+ \quad (a+(n-1)d)$ $S=(a+(n-1)d)+(a+(n-2)d)+\ldots\ldots\ldots\ldots+a$	M1	3.1a
	Reaches $2S = n\times(2a+(n-1)d)$ And so proves that $S=\frac{n}{2}[2a+(n-1)d]$ *	A1*	2.1
		(3)	
(ii)	(a) $S = 10+9.20+8.40+\ldots$		
	$64=\frac{n}{2}(20-0.8(n-1))$ o.e	M1	3.1b
	$128 = 20n-0.8n^2+0.8n$ $0.8n^2-20.8n+128=0$ $n^2-26n+160=0$ *	A1*	2.1
		(2)	
	(b) $n=10,16$	B1	1.1b
		(1)	
	(c) 10 weeks with a minimal correct reason. E.g. • He has saved up the amount by 10 weeks so he would not save for another 6 weeks • You would choose the smaller number • He starts saving negative amounts (in week 14) so 16 does not make sense	B1	2.3
		(1)	
			(7 marks)
Notes:			

(i)

B1: Correctly writes down an expression for the key terms S or S_n including $S=$ or $S_n=$

Allow a minimum of 3 correct terms including the first and last terms, and no incorrect terms.

Score for S or $S_n = a+(a+d)+\ldots\ldots\ldots\ldots+(a+(n-1)d)$ with + signs, not commas

If the series contains extra terms that should not be there E.g

$S=a+(a+d)+\ldots\ldots\ldots\ldots(a+nd)+(a+(n-1)d)$ score B0

M1: For the key step in reversing the terms and adding the two series.

Look for a minimum of two terms, including a and $a+(n-1)d$, the series reversed with evidence of adding, for example $2S=$ Condone the extra incorrect terms (see above) appearing.

Can be scored when terms are separated by commas

A1*: Shows correct work (no errors) with all steps shown leading to given answer.

There should be no incorrect terms. A minimum of 3 terms should be shown in each sum

The solution below is a variation of this.

$$S = a + (a+d) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + l$$

$$S = l + (l-d) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + a$$

$$2S = n(a+l)$$

$$S = \frac{n}{2}(a+l) = \frac{n}{2}(a+a+(n-1)d) = \frac{n}{2}(2a+(n-1)d)$$

B1 and A1 are not scored until the last line, M scored on line 3

The following scores B1 M0 A0 as the terms in the second sum are not reversed

(i) $S_n = a + (a+d) + (a+2d) \ldots\ldots a+(n-1)d$

$+$

$S_n = a + (a+(n-1)d + (a+(n-2)d) + a+(n-3)d$

$=$

$2S_n = 2a + (n-1)d + 2a+(n-1)d + 2a(n-1)d$

$2S_n = n\,[2a + (n-1)d]$

SC in (a) Scores B1 M0 A0.

They use $0+1+2+\ldots+(n-1) = \frac{n(n-1)}{2}$ which relies on the quoted proof.

13 i) a_n $a+d$ $a+2d$ (1)

$\sum_{p=1}^{n} a+(p-1)d$

$S_n = a + a+d + a+2d + \ldots\; a+(n-1)d$

$S_n = an + (0+1+2+3+\ldots\, n-1)d$

sum of 1 to $n-1$ = $\frac{n(n-1)}{2}$

$S_n = an + \frac{n(n-1)}{2}d$

$S_n = n\left(a + \frac{(n-1)d}{2}\right)$

$S_n = \frac{n}{2}(2a+(n-1)d)$

(ii) (a)

M1: Uses the information given to set up a correct equation in n.

The values of S, a and d need to be correct and used within a correct formula

Possible ways to score this include unsimplified versions $64 = \frac{n}{2}(2\times10+(n-1)\times-0.8)$,

$64 = \frac{n}{2}(10+10+(n-1)\times-0.8)$ or versions using pence rather than £'s $6400 = \frac{n}{2}(2000+(n-1)\times-80)$

Allow recovery for both marks following $64 = \frac{n}{2}(2\times10+(n-1)-0.8)$ with an invisible ×

A1*: Proceeds without error to the given answer. (Do not penalise a missing final trailing bracket)
Look for at least a line with the brackets correctly removed as well as a line with the terms in n correctly combined

E.g. $64 = \frac{n}{2}(20+(n-1)\times-0.8) \Rightarrow 64 = 10n - 0.4n^2 + 0.4n \Rightarrow 0.4n^2 - 10.4n + 64 = 0 \Rightarrow n^2 - 26n + 160 = 0$

(ii)(b)

B1: $n = 10, 16$

(ii)(c)

B1: Chooses 10 (weeks) and gives a minimal acceptable reason. The reason must focus on why the answer is 10 (weeks) rather than 16(weeks) or alternatively why it would not be 16 weeks.

(**Q13 9MA0/01, June 2022**)

Q7.

Question	Scheme	Marks	AOs
(a)	$\frac{1}{\sqrt{4-x}}=(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\times(1\pm......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n=1+nax+\frac{n(n-1)}{2}a^2x^2+$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x=-14$ and gives a valid reason. Eg explains that the expansion is not valid for $\|x\|>4$	B1	2.4
		(1)	
(b)(ii)	States $x=-\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
		(6 marks)	

(a)

M1: For the strategy of expanding $\frac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1 \pm ax)^n = 1 \pm nax \pm \frac{n(n-1)}{2}a^2x^2 +$

Condone sign slips and the "a" not being squared in term 3. Condone $a = \pm 1$
Look for an attempt at the correct binomial coefficient for their n, being combined with the correct power of ax

A1: $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$ unsimplified

FYI the simplified form is $1+\frac{x}{8}+\frac{3x^2}{128}$ Accept the terms with commas between.

A1: $\frac{1}{\sqrt{4-x}} = \frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

Note: Alternatively $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}4^{-\frac{5}{2}}(-x)^2 + ..$

M1: For $4^{-\frac{1}{2}} + ...$ **M1:** As above but allow slips on the sign of x and the value of n **A1:** Correct unsimplified (as above) **A1:** As main scheme

(b) Any evaluations of the expansions are irrelevant.
Look for a suitable value and a suitable reason for both parts.

(b)(i)

B1: Requires $x = -14$ with a suitable reason.

Eg. $x=-14$ as the expansion is only valid for $|x|<4$ or equivalent.

Eg '$x=-14$ as $|-14|>4$" or 'I cannot use $x=-14$ as $\left|\frac{-14}{4}\right|>1$'

Eg. '$x=-14$ as is outside the range $|x|<4$'

Do not allow '-14 is too big' or '$x=-14, |x|<4$' either way around without some reference to the validity of the expansion.

(b)(ii)

B1: Requires $x=-\frac{1}{2}$ with a suitable reason.

Eg. $x=-\frac{1}{2}$ as it is 'the smallest/smaller value' or '$x=-\frac{1}{2}$ as the value closest to zero' (that will give the more accurate approximation). The bracketed statement is not required.

(**Q04 9MA0/01, June 2019**)

Q8.

Question	Scheme	Marks	AOs
	${}^7C_4 a^3 (2x)^4$	M1	1.1b
	$\frac{7!}{4!3!} a^3 \times 2^4 = 15120 \Rightarrow a = \ldots$	dM1	2.1
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes:

M1: For an attempt at the correct coefficient of x^4.

The coefficient must have

- the correct binomial coefficient
- the correct power of a
- 2 or 2^4 (may be implied)

May be seen within a full or partial expansion.

Accept ${}^7C_4 a^3 (2x)^4$, $\frac{7!}{4!3!} a^3 (2x)^4$, $\binom{7}{4} a^3 (2x)^4$, $35a^3 (2x)^4$, $560a^3 x^4$, $\binom{7}{4} a^3 16x^4$ etc.

or ${}^7C_4 a^3 2^4$, $\frac{7!}{4!3!} a^3 2^4$, $\binom{7}{4} a^3 2^4$, $35a^3 2^4$, $560a^3$ etc.

or ${}^7C_3 a^3 (2x)^4$, $\frac{7!}{4!3!} a^3 (2x)^4$, $\binom{7}{3} a^3 (2x)^4$, $35a^3 (2x)^4$, $560a^3 x^4$, $\binom{7}{3} a^3 16x^4$ etc.

or ${}^7C_3 a^3 2^4$, $\frac{7!}{4!3!} a^3 2^4$, $\binom{7}{3} a^3 2^4$, $35a^3 2^4$, $560a^3$

You can condone missing brackets around the "2x" so allow e.g. $\frac{7!}{4!3!} a^3 2x^4$

An alternative is to attempt to expand $a^7 \left(1 + \frac{2x}{a}\right)^7$ to give $a^7 \left(\ldots \frac{7\times6\times5\times4}{4!} \left(\frac{2x}{a}\right)^4 \ldots\right)$

Allow M1 for e.g. $a^7 \left(\ldots \frac{7\times6\times5\times4}{4!} \left(\frac{2x}{a}\right)^4 \ldots\right)$, $a^7 \left(\ldots \binom{7}{4} \left(\frac{2x}{a}\right)^4 \ldots\right)$, $a^7 \left(\ldots 35 \left(\frac{2x}{a}\right)^4 \ldots\right)$ etc.

but condone missing brackets around the $\frac{2x}{a}$

Note that ${}^7C_3$, $\binom{7}{3}$ etc. are equivalent to ${}^7C_4$, $\binom{7}{4}$ etc. and are equally acceptable.

If the candidate attempts $(a + 2x)(a + 2x)(a + 2x)\ldots$ etc. then it must be a complete method to reach the required term. Send to review if necessary.

dM1: For $"560"a^3 = 15120 \Rightarrow a = \ldots$ Condone slips on copying the 15120 but their "560" must be an attempt at ${}^7C_4 \times 2$ or ${}^7C_4 \times 2^4$ and must be attempting the <u>cube root</u> of $\frac{15120}{"560"}$. **Depends on the first mark.**

A1: $a = 3$ and no other values i.e. ± 3 scores A0

Note that this is fairly common:

$${}^7C_4 a^3 2x^4 = 70a^3 x^4 \Rightarrow 70a^3 = 15120 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

and scores M1 dM1 A0

(**Q04 9MA0/02, Oct 2020**)

Q9.

Question	Scheme	Marks	AOs
(a)	$\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm 9x)^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-9x)^3$	M1	1.1b
	$(1-9x)^{\frac{1}{2}} = 1+\frac{1}{2}(-9x)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\pm 9x)^2+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-9x)^3$	A1	1.1b
	$(1-9x)^{\frac{1}{2}} = 1-\frac{9}{2}x-\frac{81}{8}x^2-\frac{729}{16}x^3$	A1	1.1b
		(3)	
(b)	Expansion is valid for $\lvert x\rvert < \frac{1}{9}$ and $x=-\frac{2}{9}$ is outside this range.	B1	2.4
		(1)	
			(4 marks)

Notes

(a)

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$ and obtains the correct structure for term 3 or term 4. Award for the correct coefficient with the correct power of x.

e.g. $\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ or $\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(\lambda x)^3$ where $\lambda \neq 1$

Condone missing or incorrect brackets around the x terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6.

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}, \begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$ unless these are interpreted correctly.

A1: Correct unsimplified expression as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work.

May be implied by a correct simplified expression.

OR allow this mark for at least 2 correct simplified terms from $-\frac{9}{2}x, -\frac{81}{8}x^2$ and $-\frac{729}{16}x^3$

A1: $1-\frac{9}{2}x-\frac{81}{8}x^2-\frac{729}{16}x^3$ or simplified equivalent. Correct answer with no working can score full marks. Ignore any extra terms and allow the terms to be listed or in a different order. Apply isw once a correct expansion is seen. Condone $+-$ (equivalent to listing).

Allow recovery if applicable e.g. if an "x" is lost then "reappears".

Allow decimal equivalents $1-4.5x-10.125x^2-45.5625x^3$ provided they are exact.

Allow mixed numbers $1-4\frac{1}{2}x-10\frac{1}{8}x^2-45\frac{9}{16}x^3$

Note: You may see attempts via direct expansion, but these will be scored using the main scheme, ignoring absence of powers on the 1s. The below attempts both score first M1A1.
If you are unsure, send to review.

$$(1-9x)^{\frac{1}{2}} = 1^{\frac{1}{2}} + \frac{1}{2}\left(1^{-\frac{1}{2}}\right)(-9x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(1^{-\frac{3}{2}}\right)(-9x)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(1^{-\frac{5}{2}}\right)(-9x)^3$$

$$9^{\frac{1}{2}}\left(\frac{1}{9}-x\right)^{\frac{1}{2}} = 3\left[\left(\frac{1}{9}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\left(\frac{1}{9}\right)^{-\frac{1}{2}}\right)(-x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\left(\frac{1}{9}\right)^{-\frac{3}{2}}\right)(-x)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\left(\frac{1}{9}\right)^{-\frac{5}{2}}\right)(-x)^3\right]$$

(b)

B1: Expansion is valid for $|x| < \frac{1}{9}$ or $|x| \leqslant \frac{1}{9}$ and $x = -\frac{2}{9}$ is outside this range.

Requires:

- an acceptable range of validity given
- an acceptable comparison of $-\frac{2}{9}$ or $\frac{2}{9}$ with their range leading to e.g. "not valid".

Examples of acceptable alternatives include:

- (Valid for) $|9x| < 1$ or $|9x| \leqslant 1$ and as $9x = -2$ (the expansion is) not valid.
- (Valid for) $|x| < \frac{1}{9}$ or $|x| \leqslant \frac{1}{9}$ and as $\frac{2}{9} > \frac{1}{9}$ $\left(\text{or } -\frac{2}{9} < -\frac{1}{9}\right)$ the expansion is not valid.
- (Valid for) $-\frac{1}{9} < x < \frac{1}{9}$ and $x = -\frac{2}{9}$ is too small/big (condoned as minimally acceptable).
- (Series converges for) $|-9x| \leqslant 1$ and as $-9x = 2$ the series will diverge.
- (Valid for) $|x| < \frac{1}{9}$ but $\left|-\frac{2}{9}\right| > \frac{1}{9}$ so $-\frac{2}{9}$ cannot be used.

Do not accept vague statements such as "it is too big", "it is outside the range" without any mention of what the range is. $-\frac{2}{9} < -\frac{1}{9}$ alone is insufficient evidence (without any mention of what the range is) and scores B0.
An attempt to evaluate the expansion and compare with $\sqrt{3}$ is not acceptable on its own.

(**Q02 9MA0/01, June 2024**)

Q10.

Question	Scheme	Marks	AOs
(a)	$\dfrac{4}{(2+3x)^2} = 4\times 2^{-2}\times\left(1+\dfrac{3}{2}x\right)^{-2}$	M1	3.1a
	$\left(1+\dfrac{3}{2}x\right)^{-2} =$ $1+(-2)\times\left(\dfrac{3x}{2}\right)+\dfrac{(-2)\times(-3)}{2!}\left(\dfrac{3x}{2}\right)^2+\dfrac{(-2)\times(-3)\times(-4)}{3!}\left(\dfrac{3x}{2}\right)^3$	M1 A1	1.1b 1.1b
	$\dfrac{4}{(2+3x)^2} = 1-3x+\dfrac{27}{4}x^2-\dfrac{27}{2}x^3$	A1	1.1b
		(4)	
(b)	$\lvert x\rvert < \dfrac{2}{3}$	B1	2.2a
		(1)	
			(5 marks)

Notes:

(a)

M1: Attempts to take out a factor of 2 from the bracket to obtain $k\left(1+\frac{3}{2}x\right)^{-2}$.

Condone slips/errors when taking out the factor of 2 as long as they reach the form $k\left(1+\frac{3}{2}x\right)^{-2}$.

M1: For an attempt on the binomial expansion of $(1+ax)^{-2}$.

Look for correct coefficients with correct powers of x for term 3 or term 4

$$\frac{(-2)\times(-3)}{2!}(ax)^2 \text{ or } \frac{(-2)\times(-3)\times(-4)}{3!}(ax)^3$$

where a could be 1 but must be consistent with a in their $(1+ax)^{-2}$.

Condone missing brackets around the "ax"

A1: Correct (may be unsimplified) expansion of $\left(1+\frac{3}{2}x\right)^{-2}$

A1: $1-3x+\frac{27}{4}x^2-\frac{27}{2}x^3$

Allow as a list but not e.g. $1+-3x+\frac{27}{4}x^2+-\frac{27}{2}x^3$

(b)

B1: $\lvert x\rvert<\frac{2}{3}$ oe e.g. $-\frac{2}{3}<x<\frac{2}{3}$ or $\left(-\frac{2}{3},\frac{2}{3}\right)$

(**Q06 9MA0/02/M, June 2025**)

Q11.

Question	Scheme	Marks	AOs
(a)	$\sqrt{4-9x}=2(1\pm...)^{\frac{1}{2}}$	B1	1.1b
	$\left(1-"\frac{9x}{4}"\right)^{\frac{1}{2}}=...+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left("-\frac{9x}{4}"\right)^2}{2!}$ or $...+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left("-\frac{9x}{4}"\right)^3}{3!}$	M1	1.1b
	$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{3!}$	A1	1.1b
	$\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$	A1	1.1b
		(4)	
(b)	States that the approximation will be an **overestimate** since all terms (after the first one) in the expansion are negative (since $x>0$)	B1	3.2b
		(1)	
			(5 marks)
Notes:			

(a)

B1: Takes out a factor of 4 and writes $\sqrt{4-9x}=2(1\pm...)^{\frac{1}{2}}$ or $\sqrt{4}(1\pm...)^{\frac{1}{2}}$ or $4^{\frac{1}{2}}(1\pm...)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion of $(1+ax)^{\frac{1}{2}}$ $a\neq 1$ to form term 3 or term 4 with the correct structure. Look for the correct binomial coefficient multiplied by the corresponding power of x e.g.

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(...x)^2 \text{ or } \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(...x)^3 \text{ where} ...\neq 1$$

Condone missing or incorrect brackets around the x terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}$, $\begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expression for the expansion of $\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$ e.g.

$$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)\left(\pm\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)\times\left(\frac{1}{2}-2\right)\left(-\frac{9x}{4}\right)^3}{3!}$$

which may be left unsimplified as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work. If the 2 outside this expansion is only partially applied to this expansion then score A0 but if it is applied to all terms this A1 can be implied.

OR at least 2 correct simplified terms **for the final expansion** from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

A1: $\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$ oe and condone e.g. $2+\frac{-9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:
$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$

Ignore any extra terms if found. Allow terms to be "listed" and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an "x" is lost then "reappears".

Direct expansion in (a) can be marked in a similar way:

$$\sqrt{4-9x} = (4-9x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \left(\frac{1}{2}\right)4^{-\frac{1}{2}} \times (-9x)^1 + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)4^{-\frac{3}{2}} \times \frac{(-9x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)4^{-\frac{5}{2}} \times \frac{(-9x)^3}{3!}$$

B1: For 2 or $\sqrt{4}$ or $4^{\frac{1}{2}}$ as the constant term in the expansion.

M1: Correct form for term 3 or term 4.

E.g. $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \times \frac{(...x)^2}{2!}$ or $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) \times \frac{(...x)^3}{3!}$ where $... \neq 1$

Condone missing brackets around the x terms but the binomial coefficients must be correct.
Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expansion (unsimplified as above)

OR at least 2 correct simplified terms from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

A1: $\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$ oe and condone e.g. $2 + \frac{-9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:
$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$

Ignore any extra terms if found. Allow terms to be "listed" and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an "x" is lost then "reappears".

(b)
B1: States that the approximation will be an **overestimate** due to the fact that all terms (after the first one) in the expansion are negative or equivalent statements e.g.

- Overestimate because the terms are negative
- Overestimate as the terms are being taken away (from 2)

Condone "overestimate as every term is negative"

If you think a response is worthy of credit but are unsure then use Review.
This mark depends on having obtained an expansion in (a) of the form
$k - px - qx^2 - rx^3 \quad k, p, q, r > 0$ but note that if e.g. one of the algebraic terms is zero or was "lost" or there are extra negative terms this mark is still available.

....

(**Q07 9MA0/02, June 2022**)

Q12.

Question	Scheme	Marks	AOs
(a)	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=3^{-2}(1+...x+...x^2)$	M1	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	M1	1.1b
	$\left(1+\frac{x}{3}\right)^{-2}=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	A1	1.1b
	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$	A1	2.1
		(4)	

(a)

M1: Attempts a binomial expansion by taking out a factor of 3^{-2} or $\frac{1}{3^2}$ or $\frac{1}{9}$ and achieves at least the first 3 terms in their expansion. May be seen separately e.g. $\frac{1}{9}$ and $(1+...x+...x^2)$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)(kx)$ or $\frac{(-2)(-2-1)}{2!}(kx)^2$ where $k \neq 1$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $\left(1+\frac{x}{3}\right)^{-2}$ e.g. $=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2-...$ or $1-\frac{2x}{3}+\frac{x^2}{3}-...$ Do not condone missing brackets unless they are implied by subsequent work.

Condone $\left(-\frac{x}{3}\right)^2$ for $\left(\frac{x}{3}\right)^2$

Also allow this mark for 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

<u>Direct expansion, if seen, should be marked as follows:</u>

$$\left((3+x)^{-2}=3^{-2}-2\times 3^{-3}\times x+\frac{-2(-2-1)}{2!}\times 3^{-4}\times x^2\right)$$

M1: For $(3+x)^{-2}=3^{-2}+3^{-3}\times \alpha x+3^{-4}\times \beta x^2$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)\times 3^{-3}x$ or $\frac{(-2)(-2-1)}{2!}\times 3^{-4}x^2$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $(3+x)^{-2}$ e.g. $3^{-2}-2\times 3^{-3}\times x+\frac{-2(-2-1)}{2!}\times 3^{-4}\times x^2$

Also award for at least 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

Note that M0M1A1A0 is a possible mark trait in either method

Note regarding a possible misread in parts (b) and (c)

Some candidates are misreading $\int \frac{6x}{(3+x)^2}\,dx$ in parts (b) and (c) as $\int \frac{6}{(3+x)^2}\,dx$

If parts (b) and (c) are consistently attempted with $\int \frac{6}{(3+x)^2}\,dx$ then we will allow the M marks in (b) **only**. **M1** for $x^n \to x^{n+1}$ applied to their expansion in part (a) or 6× (their expansion in part (a)) and **dM1** for substituting in 0.4 and 0.2 and subtracting either way round (may be implied).

No marks are available in part (c)

MARK PARTS (b) and (c) TOGETHER

(b)	$\int 6x\times"\left(\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}\right)"\,dx=\int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx=...$	**M1**	1.1b
	$\int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx=\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}$ oe	**A1**	1.1b
	$\left["\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}"\right]_{0.2}^{0.4}=\left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right)-\left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right)$	**dM1**	3.1a
	$=$ awrt 0.03304 or $\frac{223}{6750}$	**A1**	1.1b
		(4)	

(b)

M1: Attempts to multiply their expansion from part (a) by $6x$ or just x and attempts to integrate. Condone copying slips and slips in expanding. Look for $x^n \to x^{n+1}$ at least once having multiplied by $6x$ or x.
Ignore e.g. spurious integral signs.

A1: Correct integration, simplified or unsimplified.

$$\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18} \text{ oe e.g. } \frac{1}{9}\left(3x^2-\frac{4x^3}{3}+\frac{x^4}{2}\right),\ 6\left(\frac{x^2}{18}-\frac{2x^3}{81}+\frac{x^4}{108}\right)$$

If they have extra terms they can be ignored.
Ignore e.g. spurious integral signs.

dM1: An overall problem-solving mark for

- using part (a) by integrating $6x\times$ their binomial expansion and
- substituting in 0.4 and 0.2 and subtracting either way round (may be implied)

For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the limits 0.4 and 0.2.

This could be e.g. $\left[f(x)\right]_{0.2}^{0.4}=...$ provided the first M was scored. If the integration was correct, evidence can be taken from answer of awrt 0.0330 if limits are not seen elsewhere. **Depends on the first M mark.**

A1: awrt 0.03304 (NB allow the exact value which is $\frac{223}{6750}=0.033037037...$).

Isw following a correct answer.
Note answers which use additional terms in the expansion to give a different approximation score A0
Also note that the actual value is 0.032865…

Some may use integration by parts in (b) and the following scheme should be applied.

Integration by parts in (b):

Either by taking $u = 6x$ and $\frac{dv}{dx} = "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = 6x \times \left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - 6\int\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right)dx$$

$$= 6x\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - \left(\frac{1}{3}x^2 - \frac{2x^3}{27} + \frac{6x^4}{324}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = kx \times f(x) - k\int f(x)dx = kx \times f(x) - kg(x)$$

Where f(x) is an attempt to integrate their expansion from (a) with $x^n \to x^{n+1}$ at least once and g(x) is an attempt to integrate their f(x) with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

Or by taking $u = "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"$ and $\frac{dv}{dx} = 6x$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int 3x^2 \times \left(-\frac{2}{27} + \frac{2x}{27}\right)dx$$

$$= 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int\left(\frac{6x^3}{27} - \frac{6x^2}{27}\right)dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \left(\frac{6x^4}{108} - \frac{6x^3}{81}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = kx^2 \times f(x) - k\int x^2 g(x)dx = kx^2 \times f(x) - kh(x)$$

Where f(x) is their expansion from (a) and g(x) is an attempt to differentiate their f(x) with $x^n \to x^{n-1}$ at least once and h(x) is an attempt to integrate their $x^2 g(x)$ with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

(c)	Overall problem-solving mark (see notes)	M1	3.1a
	$u = 3 + x \Rightarrow \int_{3.2}^{3.4} f(u)\, du \Rightarrow \int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\, du = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2} du \Rightarrow \ldots\ln u + \ldots u^{-1}$	M1	1.1b
	$\int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\, du = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2} du \Rightarrow 6\ln u + 18u^{-1}$	A1	1.1b
	$\left[6\ln u + 18u^{-1}\right]_{3.2}^{3.4} = \left(6\ln 3.4 + \frac{18}{3.4}\right) - \left(6\ln 3.2 + \frac{18}{3.2}\right) = \ldots$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1
		(5)	
(c) Alt 1	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2}\, dx = \frac{\ldots x}{3+x} \pm \ldots\int(3+x)^{-1}dx = \frac{\ldots x}{3+x} \pm \ldots\ln(3+x)$ oe	M1	1.1b
	$= 6\ln(3+x) - \frac{6x}{3+x}$ oe	A1	1.1b
	$\left(6\ln(3+0.4) - \frac{6(0.4)}{3+0.4}\right) - \left(6\ln(3+0.2) - \frac{6(0.2)}{3+0.2}\right) = \ldots$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1

(c) Alt 2	Overall problem-solving mark (see notes)	**M1**	3.1a
	$\int 6x(3+x)^{-2}\ dx = \int\left(\frac{...}{(3+x)}+\frac{...}{(3+x)^2}\right)dx = ...\ln(3+x)+\frac{...}{3+x}$ oe	**M1**	1.1b
	$= 6\ln(3+x)+\frac{18}{3+x}$ oe	**A1**	1.1b
	$\left(6\ln(3+0.4)+\frac{18}{3+0.4}\right)-\left(6\ln(3+0.2)+\frac{18}{3+0.2}\right)=...$	**ddM1**	1.1b
	$6\ln\left(\frac{17}{16}\right)-\frac{45}{136}$ oe	**A1**	2.1
			(13 marks)

Notes

(c) There are various methods which can be used

M1: An overall problem-solving mark for <u>all of</u>

- using an appropriate integration technique e.g. substitution, by parts or partial fractions – note that this may not be correct but mark positively if they have tried one of these approaches
- integrates one of their terms to a natural logarithm, e.g., $\frac{a}{3+x}\to b\ln(3+x)$ or $\frac{a}{u}\to b\ln u$
- substitutes in correct limits and subtracts either way round

M1: Integrates to achieve an expression of the required form for their chosen method

- substitution: $u=x+3\to\pm\frac{a}{u}\pm b\ln u$ or e.g. $u=(x+3)^2\to\pm\frac{a}{\sqrt{u}}\pm b\ln u$
- parts: $\pm a\ln(3+x)\pm\frac{bx}{3+x}$ condone missing brackets e.g. $...\ln x+3$ for $...\ln(3+x)$
- partial fractions: $\pm a\ln(3+x)\pm\frac{b}{3+x}$ condone missing brackets e.g. $...\ln 3+x$ for $...\ln(3+x)$

A1: Correct integration for their method e.g.

- substitution: $u=x+3\to 6\ln u+18u^{-1}$ or e.g. $u=(x+3)^2\to 3\ln u+\frac{18}{\sqrt{u}}$
- parts: $6\ln(3+x)-\frac{6x}{3+x}$
- partial fractions: $6\ln(3+x)+\frac{18}{3+x}$ oe e.g. $3\ln(9+6x+x^2)+\frac{18}{3+x}$

Note that the above terms may appear "separated" but must be correct with the correct signs.
(ignore any reference to a constant of integration)
Do not condone missing brackets e.g. $6\ln x+3$ for $6\ln(3+x)$ unless they are implied by later work.

ddM1: Substitutes in the correct limits for their integral and subtracts either way round to find a value
Depends on both previous method marks.
For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the appropriate limits.
This could be e.g. $\left[f(x)\right]_{0.2}^{0.4}=...$ provided both previous M marks were scored.
Note that for substitution they may revert back to $3+x$ and so should be using 0.4 and 0.2

A1: A full and rigorous argument leading to $6\ln\left(\frac{17}{16}\right)-\frac{45}{136}$ or exact equivalent e.g. $3\ln\left(\frac{289}{256}\right)-\frac{45}{136}$ or e.g. $-6\ln\left(\frac{16}{17}\right)-\frac{45}{136}$

The brackets are not required around the $\frac{17}{16}$ and allow exact equivalents e.g. allow 1.0625 or $1\frac{1}{16}$ but not e.g. $\frac{3.4}{3.2}$. The $\frac{45}{136}$ must be exact or an exact equivalent. Also allow e.g. $6\ln\left|\frac{17}{16}\right|-\frac{45}{136}$

Ignore spurious integral signs that may appear as part of their solution.

(**Q13 9MA0/02, June 2023**)

Q13.

Question	Scheme	Marks	AOs
(a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{'1.0658'^4}$ or $A = \frac{50000}{'1.0658'^{11}}$	M1	1.1b
	$A = 24795 \rightarrow 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / (£) 24 800 is the value of the car on 1st January 2001	B1	3.4
	p / 1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p =$ awrt 1.0658

Both marks can be awarded from incorrect but consistent interpretations of t. Eg.

$32000 = Ap^5$, $50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ' =24 800' or ' ≈ 24800 '. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

A1: 24800×1.0658^4, achieves a value between 31095 and 32005 followed by $\approx 32\,000$ hence A must be $\approx 24\,800$

(b)

B1: States that A is the value of the car on 1st January 2001.
The statement must reference **the car**, its **cost/value**, and **"0" time**
Allow 'it is the initial value of the car" "it is the cost of the car at $t = 0$" "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.
The statement must reference **a yearly rate** and **an increase in value or multiplier**.
They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."
Allow "p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow ' the value of the car multiplies by p each year'
Do not allow "by how much the value of the car rises each year " or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = '24800' \times '1.0658'^t$ and proceeds to their $'1.0658'^t = k$
Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model **and** (ii) translating the situation into a correct method to find 't'

A1: $(t) =$ awrt 21.8 or 21.9 or $\log_{1.0658}\left(\dfrac{100000}{24800}\right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks

Allow different methods in part (c).

Eg Via GP a formula

M1: $24800\times'1.0658'^{n-1} = 100000 \Rightarrow '1.0658'^{n-1} = K$

dM1: Uses a correct method to find n.

A2: 2022

Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\,000$ or $t = n+1, V > 100\,000$

(So for the correct A and p this would be scored for $t = 21, V \approx £95000$ or $t = 21, V \approx £101\,000$

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest £1000 is sufficient accuracy)

A1: As before

(**Q12 9MA0/01, June 2018**)

Q14.

Question	Scheme	Marks	AOs
(a)(i)	e.g. $(u_2 =)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1 = 40$ *	B1*	2.1
(ii)	$u_3 = 40+7\cos\left(\frac{2\pi}{2}\right)-5(-1)^2\ (=28)$ **or** $u_4 = "28"+7\cos\left(\frac{3\pi}{2}\right)-5(-1)^3\ (=33)$	M1	1.1b
	$u_3 = 28$ and $u_4 = 33$	A1	1.1b
		(3)	
(b)(i)	$(u_5 =)35$	B1	2.2a
(ii)	e.g. $\sum_{r=1}^{25} u_r = 6(35+40+"28"+"33")+35$	M1	3.1a
	$=851$	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)

(i)

B1*: Correct application of the formula with $n = 1$ and proceeds correctly to achieve an answer of 40 with no errors. Note that e.g., $(u_2 =)35+7\cos\left(\frac{35\pi}{2}\right)-5(-1)^{35} = 35+0+5 = 40$ scores B0

As a minimum need to see e.g. $(u_2 =)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1 = 40,\quad 35+0+5=40,\quad 35+5=40,\quad 35-5(-1)^1=40$

(ii)

M1: A correct attempt to use the formula to find a value for u_3 or u_4

Look for $n = 2$ substituted correctly into the given formula with $u_2 = 40$. May be implied by $u_3 = 28$

Or their calculated value of u_3 used with $n = 3$ substituted correctly into the given formula to find u_4

Condone use of calculator in degree mode which gives $u_3 = 41.989...$ which may imply this mark if no working is shown. If there is **no** working and u_3 is incorrect and u_4 is correct score M0A0

A1: Both correct $u_3 = 28$ and $u_4 = 33$ If 28, 33 are listed then allow M1A1.

For both correct values only score M1A1

(b)(i)

B1: $(u_5 =)35$

(ii)

M1: Attempts a <u>**correct**</u> method to find $\sum_{r=1}^{25} u_r$

There are various ways e.g. attempts to add 35 to $6\times$the sum of their four values.

Some other examples are:

$\sum_{r=1}^{25} u_r = 7\times35+6\times40+6\times"28"+6\times"33"$, $\sum_{r=1}^{25} u_r = 7(35+40+"28"+"33")-(40+"28"+"33")$,

$\sum_{r=1}^{25} u_r = \frac{25}{4}(35+40+"28"+"33")+1$, $2(35+40+"28"+"33")=272$, $272\times3=816$, $816+35$

There may be other methods seen but the calculation must be correct for their values.

If there is no working, with incorrect u_3 and/or u_4 you will need to check if their answer implies a correct method using $6(35+40+"28"+"33")+35$

Attempts to use an AP/GP formula score M0

A1: 851 (Correct answer with no working scores both marks)

(**Q02 9MA0/02, June 2023**)

Q15.

Question	Scheme	Marks	AOs
(a)	$u_1 = 6 \Rightarrow u_2 = 6k - 5$ $u_2 = 6k - 5 \Rightarrow u_3 = k(6k-5) - 5$ $\Rightarrow k(6k-5) - 5 = -1$	M1	1.1b
	$\Rightarrow 6k^2 - 5k - 4 = 0$ *	A1*	2.1
		(2)	
	Alternative:		
	$u_3 = -1 \Rightarrow -1 = ku_2 - 5 \Rightarrow u_2 = \frac{4}{k}$ $u_1 = 6 \Rightarrow u_2 = 6k - 5 \Rightarrow \frac{4}{k} = 6k - 5$	M1	1.1b
	$\Rightarrow 6k^2 - 5k - 4 = 0$ *	A1*	2.1
(b)(i)	$k = \frac{4}{3}$	B1	2.2a
(ii)	$k = \frac{4}{3} \Rightarrow u_2 = \frac{4}{3} \times 6 - 5 \Rightarrow \sum_{r=1}^{3} u_r = 6 + \frac{4}{3} \times 6 - 5 - 1$	M1	1.1b
	$\sum_{r=1}^{3} u_r = 8$	A1	1.1b
		(3)	
			(5 marks)

Notes
(a) **M1:** Correct application of the given recurrence relation using $u_1 = 6$ to find u_2 and then u_3 in terms of k and sets $u_3 = -1$ Condone missing brackets if the intention is clear e.g. $u_2 = 6k - 5 \Rightarrow u_3 = k\,6k - 5 - 5$ **A1*:** Obtains the printed answer with no errors including the "= 0" This is a <u>given answer</u> so do not condone slips/missing brackets unless they are recovered **before** the final printed answer. **Alternative:** **M1:** Correct application of the given recurrence relation using $u_3 = -1$ to find u_2 in terms of k and then uses $u_1 = 6$ to find another expression for u_2 in terms of k and equates the 2 expressions. **A1*:** Obtains the printed answer with no errors including the "= 0" This is a <u>given answer</u> so do not condone slips unless they are recovered **before** the final printed answer. (b)(i) **B1:** Deduces the correct value of k. Ignore any working and just look for this value. Allow equivalent exact values e.g. $1\frac{1}{3}$ or $1.\dot{3}$ but not clearly rounded e.g. 1.333 It must be clear that $k = \frac{4}{3}$ is selected so if both roots are offered score B0 unless $k = \frac{4}{3}$ is clearly intended by the calculation in part (ii)

(ii)

M1: Attempts the second term by e.g. $(\text{their } k)\times 6-5$ and then adds 6 and -1 to their second term. E.g. $6+"\frac{4}{3}"\times 6-5-1$

If they use u_1 and u_3 they must be as given in the question but condone a clear mis-copy of their u_2 value.

The attempt at the second term may be implied by their value.

Note that they may use $u_3=-1$ to find u_2 e.g. $-1="\frac{4}{3}"u_2-5\Rightarrow u_2="\frac{3}{4}"(5-1)=3$

Condone slips when rearranging as long as the intention is clear.

The attempt at the second term may be seen embedded in their attempt at the sum e.g.

$\sum_{r=1}^{3}u_r=6+\underline{\frac{4}{3}\times 6-5}-1$ or e.g. $\sum_{r=1}^{3}u_r=6+\underline{\frac{3}{4}(5-1)}-1$

If they use both of their values for k allow M1.

Alternatives:

Note that $\sum_{r=1}^{3}u_r=6+6k-5-1=6k$ so you may just see an attempt at $6k$ with their $\frac{4}{3}$.

Note that $\sum_{r=1}^{3}u_r=6k^2+k-4$ so you may just see an attempt at $6k^2+k-4$ with their $\frac{4}{3}$.

A1: Correct value of 8 and no other values unless rejected.

Correct answer with no working scores both marks.

Allow recovery from an inexact value from part (i) e.g. 1.333

(**Q04 9MA0/02, June 2024**)

Q16.

Question	Scheme	Marks	AOs
	Uses the given sequence formula correctly at least once $a_2 = 4k+3,\ a_3 = k(4k+3)+3$	M1	1.1b
	$\sum_{n=1}^{3} a_n = 12 \Rightarrow 4+4k+3+k(4k+3)+3=12$	dM1	3.1a
	$4k^2+7k-2=0$ $(4k-1)(k+2)=0$	ddM1	1.1b
	$k=-2$ only	A1	2.2a
		(4)	
			(4 marks)

Notes:

M1: Uses the given sequence formula at least once

dM1: Dependent upon having used the sequence formula at least once, scored for setting $4 + \text{their } a_2 + \text{their } a_3 = 12$ where a_2 and a_3 are in terms of k.

ddM1: Dependent upon both previous marks. It is for solving a 3 term quadratic equation in k by any correct method including a calculator.

A1: cso $k=-2$ only. If $k=\frac{1}{4}$ is written down it must be rejected.

Must be $k = \ldots$ not $x = \ldots$

Note this is quite common:

$$a_2 = 4k+3,\ a_3 = k(4k+3)$$

$$\sum_{n=1}^{3} a_n = 12 \Rightarrow 4+4k+3+k(4k+3)=12 \Rightarrow 4k^2+7k-5=0$$

$$\Rightarrow k = \frac{-7\pm\sqrt{129}}{8}\left(=0.544,\ -2.29\right)$$

And scores M1dM1ddM1A0

(**Q03 9MA0/02/M, June 2025**)

Q17.

Question	Scheme	Marks	AOs
(a)	$\dfrac{3\sin 2\theta}{8\sin\theta}=\dfrac{2+2\cos 2\theta}{3\sin 2\theta}$ o.e. e.g. $(3\sin 2\theta)^2=8\sin\theta(2+2\cos 2\theta)$	M1	3.1a
	$\dfrac{3\times 2\sin\theta\cos\theta}{8\sin\theta}=\dfrac{2+2(2\cos^2\theta-1)}{3\times 2\sin\theta\cos\theta}=\dfrac{4\cos^2\theta}{3\times 2\sin\theta\cos\theta}$ or e.g. $(3\times 2\sin\theta\cos\theta)^2=8\sin\theta(2+2(2\cos^2\theta-1))=32\sin\theta\cos^2\theta$	dM1 A1	2.1 1.1b
	$\dfrac{6\sin\theta\cos\theta}{8\sin\theta}=\dfrac{4\cos^2\theta}{6\sin\theta\cos\theta}\Rightarrow\dfrac{6\cancel{\cos\theta}}{8}=\dfrac{4\cancel{\cos\theta}}{6\sin\theta}\Rightarrow\sin\theta=\dfrac{32}{36}=\dfrac{8}{9}$* or e.g. $36\sin^2\theta\cos^2\theta=32\sin\theta\cos^2\theta\Rightarrow 36\sin\theta=32\Rightarrow\sin\theta=\dfrac{8}{9}$*	A1*	2.1
		(4)	

M1: For the key step in using the ratio of $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}$

Allow any valid attempt E.g. $a_3=a_1\times\left(\dfrac{a_2}{a_1}\right)^2$, $a_2^2=a_1\times a_3$

dM1: For using $\sin 2\theta=2\sin\theta\cos\theta$ and $\cos 2\theta=2\cos^2\theta-1$ to eliminate the constant term.

A1: For any correct equation in single angles without a constant term e.g.

$$\frac{6\sin\theta\cos\theta}{8\sin\theta}=\frac{4\cos^2\theta}{6\sin\theta\cos\theta}\text{ or }(3\times 2\sin\theta\cos\theta)^2=32\sin\theta\cos^2\theta$$

A1*: Proceeds to the given answer with accurate and sufficient work.
There is no requirement to justify why $\sin\theta\neq 0$ or $\cos\theta\neq 0$

	(a) Alternative:		
	$\dfrac{3\sin 2\theta}{8\sin\theta}=\dfrac{2+2\cos 2\theta}{3\sin 2\theta}$ o.e. e.g. $(3\sin 2\theta)^2=8\sin\theta(2+2\cos 2\theta)$	M1	3.1a
	$\dfrac{3\times 2\sin\theta\cos\theta}{8\sin\theta}=\dfrac{2+2(1-2\sin^2\theta)}{3\times 2\sin\theta\cos\theta}$ $\Rightarrow 36\sin\theta\cos^2\theta=32-32\sin^2\theta$ $\Rightarrow 36\sin\theta(1-\sin^2\theta)=32-32\sin^2\theta$	dM1 A1	2.1 1.1b
	$\Rightarrow 36\sin^3\theta-32\sin^2\theta-36\sin\theta+32=0$ $36\sin^3\theta-32\sin^2\theta-36\sin\theta+32=0\Rightarrow\sin\theta=\dfrac{8}{9}$*	A1*	2.1

M1: As above

dM1: For using $\sin 2\theta=2\sin\theta\cos\theta$, $\cos 2\theta=1-2\sin^2\theta$ and $\cos^2\theta=1-\sin^2\theta$ and proceeding to a cubic (or quartic) equation in just $\sin\theta$

A1: Correct equation in $\sin\theta$ e.g. $36\sin\theta(1-\sin^2\theta)=32-32\sin^2\theta$

A1*: Solves their cubic (or quartic) by any means (including a calculator) to obtain $\sin\theta=\dfrac{8}{9}$
If the other roots (1 and −1) are found, they must be rejected.

Mark (b) and (c) together.

(b)	$r=\dfrac{3\sin 2\theta}{8\sin\theta}=\dfrac{6\cos\theta}{8}=\dfrac{6\cos(1.0949...)}{8}=0.344$	M1	3.1a
	$r=\text{awrt } 0.344$ so $\lvert r\rvert<1$ so S_∞ exists	A1	2.4
		(2)	
(c)	$"a"=8\sin\theta=\dfrac{64}{9}$	B1	2.2a
	Attempts $\dfrac{a}{1-r}-\dfrac{a\left(1-r^{10}\right)}{1-r}=\dfrac{ar^{10}}{1-r}=\dfrac{\frac{64}{9}\times"0.344"^{10}}{1-"0.344"}$	M1	1.1b
	$S_\infty-S_{10}=\text{awrt } 2.5\times10^{-4}$	A1	1.1b
		(3)	
			(9 marks)

(b)

M1: Attempts to find the value of r with $\sin\theta=\dfrac{8}{9}$

Alternatively states that $r=\dfrac{3}{4}\cos\theta$ o.e and states $r<1$

A1: Achieves $r=\text{awrt } 0.344$ and states that as $\lvert r\rvert<1$ S_∞ exists

In the alternative gives a reason e.g. as $\lvert r\rvert<1$ S_∞ exists

Some values for reference:

$$r=\frac{\sqrt{17}}{12}(=0.34359...),\ a_1=\frac{64}{9}(=7.111...),\ a_2=\frac{16\sqrt{17}}{27}(=2.443...),\ a_3=\frac{68}{81}(=0.8395...)$$

(c)

B1: Deduces the value of a to be $\dfrac{64}{9}$ but condone awrt 7.11

M1: Attempts $S_\infty-S_{10}$ using correct formulae and with their values of a and r

A1: awrt 2.5×10^{-4}

(**Q13 9MA0/02/M, June 2025**)

Q18.

Question	Scheme	Marks	AOs
(a)	Uses the sequence formula $a_{n+1}=\frac{k(a_n+2)}{a_n}$ once with $a_1=2$	M1	1.1b
	$(a_1=2)$, $a_2=2k$, $a_3=k+1$, $a_4=\frac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\frac{k(k+3)}{k+1}=2 \Rightarrow k^2+3k=2k+2 \Rightarrow k^2+k-2=0$ *	A1*	2.1
		(3)	
(b)	States that when $k=1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4}=2, a_{2/5}=-4, a_{3/6}=-1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26\times(2+-4+-1)+2+-4$	M1	3.1a
	$=-80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

(a)

M1: Applies the sequence formula $a_{n+1} = \dfrac{k(a_n + 2)}{a_n}$ seen once.

This is usually scored in attempting to find the second term. E.g. for $a_2 = 2k$ or $a_{1+1} = \dfrac{k(2+2)}{2}$

M1: Attempts to find $a_1 \to a_4$ and sets $a_1 = a_4$. Condone slips.

Other methods are available. E.g. Set $a_4 = 2$, work backwards to find a_3 and equate to $k+1$

There is no requirement to see either a_1 or any of the labels. Look for the correct terms in the correct order.

There is no requirement for the terms to be simplified

FYI $a_1 = 2, a_2 = 2k, a_3 = k+1, a_4 = \dfrac{k(k+3)}{k+1}$ and so $2 = \dfrac{k(k+3)}{k+1}$

A1*: Proceeds to the given answer with accurate work showing all necessary lines. See MS for minimum

(b)

B1: States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3.

Do not accept "the terms just repeat" or "it would mean all the terms of the sequence are 2"

There must be some reference to the fact that it does not have order 3. Accept it has order 1.

It is acceptable to state $a_2 = a_1 = 2$ and state that the sequence does not have order 3

(c)

B1: Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$

M1: Uses a clear strategy to find the sum to 80 terms. This will usually be found using multiples of the first three terms.

For example you may see $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{78} a_r\right) + a_{79} + a_{80} = 26\times(2+-4+-1)+2+-4$

or $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{81} a_r\right) - a_{81} = 27\times(2+-4+-1)-(-1)$

For candidates who find in terms of k award for $27\times 2 + 27\times(2k) + 26\times(k+1)$ or $80k + 80$

If candidates proceed and substitute $k = -2$ into $80k + 80$ to get -80 then all 3 marks are scored.

A1: -80

Note: Be aware that we have seen candidates who find the first three terms correctly, but then find

$26\frac{2}{3}\times(2+-4+-1) = 26\frac{2}{3}\times -3$ which gives the correct answer

but it is an incorrect method and should be scored B1 M0 A0

(**Q13 9MA0/01, Oct 2020**)

Q19.

Question	Scheme	Marks	AOs
(a)	$u_2 = k-12,\ u_3 = k - \frac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \frac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \frac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0$ *	A1*	2.1
		(3)	
(b)	$k = 6, \left(\frac{40}{3}\right)$	M1	1.1b
	$k = 6$ as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)10$	B1	2.2a
		(1)	
			(6 marks)
Notes			

(a)

M1: Attempts to apply the sequence formula once for either u_2 or u_3.

Usually for $u_2 = k - \frac{24}{2}$ o.e. but could be awarded for $u_3 = k - \frac{24}{\text{their } "u_2"}$

dM1: Award for

- attempting to apply the sequence formula to find both u_2 and u_3
- using $2 + 2"u_2" + "u_3" = 0 \Rightarrow$ an equation in k. The u_3 may have been incorrectly adapted

A1*: Fully correct work leading to the printed answer.

There must be

- (at least) one correct intermediate line between $2 + 2(k-12) + k - \frac{24}{k-12} = 0$ (o.e.) and the given answer that shows how the fractions are "removed". E.g. $(3k-22)(k-12) - 24 = 0$
- no errors in the algebra. The = 0 may just appear at the answer line.

(b)

M1: Attempts to solve the quadratic which is implied by sight of $k = 6$.

This may be awarded for any of

- $3k^2 - 58k + 240 = (ak \pm c)(bk \pm d) = 0$ where $ab = 3, cd = 240$ followed by $k =$
- an attempt at the correct quadratic formula (or completing the square)
- a calculator solution giving at least $k = 6$

A1: Chooses $k = 6$ and gives a minimal reason

Examples of a minimal reason are

- 6 because it is an integer
- 6 because it is a whole number
- 6 because $\frac{40}{3}$ or $13.\dot{3}$ is not an integer

(c)

B1: Deduces the correct value of u_3.

(**Q03 9MA0/01, Oct 2021**)

Q20.

Question	Scheme	Marks	AOs
(a)	$u_3 = £20000 \times 1.08^2 = (£)23328*$	B1*	1.1b
		(1)	
(b)	$20000 \times 1.08^{n-1} > 65000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1*: Uses a correct method to show that the Profit in Year 3 will be £23 328. Condone missing units

E.g. $£20000 \times 1.08^2$ or $£20000 \times 108\% \times 108\%$

This may be obtained in two steps. E.g $\frac{8}{100} \times 20000 = 1600$ followed by $\frac{8}{100} \times 21600 = 1728$ with the calculations $21600 + 1728 = 23328$ seen.

Condone calculations seen as 8% of $20000 = 1600$.

This is a show that question and the method must be seen.

It is not enough to state Year 1 = £21 600, Year 2 = £ 23 328

(b)

M1: Sets up an inequality or an equation that will allow the problem to be solved.

Allow for example N or n for $n-1$. So award for $20\,000\times1.08^{n-1}>65\,000$, $20\,000\times1.08^{n}=65\,000$ or $20\,000\times(108\%)^{n}\geqslant 65\,000$ amongst others.

Condone slips on the 20 000 and 65 000 but the 1.08 o.e. must be correct

M1: Uses a correct strategy involving logs in an attempt to solve a type of equation or inequality of the form seen above. It cannot be awarded from a sum formula

The equation/inequality must contain an index of $n-1$, N, n etc.

Again condone slips on the 20 000 and 65 000 but additionally condone an error on the 1.08, which may appear as 1.8 for example

E.g. $20000\times1.08^{n}=65000\Rightarrow n\log1.08=\log\dfrac{65000}{20000}\Rightarrow n=...$

E.g. $2000\times1.8^{n}=65000\Rightarrow\log2000+n\log1.8=\log65000\Rightarrow n=...$

A1: Interprets their decimal value and gives the correct year number. Year 17

The demand of the question dictates that solutions relying entirely on calculator technology are not acceptable, BUT allow a solution that appreciates **a correct term** formula or the entire set of calculations where you may see the numbers as part of a larger list

E.g. Uses, or implies the use of, an acceptable calculation and finds value(s)

for M1: $(n=16)\Rightarrow P=20000\times1.08^{15}$=awrt 63400 **or** $(n=17)\Rightarrow P=20000\times1.08^{16}$ = awrt 68500

M1: $(n=16)\Rightarrow P=20000\times1.08^{15}$=awrt 63400 **and** $(n=17)\Rightarrow P=20000\times1.08^{16}$ = awrt 68500

A1: 17 years following correct method and both M's

(c)

M1: Attempts to use the model with a **correct** sum formula to find the total profit for the 20 years.

You may see an attempt to find the sum of 20 terms via a list. This is acceptable provided there are 20 terms with $u_n=1.08\times u_{n-1}$ seen at least 4 times and the sum attempted.

Condone a slip on the 20 000 (e.g appearing as 2 000) and/or a slip on the 1.08 with it being the same "r" as in (b) . Do not condone 20 appearing as 19 for instance

A1: awrt £915 000 but condone missing unit

The demand of the question dictates that all stages of working should be seen. An answer without working scores M0 A0

(**Q05 9MA0/01, Oct 2021**)

Q21.

Question	Scheme	Marks	AOs
	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=\ldots$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
		(3)	

	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=\ldots$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-\ldots$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+\ldots-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-\ldots$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+\ldots=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-\ldots=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-\ldots$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\ldots\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=\ldots$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
			(3 marks)

Notes
B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be seen as part of them writing down the sequence but must be the **first** term. M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a=\frac{9}{16}$ and $r=\pm\frac{3}{4}$ A1*: Correct proof **Alternative 1:** B1: Deduces the correct value for the sum to infinity (starting at $n=1$) or the common ratio M1: Calculates the required value by subtracting the first term from their sum to infinity A1*: Correct proof **Alternative 2:** B1: Deduces the correct value of the **first** term or the common ratio. M1: Splits the series into "odds" and "evens", attempts the sum of both parts and calculates the required value by adding both sums A1*: Correct proof **Alternative 3:** B1: Deduces the correct value of the **first** term M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for "S" A1*: Correct proof

(**Q09 9MA0/02, Oct 2021**)

Q22.

Question	Scheme	Marks	AOs
(a)	$\frac{12-3k}{3k+4}=\frac{k+16}{12-3k}$	M1	3.1a
	$3k^2-62k+40=0$ *	A1*	1.1b
		(2)	
(b)(i)	$3k^2-62k+40=0 \Rightarrow k=...$	M1	1.1b
	States $k=20$ and gives a reason e.g. that this gives a values of r such that $\lvert r\rvert<1$	A1	3.2a
(ii)	$a=64$ **and** $r=-\frac{3}{4}$ (or allow $a=6$ **and** $r=\frac{5}{3}$)	B1	1.1b
	$S_\infty=\frac{"64"}{1-"\left(-\frac{3}{4}\right)"}=...$	M1	3.1a
	$S_\infty=\frac{256}{7}$	A1	1.1b
		(5)	
			(7 marks)

Notes
(a) M1: Forms a correct equation linking the three terms. Condone invisible brackets if implied by further work. Possible equations below (which are not exhaustive) should use *n*th term or sum of series formulae e.g. $\frac{12-3k}{3k+4}=\frac{k+16}{12-3k}$ or $\left(\frac{12-3k}{3k+4}\right)^2=\frac{k+16}{3k+4}$ or $(12-3k)^2=(3k+4)(k+16)$ or $(3k+4)\left(\frac{k+16}{12-3k}\right)=12-3k$ or $(12-3k)\left(\frac{12-3k}{k+16}\right)=k+16$ or $3k+4+12-3k+k+16=\frac{(3k+4)\left(1-\left(\frac{k+16}{12-3k}\right)^3\right)}{1-\frac{k+16}{12-3k}}$ (sum of three terms) A1*: Achieves the given quadratic with no errors including invisible brackets. It cannot be for just proceeding in one step from the starting equation to the given answer and usually will involve attempting to multiply out brackets or dealing with any fractions.
(b)(i) M1: Attempts to solve the given quadratic achieving at least one value for *k*. Usual rules apply for solving a quadratic and this may be achieved directly from a calculator. (May also be implied by $\frac{2}{3}$)

A1:	20 and gives correct reasoning (**if r is found anywhere in part (i) then it must be correct**): e.g. 20 since $\lvert r \rvert < 1$. e.g. since $\lvert r \rvert = 0.75 < 1$ e.g. by listing **at least two consecutive** terms for $k = 20$ (must be correct) e.g. 64, -48 do not withhold this mark if they proceed to make a comment e.g. "the numbers are getting smaller" as we are condoning this to mean they are referring to the magnitude of the numbers e.g. when $k = 20$, $r = -\frac{3}{4}$ o.e. which is between 1 and -1 (condone "it is smaller than 1"). Do not accept a reason on its own which is just simply stating that the sequence is converging or equivalent such as "spiralling". Allow reasoning which excludes $k = \frac{2}{3}$ e.g. $r = \frac{5}{3}$ which is greater than 1.
(ii)	
Work may be seen in part (i), but must be used in part (ii) to score.	
B1:	$a = 64$ **and** $r = -\frac{3}{4}$ o.e. (or allow $a = 6$ **and** $r = \frac{5}{3}$ o.e.) May be implied by later work or a correct calculation using these values to find S_∞
M1:	A full attempt to find S_∞ by using their value of k to reach a value for r such that $\lvert r \rvert < 1$ and a value for a. Condone sign slips in their calculations of a and r only. You may need to check this by substituting in their value for k if no calculations are seen. They must substitute these values in to $\frac{a}{1-r}$ correctly so e.g. $a = 64$, $r = -\frac{3}{4} \Rightarrow S_\infty = \frac{64}{1-\frac{3}{4}}$ is M0. They cannot just substitute in their k as r in the formula. Do not allow attempts to manually calculate the values of lots of terms for this mark as this would not lead to the answer. $\sum_{n=1}^{\infty} 64 \times \left(-\frac{3}{4}\right)^{n-1}$ on its own is M0.
A1:	$\frac{256}{7}$ cao. (Do not allow 36.6 as this is not S_∞) isw after a correct exact answer is seen.

(**Q09 9MA0/01, June 2023**)

Q23.

Question	Scheme		Marks	AOs
(a)	$\dfrac{9^{7-2k}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \dfrac{3^{2(7-2k)}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{3^{2(7-2k)}}$		M1	3.1a
	$\left(3^{2(7-2k)}\right)^2=3^{4k-5}\times 3^{2(k-1)}$ $\Rightarrow 28-8k=6k-7\Rightarrow k=...$	$3^{2(7-2k)-(4k-5)}=3^{2(k-1)-2(7-2k)}$ $\Rightarrow 19-8k=6k-16\Rightarrow k=...$	dM1	1.1b
	$k=\dfrac{5}{2}$*		A1*	2.1
			(3)	
(b)	$a=3^{4(2.5)-5}$ and $r=\dfrac{9^{7-2(2.5)}}{3^{4(2.5)-5}}\Rightarrow$ one of $a=243$ or $r=\dfrac{1}{3}$		M1	2.2a
	$S_\infty=\dfrac{a}{1-r}=\dfrac{"243"}{1-"\frac{1}{3}"}$		M1	1.1b
	$S_\infty=\dfrac{729}{2}(364.5)$ cao		A1	1.1b
			(3)	
				(6 marks)

Notes

(a) **Special cases:**

SC 1: For those that verify rather than prove a SC 100 is awarded for substituting $k=\dfrac{5}{2}$ into all three terms to correctly obtain 243, 81 and 27 **with** a statement that this is **geometric** with $r=\dfrac{1}{3}$ (or equivalent reason). All statements must be correct.

SC 2: Be aware that e.g. $\dfrac{9^{7-2k}}{3^{4k-5}}=\dfrac{3^{2(k-1)}}{9^{7-2k}}\Rightarrow 81^{2(7-2k)}=9^{6k-7}$ is an incorrect process (without some indication that they have intentionally squared both sides) that fortuitously leads to the correct answer and may score maximum SC 010.

M1: Uses the 3 terms to set up an equation in k **and**

- **either** reaches a common base by replacing 9 with 3^2 or by replacing 3 with $9^{0.5}$ and uses the power law of indices correctly
- **or** uses the laws of indices correctly to reach $9^{14-4k}=3^{6k-7}$ condoning slips in e.g. expanding brackets.

Writing down e.g. $2(7-2k)-(4k-5)=2(k-1)-2(7-2k)$ is sufficient to imply the M1.

dM1: Correct processing leading to a value for k.

A1*: Correct value following correct working. Allow $k=2.5$ in place of $k=\dfrac{5}{2}$

Condone missing/invisible brackets provided they are recovered correctly.

Alt 1 Using Base 9:

$$\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}}\Rightarrow\frac{9^{7-2k}}{9^{2k-2.5}}=\frac{9^{k-1}}{9^{7-2k}}\text{ o.e. scores M1}$$

$$\Rightarrow 9^{9.5-4k}=9^{3k-8}\Rightarrow 9.5-4k=3k-8\Rightarrow 7k=17.5\Rightarrow k=2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 2 Finding r in terms of k and using e.g. $u_3 = ar^2$:

$$r = \frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(7-2k)}}{3^{4k-5}} \left\{= 3^{19-8k}\right\} \text{ or } r = \frac{3^{2(k-1)}}{9^{7-2k}} = \frac{3^{2k-2}}{3^{2(7-2k)}} \left\{= 3^{6k-16}\right\}$$

$$\Rightarrow 3^{4k-5} \times \left(3^{19-8k}\right)^2 = 3^{2k-2} \text{ or } \Rightarrow 3^{4k-5} \times \left(3^{6k-16}\right)^2 = 3^{2k-2} \text{ scores M1}$$

$$\Rightarrow 3^{4k-5} \times 3^{2(19-8k)} = 3^{2k-2} \Rightarrow 33 - 12k = 2k - 2 \Rightarrow 14k = 35 \Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 3 Using Logs Way 1:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \left(9^{7-2k}\right)^2 = 3^{6k-7} \Rightarrow 9^{14-4k} = 3^{6k-7} \text{ scores M1}$$

$$\Rightarrow (14 - 4k)\log_3 9 = 6k - 7$$

$$\Rightarrow 2(14 - 4k) = 6k - 7$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 4 Using Logs Way 2:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}}$$

$$\Rightarrow (7-2k)\log_3 9 - (4k-5)\log_3 3 = (2k-2)\log_3 3 - (7-2k)\log_3 9 \quad \text{scores M1}$$

$$\Rightarrow 2(7-2k) - (4k-5) = 2k - 2 - 2(7-2k)$$

$$\Rightarrow 19 - 8k = 6k - 16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 5 Recognising that taking $\log_3$ forms an Arithmetic Sequence:

$$\{\log_3\}u_1 = 4k - 5, \ \{\log_3\}u_2 = 2(7-2k), \ \{\log_3\}u_3 = 2(k-1)$$

$$\Rightarrow 2(7-2k) - (4k-5) = 2(k-1) - 2(7-2k) \quad \text{scores M1}$$

$$\Rightarrow 19 - 8k = 6k - 16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.
There is no need to see any mention of log in this approach.

(b)

M1: Deduces expressions for the first term **and** the common ratio using $k = \frac{5}{2}$ in the correct formulae **and** finds at least one of $a = 243$ or $r = \frac{1}{3}$. Allow if seen in (a). May be implied by correct values for a and r. For reference, $a = 3^{4(2.5)-5}$ and $r = \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}$ $\left\{\text{or } r = \frac{3^{2(2.5-1)}}{9^{7-2(2.5)}}\right\}$

M1: Recalls the sum to infinity formula and substitutes their values for a and r provided $|r| < 1$
Dependent on a correct attempt to find both a and r using $k = 2.5$ but allow if neither value is correct or if they are unprocessed e.g. $\dfrac{3^{4(2.5)-5}}{1 - \dfrac{9^{7-2(2.5)}}{3^{4(2.5)-5}}}$ scores this mark.

A1: cao. Correct sum to infinity. Answer only (with no working) scores full marks. Apply isw.

Q24.

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	= 36.915 minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** th km is $6\times1.05=6\times1.05^1$ 6th km is $6\times1.05\times1.05=6\times1.05^2$ 7th km is $6\times1.05\times1.05\times1.05=6\times1.05^3$ Hence the time for the r th km is $6\times1.05^{r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20} 6\times1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 th to 20th km $=\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** Eg. Total time = 24 + $\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

(a)

M1: For using model to calculate the total time.

Sight of 24 minutes + $6\times1.05+6\times1.05^2$ or equivalent is required. Eg 24 + 6.3 + 6.615

Alternatively in seconds 24 minutes + 378 sec (6min 18 s) +396.9 (6 min 37 s)

A1*: 36 minutes 55 seconds following 36.915, 24+ 6.3+6.615 , 24 + $6\times1.05+6\times1.05^2$ or equivalent working in seconds

(b) Must be seen in (b)

B1: As seen in scheme. For making the link between the *r* th km and the index of 1.05
Or for EXPLAINING that "the time taken per km (6 mins) only starts to increase by 5% after the first 4 km"

(c) The correct sum formula $\frac{a(r^n-1)}{r-1}$ **, if seen, must be correct in part (c) for all relevant marks**

M1: For the overall strategy of finding the total time.

Award for adding 18, 24, 30.3 or awrt 36.9 and the sum of a geometric sequence

So award the mark for expressions such as $6\times4+\sum6\times1.05^n$ or $24+\frac{6(1.05^{20}-1)}{1.05-1}$

The geometric sequence formula, must be used with $r=1.05$ oe but condone slips on *a* and *n*

M1: For an attempt at using a correct sum formula for a GP to find an allowable sum
The value of *r* must be 1.05 oe such as 105% but you should allow a slip on the value of *n* used for their value of *a* (See below: We are going to allow the correct value of *n* or one less)
If you don't see a calculation it may be implied by sight of one of the values seen below

Allow for $a=6,\quad n=17$ or 16 Eg. $\frac{6(1.05^{17}-1)}{1.05-1}=(155.0)$ or $\frac{6(1.05^{16}-1)}{1.05-1}=(141.9)$

Allow for $a=6.3, n=16$ or 15 Eg $\frac{6.3(1.05^{16}-1)}{1.05-1}=(149.0)$ or $\frac{6.3(1.05^{15}-1)}{1.05-1}=(135.9)$

Allow for $a=6.615,\ n=15$ or 14 Eg $\frac{6.615(1.05^{15}-1)}{1.05-1}=(142.7)$ or $\frac{6.615(1.05^{14}-1)}{1.05-1}=(129.6)$

A1: For a correct calculation that will find the **total time**. It does not need to be processed

Allow for example, amongst others, $24+\frac{6.3(1.05^{16}-1)}{1.05-1}$, $18+\frac{6(1.05^{17}-1)}{1.05-1}$, $30.3+\frac{6.615(1.05^{15}-1)}{1.05-1}$

A1: For a total time of awrt 173 minutes and 3 seconds

This answer alone can be awarded 4 marks as long as there is some evidence of where it has come from.

....

Candidates that list values: Handy Table for Guidance

M1: For a correct overall strategy which would involve adding four sixes followed by at least 16 other values

The values which may be written in the form 6×1.05^2 or as numbers

Can be implied by $6+6+6+6+(6\times1.05)+.....+(6\times1.05^{16})$

M1: For an attempt to add the numbers from (6×1.05) to (6×1.05^{16}). This could be done on a calculator in which case expect to see awrt 149

Alternatively, if written out, look for 16 values with 8 correct or follow through correct to 1 dp

A1: Awrt 173 minutes

A1: Awrt 173 minutes and 3 seconds

Km	Time per km	Total Time
1	6.0000	
2	6.0000	12
3	6.0000	18
4	6.0000	24
5	6.3000	30.3
6	6.6150	36.915
7	6.9458	43.86075
8	7.2930	51.15379
9	7.6577	58.81148
10	8.0406	66.85205
11	8.4426	75.29465
12	8.8647	84.15939
13	9.3080	93.46736
14	9.7734	103.2407
15	10.2620	113.5028
16	10.7751	124.2779
17	11.3139	135.5918
18	11.8796	147.4714
19	12.4736	159.945
20	13.0972	173.0422

(**Q11 9MA0/01, June 2019**)

Q25.

Question	Scheme	Marks	AOs
(a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2\times"17.4" = ...$	M1	3.4
	$= 62.8$ (km h $^{-1}$)	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28\times"1.3265^4" = ...$ or $\dfrac{115}{"1.3265"}$	M1	3.4
	$= 86.7$ (km h $^{-1}$)	A1	1.1b
		(3)	
		(6 marks)	
Notes:			

(a)

M1: Translates the problem into maths using n^{th} term $= a+(n-1)d$ and attempts to find d

Look for either $115 = 28+5d \Rightarrow d = ...$ or an attempt at $\frac{115-28}{5}$ condoning slips

It is implied by use of $d = 17.4$ Note that $115 = 28+6d \Rightarrow d = ...$ is M0

M1: Uses the model to find the fastest speed the car can go in 3rd gear using $28+2\text{"}d\text{"}$ or equivalent.

This can be awarded following an incorrect method of finding "d"

A1: 62.8 km/ h Lack of units are condoned. Allow exact alternatives such as $\frac{314}{5}$

(b)

M1: Translates the problem into maths using n^{th} term $= ar^{n-1}$ and attempts to find r

It must use the 1st and 6th gear and not the 3rd gear found in part (a)

Look for either $115 = 28r^5 \Rightarrow r = ...$ o.e. or $\sqrt[5]{\frac{115}{28}}$ condoning slips.

It is implied by stating or using $r =$ awrt 1.33

M1: Uses the model to find the fastest speed the car can go in 5th gear using $28\times\text{"}r^4\text{"}$ or $\frac{115}{\text{"}r\text{"}}$ o.e.

This can be awarded following an incorrect method of finding "r"

A common misread seems to be finding the fastest speed the car can go in 3rd gear as in (a).

Providing it is clear what has been done, e.g. $u_3 = 28\times\text{"}r^2\text{"}$ it can be awarded this mark.

A1: awrt 86.7 km/h Lack of units are condoned. Expressions must be evaluated.

(**Q05 9MA0/01, Oct 2020**)

Q26.

Question	Scheme	Marks	AOs
(a) Way 1	$\{y = x^x \Rightarrow\}$ $\ln y = x\ln x$	B1	1.1a
	$\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $\frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = e^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	(5)	
(a) Way 2	$\{y = x^x \Rightarrow\}$ $y = e^{x\ln x}$	B1	1.1a
	$\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x\ln x}$	M1	1.1b
		A1	2.1
	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $\frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = e^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	(5)	
(b) Way 1	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}$ $x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone) Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
	Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
		(2)	
		(11 marks)	

Note	**A common solution** A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x\log x \Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x$ $\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda\frac{1}{y}\frac{dy}{dx}$; $\lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\frac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...$; $k \neq 0$

Question	Scheme	Marks	AOs
(b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(b) Way 3	For $\ln y = x\ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(b) Way 4	For $\log y = x\log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question

(a)	Way 1
B1:	$\ln y = x\ln x$. Condone $\log_x y = x\log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{dy}{dx}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1 + \ln x)$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x\ln x \to x = 0.368$ with no intermediate working
(a)	Way 2
B1:	$y = e^{x\ln x}$
M1:	For either $y = e^{x\ln x} \Rightarrow \frac{dy}{dx} = f(\ln x)e^{x\ln x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x\ln x}$ or $\frac{dy}{dx} = (1 + \ln x)e^{x\ln x}$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\}$ $\ln y = x\ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1}$ or awrt 0.368

Notes for Question Continued	
(b)	**Way 1**
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} =$ awrt 1.8... and $1.6^{1.6} =$ awrt 2.1..., reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	**Way 2**
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	**Way 3**
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	**Way 4**
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...,\ x_3 = 1.46626...,\ x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for "Converges in a cob-web pattern" or "Converges up and down to α"

(**Q11 9MA0/02, June 2019**)

Q27.

Question	Scheme	Marks	AOs
(a)(i)	$50x^2+38x+9 \equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B = \ldots$ or $C = \ldots$	M1	1.1b
	$B=1$ and $C=2$	A1	1.1b
(a)(ii)	E.g. $x=0$ $x=0 \Rightarrow 9=2A+B+4C$ $\Rightarrow 9=2A+1+8 \Rightarrow A=\ldots$	M1	2.1
	$A=0$*	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2}=(5x+2)^{-2}=2^{-2}\left(1+\frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2}=2^{-2}+\ldots$	M1	3.1a
	$\left(1+\frac{5}{2}x\right)^{-2}=1-2\left(\frac{5}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2+\ldots$	M1	1.1b
	$2^{-2}\left(1+\frac{5}{2}x\right)^{-2}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots$	A1	1.1b
	$\frac{1}{(1-2x)}=(1-2x)^{-1}=1+2x+\frac{-1(-1-1)}{2!}(2x)^2+.$	M1	1.1b
	$\frac{1}{(5x+2)^2}+\frac{2}{1-2x}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots+2+4x+8x^2+\ldots$	dM1	2.1
	$=\frac{9}{4}+\frac{11}{4}x+\frac{203}{16}x^2+\ldots$	A1	1.1b
(b)(ii)	$\lvert x\rvert<\frac{2}{5}$	B1	2.2a
		(7)	
			(11 marks)
	Notes		

(a)(i)

M1: **Uses a correct identity** and makes progress using an appropriate strategy (e.g. sub $x=\frac{1}{2}$) to find a value for B or C. May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A.

Amongst many different methods are:

Compare terms in x^2 $\Rightarrow 50=-10A+25C$ which would be implied by $50=-10A+25\times"2"$

Compare constant terms or substitute $x=0 \Rightarrow 9=2A+B+4C$ implied by $9=2A+1+4\times 2$

A1*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.
Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0

(b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an expression of the form $(5x+2)^{-2} = 2^{-2}(1+*x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2} + \ldots$

M1: Correct attempt at the binomial expansion of $(1+*x)^{-2}$ up to the term in x^2

Look for $1+(-2)*x+\frac{(-2)(-3)}{2}*x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3. ....

Alt Look for correct structure for 2nd and 3rd terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their 'B'

A direct expansion would look like $(2+5x)^{-2} = 2^{-2} + (-2)2^{-3}\times 5x + \frac{(-2)(-3)}{2}2^{-4}\times(5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1+(-1)*x+\frac{(-1)(-2)}{2}*x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.
There must be some attempt to use their values of B and C

A1: Correct expression or correct values for p, q and r.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5} < x < \frac{2}{5}$ or $x \in \left(-\frac{2}{5}, \frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered

(**Q09 9MA0/01, Oct 2021**)

Q28.

Question	Scheme	Marks	AOs
(a)	$(1+8x)^{\frac{1}{2}} = 1+\frac{1}{2}\times 8x + \frac{\frac{1}{2}\times -\frac{1}{2}}{2!}\times(8x)^2 + \frac{\frac{1}{2}\times -\frac{1}{2}\times -\frac{3}{2}}{3!}\times(8x)^3$	M1 A1	1.1b 1.1b
	$= 1+4x-8x^2+32x^3+\ldots$	A1	1.1b
		(3)	
(b)	Substitutes $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x=\frac{1}{32}$ is substituted into $1+4x-8x^2+32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
			(5 marks)
Notes:			

(a)

M1: Attempts the binomial expansion with $n=\frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4.

Award for the correct coefficient with the correct power of x. Do not accept ${}^{n}C_{r}$ notation for coefficients.

For example look for term 3 in the form $\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(*x)^{2}$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(*x)^{3}$

A1: Correct (unsimplified) expression. May be implied by correct simplified expression

A1: $1+4x-8x^{2}+32x^{3}$

Award if there are extra terms (even if incorrect).

Award if the terms are listed $1,\ 4x,-8x^{2},32x^{3}$

(b)

M1: Score for substituting $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to obtain $\frac{\sqrt{5}}{2}$ or equivalent such as $\sqrt{\frac{5}{4}}$

Alternatively award for substituting $x=\frac{1}{32}$ into **both sides** and making a connection between the two sides by use of an = or $\approx$.

E.g. $\left(1+\frac{8}{32}\right)^{\frac{1}{2}}=1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^{2}+32\times\left(\frac{1}{32}\right)^{3}$ following through on their expansion

Also implied by $\frac{\sqrt{5}}{2}=\frac{1145}{1024}$ for a correct expansion

It is not enough to state substitute $x=\frac{1}{32}$ into " the expansion" or just the rhs "$1+4x-8x^{2}+32x^{3}$"

A1ft: Requires a full (and correct) **explanation** as to how the expansion can be used to estimate $\sqrt{5}$

E.g. Calculates $1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^{2}+32\times\left(\frac{1}{32}\right)^{3}$ and multiplies by 2.

This can be scored from an incorrect binomial expansion or a binomial expansion with more terms.

The explanation could be mathematical. So $\frac{\sqrt{5}}{2}=\frac{1145}{1024}\rightarrow\sqrt{5}=\frac{1145}{512}$ is acceptable.

SC: For 1 mark, M1,A0 score for a statement such as "substitute $x=\frac{1}{32}$ into both sides of part (a) and make $\sqrt{5}$ the subject"

(**Q01 9MA0/01, Oct 2020**)

Q29.

Question	Scheme	Marks	AOs
(a)	$S_{n}=a+ar+ar^{2}+........+ar^{n-1}$	B1	1.2
	$rS_{n}=ar+ar^{2}+ar^{3}+........+ar^{n}\Rightarrow S_{n}-rS_{n}=...$	M1	2.1
	$S_{n}-rS_{n}=a-ar^{n}$	A1	1.1b

	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4\times\dfrac{a(1-r^5)}{1-r}$ or $4\times\dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1-r$)	M1	3.1a
	$1-r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5-1)(r^5-3) = 0 \Rightarrow r^5 = ...$ or e.g. $1-r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = ...$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

B1: Writes out the sum or lists terms. Condone the omission of S.
The sum must include the first and last terms and (at least) two other correct terms and no incorrect terms e.g. ar^n
Note that the sum may be seen embedded within their working.

M1: For the key step in attempting to multiply the first series by r and subtracting.

A1: $S_n - rS_n = a - ar^n$ either way around but condone one side being prematurely factorised (but not both) **following correct work but this could follow B0 if insufficient terms were shown.**

A1*: Depends on all previous marks. Proceeds to given result showing all steps including seeing both sides unfactorised at some point in their working.
Note: If terms are listed rather than added then allow the first 3 marks if otherwise correct but withhold the final mark.

(b)

M1: For the correct strategy of producing an equation in just r^{10} and r^5 (and possibly $(1-r)$) with the "4" on either side using the result from part (a) and makes progress to at least cancel through by a
Some candidates retain the "$1-r$" and start multiplying out e.g. $(1-r)(1-r^{10})$ and this mark is still available as long as there is progress in cancelling the "a".

A1: Correct equation with the a and the $1-r$ cancelled. Allow any correct equation in just r^5 and r^{10}

dM1: Depends on the first M. Solves as far as $r^5 = ...$ by solving a 3 term quadratic in r^5 by a valid method – see general guidance or by difference of 2 squares – see above

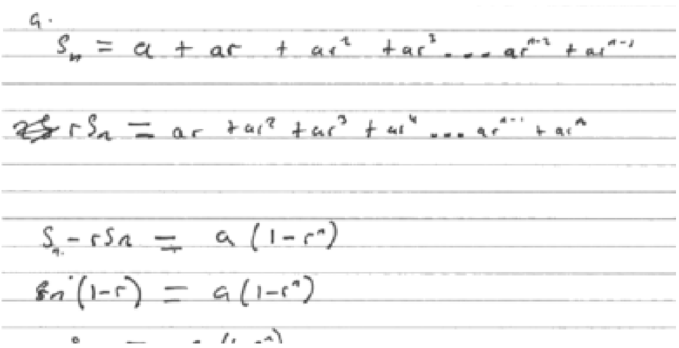
A1: $r = \sqrt[5]{3}$ oe only. The solution $r = 1$ if found must be rejected here.

(b) Note: For candidates who use $S_5 = 4S_{10}$ expect to see:

$4\times\dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r} \Rightarrow 4(1-r^{10}) = (1-r^5)$ M1A0

$4r^{10} - r^5 - 3 = 0 \Rightarrow (4r^5+3)(r^5-1) = 0 \Rightarrow r^5 = ...$ or $4(1-r^5)(1+r^5) = (1-r^5) \Rightarrow r^5 = ...$ dM1A0

Example for (a)


a.
$S_n = a + ar + ar^2 + ar^3 ... ar^{n-2} + ar^{n-1}$
$rS_n = ar + ar^2 + ar^3 + ar^4 ... ar^{n-1} + ar^n$
$S_n - rS_n = a(1-r^n)$
$S_n(1-r) = a(1-r^n)$

This scores B1M1A1A0:

B1: Writes down the sum including first and last terms and at least 2 other correct terms and no incorrect terms

M1: Multiplies by *r* and subtracts

A1: Correct equation (we allow one side to be prematurely factorised)

A0: One side was prematurely factorised

(**Q15 9MA0/02, Oct 2020**)

Q30.

Question	Scheme	Marks	AOs
(i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \ldots$		
	$= \dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \dfrac{20}{1-\frac{1}{2}} - \dfrac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40 - 37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = \right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times\ldots\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	

(ii) Way 2	$\left\{\sum_{n=1}^{48}\log_5\left(\frac{n+2}{n+1}\right)=\right\}\ \sum_{n=1}^{48}(\log_5(n+2)-\log_5(n+1))$	M1	1.1b
	$=(\log_5 3+\log_5 4+......+\log_5 50)-(\log_5 2+\log_5 3+......+\log_5 49)$	M1	3.1a
	$=\log_5 50-\log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25)=2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes for Question	
(i)	**Way 1**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1<$ their $r<1$) and their value for a
M1:	Finds the infinite sum by using a complete strategy of applying $\frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$.
A1:	2.5 o.e.
(i)	**Way 2**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1<$ their $r<1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{10}{1-\frac{1}{2}}-(10+5+2.5)$ or $\frac{10}{1-\frac{1}{2}}-\frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 3**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1<$ their $r<1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{20}{1-\frac{1}{2}}-(20+10+5+2.5)$ or $\frac{20}{1-\frac{1}{2}}-\frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
Note:	Give M1 M1 A1 for a correct answer of 2.5 from no working in (i)
(ii)	**Way 1**
M1:	Some evidence of applying the addition law of logarithms as part of a valid proof
M1:	Begins to solve the problem by just writing (or by combining) at least three terms including • **either** the first two terms and the last term • **or** the first term and the last two terms

Note:	The 2nd mark can be gained by writing any of • listing $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right), \log_5\left(\frac{50}{49}\right)$ or $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{49}{48}\right), \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + ... + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + ... + \log_5\left(\frac{49}{48}\right) + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2} \times \frac{4}{3} \times ... \times \frac{50}{49}\right)$ *{this will also gain the 1st M1 mark}* • $\log_5\left(\frac{3}{2} \times ... \times \frac{49}{48} \times \frac{50}{49}\right)$ *{this will also gain the 1st M1 mark}*
A1*:	Correct proof leading to a correct answer of 2
Note:	Do not allow the 2nd M1 if $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right)$ are listed and $\log_5\left(\frac{50}{49}\right)$ is used for the first time in their applying the formula $S_{48} = \frac{48}{2}\left(\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{50}{49}\right)\right)$
Note:	<u>**Listing all 48 terms**</u> Give M0 M1 A0 for $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \log_5\left(\frac{5}{4}\right) + ... + \log_5\left(\frac{50}{49}\right) = 2$ {lists all terms} Give M0 M0 A0 for $0.2519... + 0.1787... + 0.1386... + ... + 0.0125... = 2$ {all terms in decimals}

Notes for Question	
(ii)	**Way 2**
M1:	Uses the subtraction law of logarithms to give $\log_5\left(\frac{n+2}{n+1}\right) \to \log_5(n+2) - \log_5(n+1)$
M1:	Begins to solve the problem by writing at least three terms for each of $\log_5(n+2)$ and $\log_5(n+1)$ including • **either** the first two terms and the last term for both $\log_5(n+2)$ and $\log_5(n+1)$ • **or** the first term and the last two terms for both $\log_5(n+2)$ and $\log_5(n+1)$
Note:	This mark can be gained by writing any of • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 + ... + \log_5 49 + \log_5 50) - (\log_5 2 + ... + \log_5 48 + \log_5 49)$ • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 - \log_5 2) + (\log_5 4 - \log_5 3) + ... + (\log_5 50 - \log_5 49)$ • $\log_5 3 - \log_5 2, ..., \log_5 49 - \log_5 48, \log_5 50 - \log_5 49$
A1*:	Correct proof leading to a correct answer of 2
Note:	The base of 5 can be omitted for the M marks in part (ii), but the base of 5 must be included in the final line (as shown on the mark scheme) of their solution.
Note:	If a student uses a mixture of a Way 1 or Way 2 method, then award the best Way 1 mark only or the best Way 2 mark only.
Note:	Give M1 M0 A0 (1st M for implied use of subtraction law of logarithms) for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 91.8237... - 89.8237... = 2$
Note:	Give M1 M1 A1 for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$ $= \log_5(3 \times 4 \times .... \times 50) - \log_5(2 \times 3 \times .... \times 49)$ $= \log_5\left(\frac{50!}{2}\right) - \log_5(49!)$ or $= \log_5(25 \times 49!) - \log_5(49!)$ $= \log_5 25 = 2$

(**Q08 9MA0/02, June 2019**)

Q31.

Question	Scheme	Marks	AOs
	$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4} \times \frac{1}{x} \, dx$	M1	1.1b
	$= \frac{x^4}{4}\ln x - \frac{x^4}{16} (+c)$	M1 A1	1.1b 1.1b
	$\int_1^{e^2} x^3 \ln x \, dx = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2} = \left(\frac{e^8}{4}\ln e^2 - \frac{e^8}{16}\right) - \left(-\frac{1^4}{16}\right)$	M1	2.1
	$= \frac{7}{16}e^8 + \frac{1}{16}$	A1	1.1b
		(5)	
			(5 marks)
Notes:			

M1: Integrates by parts the right way round.

Look for $kx^4 \ln x - \int kx^4 \times \frac{1}{x} \, dx$ o.e. with $k > 0$. Condone a missing dx

M1: Uses a correct method to integrate an expression of the form $\int kx^4 \times \frac{1}{x} \, dx \to c\, x^4$

A1: $\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \frac{x^4}{16}$ $(+c)$ which may be left unsimplified

M1: Attempts to substitute 1 and e^2 into an expression of the form $\pm px^4 \ln x \pm qx^4$, subtracts and uses $\ln e^2 = 2$ (which may be implied).

A1: $\frac{7}{16}e^8 + \frac{1}{16}$ o.e. Allow $0.4375e^8 + 0.0625$ or uncancelled fractions. NOT ISW: $7e^8 + 1$ is A0

You may see attempts where substitution has been attempted.

E.g. $u = \ln x \Rightarrow x = e^u$ and $\frac{dx}{du} = e^u$

M1: Attempts to integrate the correct way around condoning slips on the coefficients

$$\int x^3 \ln x \, dx = \int e^{4u} u \, du = \frac{e^{4u}}{4} u - \int \frac{e^{4u}}{4} \, du$$

M1 A1: $\int x^3 \ln x \, dx = \frac{e^{4u}}{4} u - \frac{e^{4u}}{16} (+c)$

M1 A1: Substitutes 0 and 2 into an expression of the form $\pm pue^{4u} \pm qe^{4u}$ and subtracts

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It is possible to use integration by parts "the other way around"

To do this, candidates need to know or use $\int \ln x \, dx = x \ln x - x$

FYI $I = \int x^3 \ln x \, dx = x^3 (x \ln x - x) - \int (x \ln x - x) \times 3x^2 \, dx = x^3 (x \ln x - x) - 3I + \frac{3}{4} x^4$

Hence $4I = x^4 \ln x - \frac{1}{4} x^4 \Rightarrow I = \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4$

Score M1 for a full attempt at line 1 (condoning bracketing and coefficient slips) followed by M 1 for line 2 where terms in I o.e. to form the answer.

(**Q12 9MA0/01, June 2022**)

Q32.

Question	Scheme	Marks	AOs
	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, \ldots; u_{n+1}=\frac{1}{u_n}, u_1=\frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=\frac{16}{2}(2(8)+15(5))+\frac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=728+131070=131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=(3\times16)+\frac{16}{2}(2(5)+15(5))+\frac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=48+680+131070=131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum $=10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1	3.1a
		M1	1.1b
		M1	1.1b
		A1*	2.1
		(4)	
(ii)	$\left\{u_1=\frac{2}{3}\right\}, u_2=\frac{3}{2}, u_3=\frac{2}{3}, \ldots$ (*can be implied by later working*)	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}\ 50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3}$ $\left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1	1.1b
		(3)	
		(7 marks)	

Notes for Question	
(i)	
M1:	Uses a correct methodical strategy to enable the given sum, $\sum_{r=1}^{16}(3+5r+2^r)$ to be found Allow M1 for any of the following: • expressing the given sum as either $\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}(2^r)$, $\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}(2^r)$ or $\sum_{r=1}^{16}3+5\sum_{r=1}^{16}r+\sum_{r=1}^{16}(2^r)$ • attempting to find both $\sum_{r=1}^{16}(3+5r)$ and $\sum_{r=1}^{16}(2^r)$ separately • (3×16) and attempting to find both $\sum_{r=1}^{16}(5r)$ and $\sum_{r=1}^{16}(2^r)$ separately
M1:	**Way 1:** Correct method for finding the sum of an AP with $a=8$, $d=5$, $n=16$ **Way 2:** (3×16) and a correct method for finding the sum of an AP
M1:	Correct method for finding the sum of a GP with $a=2$, $r=2$, $n=16$
A1*:	For all steps fully shown (with correct formulae used) leading to 131 798
Note:	**Way 1:** Give 2nd M1 for writing $\sum_{r=1}^{16}(3+5r)$ as $\frac{16}{2}(8+83)$
Note:	**Way 2:** Give 2nd M1 for writing $\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)$ as $48+\frac{16}{2}(5+80)$ or $48+680$
Note:	Give 3rd M1 for writing $\sum_{r=1}^{16}(2^r)$ as $\frac{2(1-2^{16})}{1-2}$ or $2(2^{16}-1)$ or $(2^{17}-2)$

(i)	
Way 3	
M1:	At least 6 correct terms **and** 16 terms shown
M1:	At least 10 correct terms (may not be 16 terms)
M1:	At least 15 correct terms (may not be 16 terms)
A1*:	All 16 terms correct **and** an indication that the sum is 131 798
(ii)	
M1:	For some indication that the next two terms of this sequence are $\frac{3}{2}, \frac{2}{3}$
M1:	For deducing that the sum can be found by applying $50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$, o.e.
A1:	Obtains $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$ or an exact equivalent
Note:	Allow 1st M1 for $u_2=\frac{3}{2}$ (or equivalent) **and** $u_3=\frac{2}{3}$ (or equivalent)
Note:	Allow 1st M1 for the first 3 terms written as $\frac{2}{3}, \frac{3}{2}, \frac{2}{3}, ...$
Note:	Allow 1st M1 for the 2nd and 3rd terms written as $\frac{3}{2}, \frac{2}{3}, ...$ in the correct order
Note:	Condone $\frac{2}{3}$ written as 0.66 or awrt 0.67 for the 1st M1 mark
Note:	Give A0 for 108.3 or 108.333... without reference to $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$

(**Q04 9MA0/02, June 2018**)

Q33.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{2^2+8^2}=\sqrt{68}=2\sqrt{17}$	B1	1.1b
	$2\cos\theta+8\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ $2=R\cos\alpha \quad 8=R\sin\alpha$ $\tan\alpha=\frac{8}{2}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=$ awrt 1.326	A1	2.2a
		(3)	
(b)(i)	$4.5\times"2\sqrt{17}"$	M1	1.1b
	$9\sqrt{17}$	A1	2.2a
(ii)	awrt 1.33	B1ft	2.2a
		(3)	
			(6 marks)

Notes

(a)

B1: $R=2\sqrt{17}$ or $\sqrt{68}$.

$\pm2\sqrt{17}$ or $\pm\sqrt{68}$ score B0

(Condone if this comes from e.g., $8=R\cos\alpha \quad 2=R\sin\alpha$)

Decimal answers score B0 unless the exact value is seen then apply isw.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{8}{2}$, $\cos\alpha=\pm\frac{2}{"\sqrt{68}"}$, $\sin\alpha=\pm\frac{8}{"\sqrt{68}"}$

May be implied by awrt 1.33 radians or 76 degrees

A1: awrt 1.326 for α. Apply isw if this is then subsequently rounded to e.g. 1.33

(b)(i)

M1: For a value of $\pm4.5\times$ their R or allow $\pm4.5R$ (with the letter R)

But not embedded in an expression e.g. $9\sqrt{17}\cos(\theta-\alpha)$ unless extracted later.

Note that the sum may be found as $9\cos x+36\sin x$ with the maximum then found using calculus

e.g. $S=9\cos x+36\sin x\Rightarrow\frac{dS}{dx}=-9\sin x+36\cos x=0\Rightarrow\tan x=4\Rightarrow\sin x=\frac{4}{\sqrt{17}},\ \cos x=\frac{1}{\sqrt{17}}$

$\Rightarrow9\cos x+36\sin x=9\sqrt{17}$. This will score M1 once they reach $\pm4.5\times$ their R

May be implied by $9\sqrt{17}$ or awrt 37.1 (which may come from a graphical method)

May also see e.g. $\text{Max}(9\cos x+36\sin x)=\sqrt{9^2+36^2}=...$

A1: $9\sqrt{17}$ or exact equivalent e.g. $\sqrt{1377}$, $4.5\sqrt{68}$, $4.5(2\sqrt{17})$ and apply isw once a correct answer is seen

(ii)

B1ft: awrt 1.33 (or follow through on their α even if in degrees (76), no matter how accurate)

(**Q08 9MA0/02, June 2023**)

Q34.

Question	Scheme	Marks	AOs
	Attempts $S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm\frac{1}{\sqrt{2}}$ (so $k = 2$)	A1	1.1b
			(4 marks)

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty = \frac{8}{7} \times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r = \pm\frac{1}{\sqrt{2}}$ giving $k = 2$

(**Q10 9MA0/02, Specimen papers**)

Q35.

Question	Scheme	Marks	AOs
(a)	$\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2+\ldots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2+..\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used	B1	2.4
		(1)	
			(5 marks)

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1\pm\ldots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2+\ldots$

A1: Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^2+$ which may be left unsimplified

A1: $\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$

(b)

B1: The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used

(**Q07 9MA0/02, Specimen papers**)

Q36.

Question	Scheme	Marks	AOs
(a)	$\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5}=1+0.5\times(4x)+\frac{0.5\times-0.5}{2}\times(4x)^2$ $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$ $(1+4x)^{0.5}=1+2x-2x^2$ and $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	M1 M1 A1	1.1b 1.1b 1.1b
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=(1+2x-2x^2......)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2...\right)$ $=1+\frac{1}{2}x+\frac{3}{8}x^2+2x+x^2-2x^2+...$ $=A+Bx+Cx^2$	dM1	2.1
	$=1+\frac{5}{2}x-\frac{5}{8}x^2......$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x\rvert<\frac{1}{4}$ Should not use $x=\frac{1}{2}$ as $\frac{1}{2}>\frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x=\frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}}\approx1+\frac{5}{2}x-\frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}}=\frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	
			(10 marks)

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions

This could be achieved by $\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5}=1+0.5\times(4x)+\frac{(0.5)\times(-0.5)}{2}\times(4x)^2$

There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1+2x\pm0.5x^2$

M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$

There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1\pm0.5x\pm0.375x^2$

A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end

dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on the multiplication on one term only. It is dependent upon having scored the first B and one of the other two M's

In the alternative it is for multiplying $\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x|>\frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2}>\frac{1}{4}$ or stating that the expansion is only valid when $|x|<\frac{1}{4}$

Condone, for this mark a candidate who substitutes $x=\frac{1}{2}$ into the $4x$ **and** states it is not valid as $2>1$ oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x=\frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}}\approx 1+\frac{5}{2}x-\frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}}=\frac{1183}{968}$ oe $\sqrt{6}=2\times\frac{1183}{968}$

A1: $\sqrt{6}=\frac{1183}{484}$ or $\sqrt{6}=\frac{2904}{1183}$ $\quad\sqrt{6}=2\times\frac{1183}{968}=\frac{1183}{484}$ would imply all 3 marks

Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5}\approx\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5}=1+(0.5)(-x)+\frac{(0.5)\times(-0.5)}{2}(-x)^2$

Or

B1: $\sqrt{\frac{1+4x}{1-x}}=\sqrt{1+\frac{5x}{1-x}}=\left(1+5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

Or

B1: $\sqrt{\frac{1+4x}{1-x}}\times\frac{\sqrt{1-x}}{\sqrt{1-x}}=\sqrt{(1+3x-4x^2)}\times(1-x)^{-1}=\left(1+\left(3x-4x^2\right)\right)^{\frac{1}{2}}\times(1-x)^{-1}$

(**Q11 9MA0/01, June 2018**)