Name: ____

Sequence and Series Exam Questions

Date:

Time:

Total marks available: 231

Total marks achieved: ______

Questions

Q1.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5 + 2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

$$r = \frac{5+2\sin\theta}{12\cos\theta} = \frac{6\tan\theta}{5+2\sin\theta}$$

$$(5+2\sin\theta)^2 = 6 \times \frac{\sin\theta}{\cos\theta} \times 12\cos\theta$$

$$25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$$

$$4\sin^2\theta - 52\sin\theta + 25 = 0 \quad \text{shown.}$$

(3)

Given that θ is an obtuse angle measured in radians, $\therefore \frac{\pi}{2} < \theta < \pi$

(b) solve the equation in part (a) to find the exact value of θ

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

$$(2\sin\theta - 25)(2\sin\theta - 1) = 0$$

$\therefore \sin\theta = \frac{25}{2}$ No real Solutions

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}, \ \pi - \frac{\pi}{6}$

reject because not within the domain of θ

$\underline{\underline{\frac{5\pi}{6}}}$

(2)

(c) show that the sum to infinity of the series can be expressed in the form

$$k(1 - \sqrt{3})$$

where k is a constant to be found.

$a = T_1 = 12\cos\left(\frac{5\pi}{6}\right) = -6\sqrt{3}$

$T_2 = 5 + 2\sin\left(\frac{5\pi}{6}\right) = 6$

$T_3 = 6\tan\left(\frac{5\pi}{6}\right) = -2\sqrt{3}$

$r = \frac{6}{-6\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$S_\infty = \frac{-6\sqrt{3}}{1-\left(-\frac{\sqrt{3}}{3}\right)} = \frac{-6\sqrt{3}}{\frac{3+\sqrt{3}}{3}}$

$S_\infty = -6\sqrt{3} \times \frac{3}{3+\sqrt{3}}$

$S_\infty = 9 - 9\sqrt{3}$

$S_\infty = 9(1-\sqrt{3})$

$\therefore k = 9$

(5)

(Total for question = 10 marks)

(Q15 9MA0/02, June 2022)

Q2.

A sequence of terms a_1, a_2, a_3, ... is defined by

$$a_1 = 3$$

$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

i) $a_1 = 3$

$a_2 = 8 - 3 = 5$

$a_3 = 8 - 5 = 3$

$a_4 = 8 - 3 = 5$

The sequence is hence periodic

ii) Repeats every 2 terms so it is an order of 2

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

$$\sum_{1}^{85} a_n = \frac{84}{2}(3+5) + 3 = \underline{\underline{339}}$$

(2)

(Total for question = 4 marks)

(**Q03 9MA0/02, June 2022**)

Q3.

In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

$a = 16$

$T_{21} = 24 = 16 + 20d$

$8 = 20d$

$\therefore d = \frac{2}{5}$

(2)

(b) Hence find the sum of the first 500 terms of the series.

$S_{500} = \frac{500}{2}\left[2(16) + 499\left(\frac{2}{5}\right)\right]$

$S_{500} = \underline{\underline{57900}}$

(2)

(Total for question = 4 marks)

(**Q01 9MA0/02, Oct 2021**)

Q4.

Jamie takes out an interest free loan of £8100

Jamie makes a payment every month to pay back the loan.

Jamie repays £400 in month 1, £390 in month 2, £380 in month 3, and so on, so that the amounts repaid each month form an arithmetic sequence.

(a) Show that Jamie repays £290 in month 12

$T_{12} = 400 + 11(-10)$

$T_{12} = £290$

(1)

After Jamie's *N*th payment, the loan is completely paid back.

(b) Show that $N^2 - 81N + 1620 = 0$

$8100 = \frac{N}{2}\left[2(400) + (N-1)(-10)\right]$

$16200 = N(800 + 10 - 10N)$

$16200 = 810N - 10N^2$

$10N^2 - 810N + 16200 = 0$

$N^2 - 81N + 1620 = 0$ shown.

(2)

(c) Hence find the value of *N*.

$(N-45)(N-36) = 0$

$\therefore N = 45$ reject $\quad N = 36$

(2)

(Total for question = 5 marks)

(**Q02 9MA0/02, June 2024**)

Q5.

The first three terms of an arithmetic sequence are

$$6k, \ 10 \text{ and } 2k$$

where k is a constant.

(a) Find the value of k.

$d = 2k - 10 = 10 - 6k$

$6k + 2k = 10 + 10$

$8k = 20$

$k = \frac{5}{2}$

(2)

(b) Hence find the value of the sum of the first 50 terms of this sequence.

$a = T_1 = 6 \times \frac{5}{2} = 15$

$T_2 = 10$

$T_3 = 5$

$\therefore d = -5$

$S_{50} = \frac{50}{2}\left[2(15) + 49(-5)\right]$

$S_{50} = -5375$

(3)

(Total for question = 5 marks)

(Q03 9MA0/01, June 2025)

Q6.

(i) In an arithmetic series, the first term is *a* and the common difference is *d*.

Show that

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S = a + a+d + a+2d + a+3d + \dots + a+(n-2)d + a+(n-1)d$

$+$

$S = a+(n-1)d + a+(n-2)d + a+(n-3)d + \dots + a+2d + a+d + a$

$2S = n(2a + (n-1)d)$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.
Given that James takes n weeks to save exactly £64
(a) show that

$$n^2 - 26n + 160 = 0$$

$T_1 = 10 \quad T_2 = 9.20 \quad T_3 = 8.40$

$d = 9.2 - 10 = -0.8$

$64 = \frac{n}{2}\left[2(10) - 0.8(n-1)\right]$

$128 = n\left[20 + 0.8 - 0.8n\right]$

$128 = 20.8n - 0.8n^2$

$0.8n^2 - 20.8n + 128 = 0$

$n^2 - 26n + 160 = 0$ Shown.

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

$(n-16)(n-10) = 0$

$\therefore n = 16 \quad n = 10$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

He has saved the necessary amount by the 10th week

so there is no need to save until the 16th week

It thus takes James 10 weeks

(1)

(Total for question = 7 marks)

(**Q13 9MA0/01, June 2022**)

Q7.

(a) Find the first three terms, in ascending powers of *x*, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

$$\frac{1}{\sqrt{4-x}} \Rightarrow (4-x)^{-1/2} \Rightarrow \left[4\left(1-\frac{x}{4}\right)\right]^{-1/2} \Rightarrow 4^{-1/2}\left(1-\frac{x}{4}\right)^{-1/2} \Rightarrow \frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2}$$

$$\frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2} = \frac{1}{2}\left[1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{4}\right)^2\left(\frac{1}{2}\right) + \cdots\right]$$

$$\frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2} = \frac{1}{2}\left[1 + \frac{x}{8} + \frac{3}{128}x^2 + \cdots\right]$$

$$\frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2} = \frac{1}{2} + \frac{x}{16} + \frac{3x^2}{256} + \cdots$$

(4)

The expansion can be used to find an approximation to $\sqrt{2}$
Possible values of *x* that could be substituted into this expansion are:

- $x = -14$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{18}} = \frac{\sqrt{2}}{6}$
- $x = 2$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $x = -\frac{1}{2}$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{\frac{9}{2}}} = \frac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of *x* should not be used

$\left|-\frac{x}{4}\right| < 1 \therefore |x| < 4$

$x = -14$ cannot be used because $|-14| = 14$, $14 > 4$

(1)

(ii) state, giving a reason, which of the three values of *x* would lead to the most accurate approximation to $\sqrt{2}$

$x = -\frac{1}{2}$ will give more accurate approximation because $-\frac{1}{2}$ is the Closest to zero.

(1)

(Total for question = 6 marks)

(**Q04 9MA0/01, June 2019**)

Q8.

In the binomial expansion of

$(a + 2x)^7$ where a is a constant

the coefficient of x^4 is 15 120

Find the value of a.

$${}^7C_4 \, (a)^{7-4} (2x)^4 = 15120x^4$$

$$35a^3 \times 16x^4 = 15120x^4$$

$$a^3 = 27$$

$$a = 3$$

(Total for question = 3 marks)

(**Q04 9MA0/02, Oct 2020**)

Q9.

(a) Find, in ascending powers of x, the first four terms of the binomial expansion of

$$(1-9x)^{\frac{1}{2}}$$

giving each term in simplest form.

$$(1-9x)^{1/2} = 1 + \left(\frac{1}{2}\right)(-9x) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(-9x)^2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-9x)^3\left(\frac{1}{6}\right) + \dots$$

$$(1-9x)^{1/2} = 1 - \frac{9}{2}x - \frac{81}{8}x^2 - \frac{729}{16}x^3 + \dots$$

(3)

(b) Give a reason why $x=-\frac{2}{9}$ should **not** be used in the expansion to find an approximation to √3

$|-9x| < 1$

$|9x| < 1$

$|x| < \frac{1}{9}$

(1)

$\left|-\frac{2}{9}\right| = \frac{2}{9}$

(Total for question = 4 marks)

$\frac{2}{9} > \frac{1}{9}$ hence not suitable to find approximation to $\sqrt{3}$

(**Q02 9MA0/01, June 2024**)

Q10.

(a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\frac{4}{(2+3x)^2}$$

writing each term in simplest form.

$$4(2+3x)^{-2} \Rightarrow 4\left[2\left[1+\frac{3}{2}x\right]\right]^{-2} \Rightarrow 4\times 2^{-2}\left[1+\frac{3}{2}x\right]^{-2} = \left(1+\frac{3}{2}x\right)^{-2}$$

$$\left(1+\frac{3}{2}x\right)^{-2} = 1 + (-2)\left(\frac{3}{2}x\right) + (-2)(-3)\left(\frac{3}{2}x\right)^2\left(\frac{1}{2}\right) + (-2)(-3)(-4)\left(\frac{3}{2}x\right)^3\left(\frac{1}{6}\right) + \cdots$$

$$\left(1+\frac{3}{2}x\right)^{-2} = 1 - 3x + \frac{27}{4}x^2 - \frac{27}{2}x^3 + \cdots$$

(4)

(b) Find the range of values of x for which this expansion is valid.

$$\left|\frac{3}{2}x\right| < 1$$

$$|x| < \frac{2}{3}$$

(1)

(Total for question = 5 marks)

(**Q06 9MA0/02/M, June 2025**)

Q11.

(a) Find the first four terms, in ascending powers of *x*, of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

$$(4-9x)^{1/2} \Rightarrow \left[4\left(1-\tfrac{9}{4}x\right)\right]^{1/2} \Rightarrow 4^{1/2}\left(1-\tfrac{9}{4}x\right)^{1/2} = 2\left(1-\tfrac{9}{4}x\right)^{1/2}$$

$$2\left(1-\tfrac{9}{4}x\right)^{1/2} = 2\left[1+\left(\tfrac{1}{2}\right)\left(-\tfrac{9}{4}x\right)+\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(-\tfrac{9}{4}x\right)^2\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\left(-\tfrac{9}{4}x\right)^3\left(\tfrac{1}{6}\right)+\dots\right]$$

$$2\left(1-\tfrac{9}{4}x\right)^{1/2} = 2\left[1-\tfrac{9}{8}x-\tfrac{81}{128}x^2-\tfrac{729}{1024}x^3+\dots\right]$$

$$2\left(1-\tfrac{9}{4}x\right)^{1/2} = 2-\tfrac{9}{4}x-\tfrac{81}{64}x^2-\tfrac{729}{512}x^3+\dots$$

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

The approximation will be an over estimate because the next terms in the binomial expansion that have not been accounted for will be negative.

(1)

(Total for question = 5 marks)

(**Q07 9MA0/02, June 2022**)

Q12.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the first three terms, in ascending powers of *x*, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

$$(3 + x)^{-2} \Rightarrow \left[3\left(1 + \frac{x}{3}\right)\right]^{-2} \Rightarrow 3^{-2}\left[1 + \frac{x}{3}\right]^{-2} \Rightarrow \frac{1}{9}\left(1 + \frac{x}{3}\right)^{-2}$$

$$\frac{1}{9}\left(1 + \frac{x}{3}\right)^{-2} = \frac{1}{9}\left[1 + (-2)\left(\frac{x}{3}\right) + (-2)(-3)\left(\frac{x}{3}\right)^{2}\left(\frac{1}{2}\right) + \dots\right]$$

$$\frac{1}{9}\left(1 + \frac{x}{3}\right)^{-2} = \frac{1}{9}\left[1 - \frac{2}{3}x + \frac{1}{3}x^{2} + \dots\right]$$

$$\frac{1}{9}\left(1 + \frac{x}{3}\right)^{-2} = \frac{1}{9} - \frac{2}{27}x + \frac{1}{27}x^{2} + \dots$$

(4)

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\,dx$$

giving your answer to 4 significant figures.

$$I = \int_{0.2}^{0.4} 6x\,(3+x)^{-2}\,dx$$

$$I = \int_{0.2}^{0.4} 6x\left(\frac{1}{9} - \frac{2}{27}x + \frac{1}{27}x^2\right)dx$$

$$I = \int_{0.2}^{0.4} \frac{2}{3}x - \frac{4}{9}x^2 + \frac{2}{9}x^3 \; dx$$

$$I = \left[\frac{1}{2}\times\frac{2}{3}x^2 - \frac{4}{9}\times\frac{1}{3}x^3 + \frac{2}{9}\times\frac{1}{4}x^4\right]_{0.2}^{0.4}$$

$$I = \left[\frac{x^2}{3} - \frac{4x^3}{27} + \frac{x^4}{18}\right]_{0.2}^{0.4}$$

$$I = \left[\frac{(0.4)^2}{3} - \frac{4(0.4)^3}{27} + \frac{(0.4)^4}{18}\right] - \left[\frac{(0.2)^2}{3} - \frac{4(0.2)^3}{27} + \frac{(0.4)^4}{18}\right]$$

$$I = \frac{223}{6750} \approx 0.03304 \quad (4\text{ s.f.})$$

(4)

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\,dx$$

giving your answer in the form *a* ln *b* + *c*, where *a*, *b* and *c* are constants to be found.

$u = 3 + x \qquad u_2 = 3 + 0.4 = 3.4$

$\frac{du}{dx} = 1 \qquad u_1 = 3 + 0.2 = 3.2$

$x = u - 3$

$\therefore du = dx$

$$I = \int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,du$$

$$I = \int_{3.2}^{3.4} \frac{6u}{u^2} - \frac{18}{u^2}\,du$$

$$I = \int_{3.2}^{3.4} \frac{6}{u} - 18u^{-2}\,du$$

$$I = \left[6\ln u - \frac{18u^{-1}}{-1}\right]_{3.2}^{3.4}$$

$$I = \left[6\ln u + \frac{18}{u}\right]_{3.2}^{3.4}$$

$$I = \left[6\ln 3.4 + \frac{18}{3.4}\right] - \left[6\ln 3.2 + \frac{18}{3.2}\right]$$

$$I = 6\ln\frac{3.4}{3.2} + \frac{18}{3.4} - \frac{18}{3.2}$$

$$I = 6\ln\frac{17}{16} - \frac{45}{136}$$

(5)

(Total for question = 13 marks)

(Q13 9MA0/02, June 2023)

Q13.

The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

i) At $t=4$, $V=32000$ At $t=11$, $V=50000$

$32000 = Ap^4$ — ① $50000 = Ap^{11}$ — ②

② ÷ ① $\Rightarrow$ $\dfrac{50000}{32000} = \dfrac{Ap^{11}}{Ap^4}$

$\dfrac{25}{16} = p^7$

$\therefore p = \left(\dfrac{25}{16}\right)^{1/7} \approx \underline{\underline{1.0658}}$ (4 d.p.)

ii) $A = \dfrac{32000}{1.0658^4}$

$A = 24796.80 \approx \underline{\underline{24800}}$

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,
(ii) the value of the constant p.

i) A represents the value of the car on 1st January 2001

ii) p represents the proportional increase in the value of the car each year.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

$$100000 = Ap^t$$

$$\frac{100\,000}{A} = p^t$$

$$\ln\left|\frac{100\,000}{A}\right| = \ln p^t$$

$$\ln\left|\frac{100\,000}{A}\right| = t\ln p$$

$$t = \frac{\ln\left|\frac{100\,000}{A}\right|}{\ln p}$$

$$t = \frac{\ln\left|\frac{100\,000}{24800}\right|}{\ln 1.0658}$$

$$t = 21.872$$

Value exceeds £100,000 21 years after 2001 hence in the year $\underline{\underline{2022}}$

(4)

(Total for question = 10 marks)

(**Q12 9MA0/01, June 2018**)

Q14.

A sequence u_1, u_2, u_3 ... is defined by

$$u_1 = 35$$
$$u_{n+1} = u_n + 7\cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that $u_2 = 40$

(ii) Find the value of u_3 and the value of u_4

i) $u_2 = 35 + 7\cos\frac{\pi}{2} - 5(-1)^1$

$u_2 = 40$

ii) $u_3 = 40 + 7\cos\pi - 5(-1)^2 = 28$

$u_4 = 28 + 7\cos\frac{3}{2}\pi - 5(-1)^3 = 33$

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

i) $u_5 = 35$

ii) $\sum_{1}^{25} u_r = \frac{24}{4}(35 + 40 + 28 + 33) + 35 = 851$

(3)

(Total for question = 6 marks)

(**Q02 9MA0/02, June 2023**)

Q15.

A sequence u_1, u_2, u_3, ... is defined by

$$u_{n+1} = ku_n - 5$$

$$u_1 = 6$$

where k is a positive constant.

Given that $u_3 = -1$

(a) show that

$$6k^2 - 5k - 4 = 0$$

$u_2 = 6k - 5$

$u_3 = -1 = k(6k-5) - 5$

$-1 = 6k^2 - 5k - 5$

$6k^2 - 5k - 4 = 0$ shown.

(2)

(b) Hence

(i) find the value of k,

(ii) find the value of $\sum_{r=1}^{3} u_r$

i) $(3k-4)(2k+1) = 0$

$\therefore$ $\underline{\underline{k = \frac{4}{3}}}$ $k = -\frac{1}{2}$ reject because $k > 0$

ii) $u_1 = 6$

$u_2 = 6k - 5 = 6\left(\frac{4}{3}\right) - 5 = 3$

$u_3 = -1$

$\therefore$ $\sum_{1}^{3} u_r = 6 + 3 - 1 = \underline{\underline{8}}$

(3)

(Total for question = 5 marks)

(**Q04 9MA0/02, June 2024**)

Q16.

A sequence of terms a_1, a_2, a_3,… is defined by

$$a_1 = 4$$

$$a_{n+1} = ka_n + 3$$

where k is a constant.

Given that

- $\sum_{n=1}^{3} a_n = 12$
- all terms of the sequence are different

find the value of k

$a_2 = 4k + 3$

$a_3 = k(4k+3) + 3$

$a_3 = 4k^2 + 3k + 3$

$\therefore\ 12 = 4 + 4k + 3 + 4k^2 + 3k + 3$

$4k^2 + 7k - 2 = 0$

$\therefore\ (4k-1)(k+2) = 0$

$\therefore\ k = \frac{1}{4}$ reject because when $k = \frac{1}{4}$ all terms in the sequence are the same. $\underline{\underline{k = -2}}$

(Total for question = 4 marks)

(**Q03 9MA0/02/M, June 2025**)

Q17.

In this question you must show all stages of your working.

Given that the first three terms of a geometric sequence are

$$8\sin\theta \qquad 3\sin 2\theta \qquad 2+2\cos 2\theta$$

and $0<\theta<\dfrac{\pi}{2}$

(a) show that

$$\sin\theta = \frac{8}{9}$$

$$\frac{3\sin 2\theta}{8\sin\theta} = \frac{2+2\cos 2\theta}{3\sin 2\theta}$$

$$(3\sin 2\theta)^2 = 8\sin\theta\,(2+2\cos 2\theta)$$

$$9\sin^2 2\theta = 16\sin\theta + 16\sin\theta\cos 2\theta$$

$$9[2\sin\theta\cos\theta]^2 = 16\sin\theta + 16\sin\theta\,(\cos^2\theta - \sin^2\theta)$$

$$36\sin^2\theta\cos^2\theta = 16\sin\theta + 16\sin\theta\,(1-2\sin^2\theta)$$

$$36\sin^2\theta(1-\sin^2\theta) = 16\sin\theta + 16\sin\theta - 32\sin^3\theta$$

$$36\sin^2\theta - 36\sin^4\theta = 32\sin\theta - 32\sin^3\theta$$

$$36\sin^4\theta - 32\sin^3\theta - 36\sin^2\theta + 32\sin\theta = 0$$

$$4\sin\theta\,(9\sin^3\theta - 8\sin^2\theta - 9\sin\theta + 8) = 0$$

$4\sin\theta = 0$ $\qquad 9\sin^3\theta - 8\sin^2\theta - 9\sin\theta + 8 = 0$

$$(\sin\theta+1)(\sin\theta-1)(9\sin\theta-8) = 0$$

$\therefore \sin\theta = -1 \qquad \sin\theta = 1 \qquad \sin\theta = \dfrac{8}{9}$

Shown.

(4)

Hence, or otherwise

(b) prove that S_∞ exists

$a = T_1 = 8\sin\theta = 8 \times \frac{8}{9} = \frac{64}{9}$

$T_2 = 3\sin 2\theta = 6\sin\theta\cos\theta$

$T_2 = 6\sin\theta \times \sqrt{1-\sin^2\theta}$

$T_2 = 6 \times \frac{8}{9} \times \sqrt{1-\left(\frac{8}{9}\right)^2}$

$T_2 = \frac{16}{3}\sqrt{\frac{17}{81}}$

$r = \dfrac{\frac{16}{3}\sqrt{\frac{17}{81}}}{64/9} = \frac{3}{4}\sqrt{\frac{17}{81}} = \frac{\sqrt{17}}{12} \approx 0.3436$

$\left|\frac{\sqrt{17}}{12}\right| < 1$ hence S_∞ does exist

(2)

(c) find the value of $S_\infty - S_{10}$ giving your answer to 2 significant figures.

$S_\infty - S_{10}$

$\sum_1^\infty - \sum_1^{10}$

$$\frac{\frac{64}{9}}{1-\frac{\sqrt{17}}{12}} - \frac{\frac{64}{9}\left[1-\left(\frac{\sqrt{17}}{12}\right)^{10}\right]}{1-\frac{\sqrt{17}}{12}} \approx \underline{\underline{2.5 \times 10^{-4}}}$$

(3)

(Total for question = 9 marks)

(**Q13 9MA0/02/M, June 2025**)

Q18.

A sequence of numbers a_1, a_2, a_3, ... is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

$$a_2 = \frac{k(2+2)}{2} = \frac{4k}{2} = 2k$$

$$a_3 = \frac{k(2k+2)}{2k} = \frac{2k^2}{2k} + \frac{2k}{2k} = k+1$$

$$a_1 = a_4 = \frac{k(k+1+2)}{k+1} = \frac{k(k+3)}{k+1}$$

$$2 = \frac{k^2+3k}{k+1}$$

$$2k + 2 = k^2 + 3k$$

$$k^2 + k - 2 = 0 \quad \text{Shown.}$$

(3)

(b) For this sequence explain why $k \neq 1$

$$k^2 + k - 2 = 0$$

$$(k-1)(k+2) = 0$$

$\therefore$ $k = 1$ reject $\quad$ $\underline{\underline{k = -2}}$

because when $k=1$, the terms in the sequence are all 2 which would not be a periodic sequence of order 3

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

$a_1 = 2$

$a_2 = 2k = 2(-2) = -4$

$a_3 = k+1 = -2+1 = -1$

$$\sum_{1}^{80} a_r = \frac{78}{3}(2-4-1) + 2 - 4 = \underline{\underline{-80}}$$

(3)

(Total for question = 7 marks)

(**Q13 9MA0/01, Oct 2020**)

Q19.

The sequence u_1, u_2, u_3,... is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

$u_1 = 2$

$u_2 = k - \frac{24}{2} = k - 12$

$u_3 = k - \frac{24}{k-12} = \frac{k(k-12) - 24}{k-12} = \frac{k^2 - 12k - 24}{k-12}$

$2 + 2(k-12) + \frac{k^2-12k-24}{k-12} = 0$

$2[k-12] + 2(k-12)^2 + k^2 - 12k - 24 = 0$

$2k - 24 + 2(k^2 - 24k + 144) + k^2 - 12k - 24 = 0$

$2k - 24 + 2k^2 - 48k + 288 + k^2 - 12k - 24 = 0$

$3k^2 - 58k + 240 = 0$ shown.

(3)

(b) Find the value of k, giving a reason for your answer.

$(3k - 40)(k - 6) = 0$

$\therefore k = \frac{40}{3}$ reject $\underline{\underline{k = 6}}$

because k must be an integer

(2)

(c) Find the value of u_3

$u_3 = k - \frac{24}{k-12}$

$6 - \frac{24}{6-12}$

$6 - \frac{24}{-6}$

$u_3 = 6 + 4 = \underline{\underline{10}}$

(1)

(Total for question = 6 marks)

(**Q03 9MA0/01, Oct 2021**)

Q20.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

$$20\,000 \times 1.08^2 = \underline{\underline{£23,328}}$$

(1)

(b) find the first year when the yearly profit will exceed £65 000

$$65000 = 20000 \times 1.08^n$$

$$\frac{13}{4} = 1.08^n$$

$$\ln 3.25 = \ln 1.08^n$$

$$\ln 3.25 = n \ln 1.08$$

$$n = \frac{\ln 3.25}{\ln 1.08}$$

$n = 15.3$ Thus when $n = 16$ will be first time when profits exceed £65000

The year this happens will be $n+1$, $16+1 = 17$

Thus in $\underline{\underline{\text{Year } 17}}$

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

$$S_{20} = \frac{20000\left[1.08^{20} - 1\right]}{1.08 - 1}$$

$$S_{20} \simeq \underline{\underline{£915\,000}}$$

(2)

(Total for question = 6 marks)

(**Q05 9MA0/01, Oct 2021**)

Q21.

Show that

$$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^{n}\cos(180n)^{\circ}=\frac{9}{28}$$

$T_1 = \left(\frac{3}{4}\right)^1 \cos 180 = -\frac{3}{4}$

$T_2 = \left(\frac{3}{4}\right)^2 \cos 360 = \frac{9}{16}$

$T_3 = \left(\frac{3}{4}\right)^3 \cos 540 = -\frac{27}{64}$

$r = \frac{9}{16} \div -\frac{3}{4} = -\frac{3}{4}$

$a = \frac{9}{16}$

We want from term 2 to infinity

So we can use T_2 as a

$\sum_{2}^{\infty}\left(\frac{3}{4}\right)^{n}\cos 180n = \frac{9/16}{1-(-3/4)} = \underline{\underline{\frac{9}{28}}}$

(Total for question = 3 marks)

(**Q09 9MA0/02, Oct 2021**)

Q22.

The first three terms of a geometric sequence are

$$3k + 4 \qquad 12 - 3k \qquad k + 16$$

where k is a constant.

(a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0$$

$$r = \frac{12 - 3k}{3k + 4} = \frac{k + 16}{12 - 3k}$$

$$(12 - 3k)^2 = (k + 16)(3k + 4)$$

$$144 - 72k + 9k^2 = 3k^2 + 52k + 64$$

$$6k^2 - 124k + 80 = 0$$

$$3k^2 - 62k + 40 = 0 \quad \text{shown.}$$

(2)

Given that the sequence converges,

(b) (i) find the value of k, giving a reason for your answer,

(ii) find the value of S_∞

(5)

i) $3k^2 - 62k + 40 = 0$

$(k-20)(3k-2) = 0$

$k = 20 \quad k = \frac{2}{3}$

$r = \frac{k+16}{12-3k}$

At $k = 20$

$r = \frac{20+16}{12-3(20)}$

$r = -\frac{3}{4}$

$\left|-\frac{3}{4}\right| = \frac{3}{4}$

$\frac{3}{4} < 1$

At $k = \frac{2}{3}$

$r = \frac{2/3 + 16}{12 - 3(2/3)}$

$r = \frac{5}{3}$

$\frac{5}{3} > 1$

reject $k = 2/3$

$k = 20$ leads to a converging sequence.

ii) $S_\infty = \frac{3k+4}{1-(-3/4)} = \frac{3(20)+4}{1.75}$

$S_\infty = \frac{256}{7}$

(Total for question = 7 marks)

(**Q09 9MA0/01, June 2023**)

Q23.

The first 3 terms of a geometric sequence are

$$3^{4k-5} \qquad 9^{7-2k} \qquad 3^{2(k-1)}$$

where k is a constant.

(a) Using algebra and making your reasoning clear, prove that $k = \frac{5}{2}$

$$r = \frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}}$$

$$\frac{3^{2(7-2k)}}{3^{4k-5}} = \frac{3^{2k-2}}{3^{2(7-2k)}}$$

$$3^{14-4k-4k+5} = 3^{2k-2-14+4k}$$

$$19-8k = 6k-16$$

$$35 = 14k$$

$$k = \frac{35}{14} = \frac{5}{2} \quad \text{shown.}$$

(3)

(b) Hence find the sum to infinity of the geometric sequence.

$$r = 3^{6k-16} = 3^{6(5/2)-16} = \frac{1}{3}$$

$$a = 3^{4k-5} = 3^{4(2.5)-5} = 243$$

$$S_\infty = \frac{243}{1-1/3} = \frac{729}{2}$$

(3)

(Total for question = 6 marks)

(Q09 9MA0/01, June 2024)

Q24.

A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

$4 \times 6 = 24$ minutes (1st 4km)

$1.05 \times 6 = 6.3$ minutes (5th km)

$6.3 \times 1.05 = 6.615$ minutes (6th km)

1st 6km $= 24 + 6.3 + 6.615 = 36.915$ minutes

$0.915 \times 60 = 54.9 \approx 55$ seconds

1st 6km = 36 minutes 55 seconds

(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leq r \leq 20$, is

$$6 \times 1.05^{r-4}$$

5th km : 6×1.05

6th km : 6×1.05^2

⋮

rth km : $6 \times 1.05^{r-4}$

Shown.

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.

$$\text{Total} = 24 + \sum_{r=5}^{20} 6 \times 1.05^{r-4}$$

$$a = 6 \times 1.05 = 6.3$$

$$r = 1.05$$

$$S_{16} = \frac{6.3(1.05^{16} - 1)}{1.05 - 1}$$

$$\text{Total} = 24 + 126(1.05^{16} - 1)$$

Total = 173.0422 minutes

$0.0422 \times 60 = 2.532$ seconds $\simeq$ 3 seconds

$\therefore$ Total Time = 173 minutes 3 seconds

(4)

(Total for question = 7 marks)

(**Q11 9MA0/01, June 2019**)

Q25.

A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

$a = 28$

$T_6 = a + 5d$

$115 = 28 + 5d$

$d = \frac{115 - 28}{5} = 17.4$

$\therefore \quad T_3 = 28 + 2(17.4) = \underline{\underline{62.8 \text{ km}h^{-1}}}$

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

$a = 28$

$T_6 = ar^5$

$115 = 28r^5$

$r = \left(\frac{115}{28}\right)^{1/5} \approx 1.3265$

$T_5 = ar^4$

$\therefore \quad 28 \times \left(\frac{115}{28}\right)^{4/5}$

$T_5 = 86.6941671 \ldots$

$T_5 \approx \underline{\underline{86.7 \text{ km}h^{-1}}}$

(3)

(Total for question = 6 marks)

(**Q05 9MA0/01, Oct 2020**)

Q26.

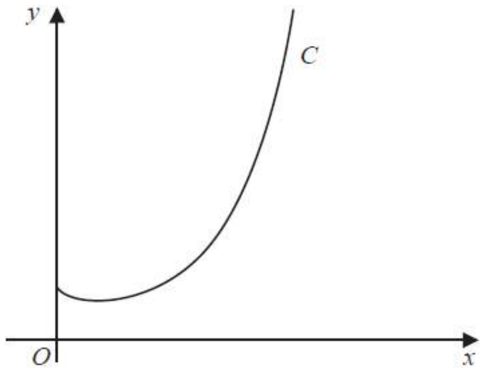

Figure 8

Figure 8 shows a sketch of the curve *C* with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the *x* coordinate of the turning point of *C*.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

$y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

using implicit differentiation on left and product rule on right

$\frac{1}{y}\frac{dy}{dx} = 1(\ln x) + \frac{1}{x} \times x$

$\frac{dy}{dx} = y \ln x + y$

Turning Point at $\frac{dy}{dx} = 0$, $y \ln x + y = 0$

$y(\ln x + 1) = 0$

$y = 0 \quad \ln x = -1$

$e^{\ln x} = e^{-1}$

$x = e^{-1}$

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

$y = x^x$

At $x = 1.5$

$y = 1.5^{1.5} = 1.837117307...$

At $x = 1.6$

$y = 1.6^{1.6} = 2.121250571...$

$1.837117307... < 2$

$2.121250571... > 2$ and C is continuous hence $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

$x_1 = 1.5$

$x_2 = 2(1.5)^{1-1.5}$

$x_2 = 2(1.5)^{-0.5} = 1.632993162...$

$x_3 = 2(1.632993162)^{1-1.632993162} = 1.466264596....$

$x_4 = 2(1.466264596)^{1-1.466264596} = 1.673135301...$

$x_4 \simeq \underline{\underline{1.673}}$ (3 d.p.)

(2)

(d) describe the long-term behaviour of x_n

x_n oscillates between 1 and 2 with a period of 2.

(2)

(Total for question = 11 marks)

(**Q11 9MA0/02, June 2019**)

Q27.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

i) $50x^2 + 38x + 9 = A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$

At $x = 1/2$

$\frac{50}{4} + 19 + 9 = C(4.5)^2$

$C = 2$

At $x = -2/5$

$50\left(\frac{4}{25}\right) - \frac{76}{5} + 9 = B\left(1 - 2\left(-2/5\right)\right)$

$B = \frac{5}{9}\left(8 + 9 - \frac{76}{5}\right)$

$B = \underline{1}$

ii) At $x = 0$

$9 = A(2)(1) + B + C(2)^2$

$9 = 2A + 1 + 8$

$0 = 2A$

$\therefore A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + ...$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

i) $f(x) = \dfrac{1}{(5x+2)^2} + \dfrac{2}{1-2x}$

$f(x) = (5x+2)^{-2} + 2(1-2x)^{-1}$

$(2+5x)^{-2} + 2(1-2x)^{-1}$

$\left[2\left(1+\frac{5}{2}x\right)\right]^{-2} + 2(1-2x)^{-1}$

$2^{-2}\left(1+\frac{5}{2}x\right)^{-2} + 2(1-2x)^{-1}$

$f(x) = \frac{1}{4}\left(1+\frac{5}{2}x\right)^{-2} + 2(1-2x)^{-1}$

$f(x) = \frac{1}{4}\left[1 + (-2)\left(\frac{5}{2}x\right) + (-2)(-3)\left(\frac{5}{2}x\right)^2\left(\frac{1}{2}\right)\right] + 2\left[1 + (-1)(-2x) + (-1)(-2)(-2x)^2\left(\frac{1}{2}\right)\right]$

$f(x) = \frac{1}{4}\left[1 - 5x + \frac{75}{4}x^2\right] + 2\left[1 + 2x + 4x^2\right]$

$f(x) = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + 2 + 4x + 8x^2 + ...$

$f(x) = \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + ...$

ii) Valid for $\left|\frac{5}{2}x\right| < 1$ and $|-2x| < 1$

$\underbrace{|x| < \frac{2}{5}}$ and $|x| < \frac{1}{2}$

This condition is more restrictive hence we will select it

$|x| < \frac{2}{5}$

(7)

(Total for question = 11 marks)

(**Q09 9MA0/01, Oct 2021**)

Q28.

(a) Find the first four terms, in ascending powers of *x*, of the binomial expansion of

$$(1+8x)^{\frac{1}{2}}$$

giving each term in simplest form.

$$1 + \left(\tfrac{1}{2}\right)(8x) + \left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)(8x)^2\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)(8x)^3\left(\tfrac{1}{6}\right)$$

$$1 + 4x - 8x^2 + 32x^3 + \ldots$$

(3)

(b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$.

There is no need to carry out the calculation.

Subsitute $x = \frac{1}{32}$ into $(1+8x)^{1/2}$

This results in $\frac{\sqrt{5}}{2}$

Then Substitute $x = \frac{1}{32}$ into $1 + 4x - 8x^2 + 32x^3$

To get approximation for $\sqrt{5}$, multiply the result by 2.

(2)

(Total for question = 5 marks)

(**Q01 9MA0/01, Oct 2020**)

Q29.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio *r* and first term *a*.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r}$$

$S_n = a + ar + ar^2 + \ldots + ar^{n-3} + ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$

$S_n(1-r) = a - ar^n$

$\therefore \quad S_n = \dfrac{a(1-r^n)}{1-r}$ Shown.

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of *r*.

If $S_{10} = 4S_5$

$\dfrac{a(1-r^{10})}{1-r} = \dfrac{4a(1-r^5)}{1-r}$

$1-r^{10} = 4(1-r^5)$

$1-r^{10} = 4-4r^5$

$r^{10} - 4r^5 + 3 = 0$

$(r^5-3)(r^5-1) = 0$

$\therefore \ r = 3^{1/5} \qquad r = 1$

$r = \sqrt[5]{3}$ reject because $r \neq 1$ to prevent $1-r$ which is the denominator being 0

(4)

(Total for question = 8 marks)

(**Q15 9MA0/02, Oct 2020**)

Q30.

(i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

$$\sum_{4}^{\infty} = \sum_{1}^{\infty} - \sum_{1}^{3}$$

$a = T_1 = 20\left(\frac{1}{2}\right)^1 = 10$

$T_2 = 20\left(\frac{1}{2}\right)^2 = 5$

$T_3 = 20\left(\frac{1}{2}\right)^3 = 2.5$

$r = \frac{5}{10} = 0.5$

$S_\infty = \sum_{1}^{\infty} = \frac{10}{1-0.5} = 20$

$S_3 = \sum_{1}^{3} = 10 + 5 + 2.5 = 17.5$

$\therefore \sum_{4}^{\infty} 20\left(\frac{1}{2}\right)^r = 20 - 17.5 = \underline{\underline{2.5}}$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

$\sum_{1}^{48} \log_5 |n+2| - \sum_{1}^{48} \log_5 |n+1|$

$\left(\log_5 3 + \log_5 4 + \ldots + \log_5 49 + \log_5 50\right) - \left(\log_5 2 + \log_5 3 + \ldots + \log_5 48 + \log_5 49\right)$

$\log_5 50 - \log_5 2$

$\log_5 \frac{50}{2}$

$\log_5 25$

$\log_5 5^2$

$2 \log_5 5$

$2 \times 1 = \underline{\underline{2}}$ Shown.

(3)

(Total for question = 6 marks)

(**Q08 9MA0/02, June 2019**)

Q31.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where *a* and *b* are rational constants to be found.

$$\int u\,dv = uv - \int v\,du$$

$u = \ln x \quad dv = x^3\,dx$

$\frac{du}{dx} = \frac{1}{x} \quad \int dv = \int x^3\,dx$

$\therefore du = \frac{dx}{x} \quad v = \frac{x^4}{4}$

$$I = \frac{x^4}{4}\ln x - \int \frac{x^4}{4} \times \frac{dx}{x}$$

$$I = \frac{x^4}{4}\ln x - \int \frac{x^3}{4}\,dx$$

$$I = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2}$$

$$I = \left[\frac{(e^2)^4}{4}\ln e^2 - \frac{(e^2)^4}{16}\right] - \left[\frac{1}{4}\ln 1 - \frac{1}{16}\right]$$

$$I = \left(\frac{e^8}{4} \times 2 - \frac{e^8}{16} + \frac{1}{16}\right.$$

$$I = \frac{7}{16}e^8 + \frac{1}{16}$$

(Total for question = 5 marks)

(**Q12 9MA0/01, June 2022**)

Q32.

(i) Show that $\sum_{r=1}^{16}(3 + 5r + 2^r) = 131\,798$

$$\sum_{1}^{16} 3 + \sum_{1}^{16} 5r + \sum_{1}^{16} 2^r$$

$$16(3) + \frac{16}{2}\left[2(5) + 15(5)\right] + \frac{2\left[2^{16} - 1\right]}{2 - 1}$$

$$48 + 680 + 131070$$

$$\underline{\underline{131\,798}}$$

(4)

(ii) A sequence u_1, u_2, u_3, ... is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

$u_1 = 2/3$

$u_2 = 3/2$

$u_3 = 2/3$

$u_4 = 3/2$

$$\sum_{1}^{100} u_r = \frac{100}{2}\left(\frac{2}{3} + \frac{3}{2}\right) = \underline{\underline{\frac{325}{3}}}$$

(3)

(Total for question = 7 marks)

(**Q04 9MA0/02, June 2018**)

Q33.

(a) Express 2 cos θ + 8 sin θ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

$R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$2\cos\theta + 8\sin\theta$

$R\sin\alpha = 8 \qquad R\cos\alpha = 2$

$\alpha = \tan^{-1}\left(\frac{8}{2}\right) \qquad R = \sqrt{8^2+2^2}$

$\alpha = \underline{1.326}$ (3 d.p.) $\qquad R = \underline{2\sqrt{17}}$

$2\sqrt{17}\cos(\theta - 1.326)$

(3)

The first three terms of an arithmetic sequence are

$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

i) $S_9 = \frac{9}{2}\left[2\cos x + 8\sin x\right] \qquad a = \cos x, \; d = \sin x$

$S_9 = 4.5\left[2\sqrt{17}\cos(x - 1.326)\right]$

$S_9 = 9\sqrt{17}\cos(x - 1.326)$

$S_{9_{max}} = 9\sqrt{17} \times 1 = 9\sqrt{17}$

ii) $S_{9_{max}}$ occurs at $\cos(x - 1.326) = 1$

$x - 1.326 = 0$

$\underline{x = 1.326}$

(3)

(Total for question = 6 marks)

(**Q08 9MA0/02, June 2023**)

Q34.

In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm \frac{1}{\sqrt{k}}$, where k is an integer to be found.

$$S_\infty = \frac{8}{7} \times S_6$$

$$\frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$$

$$7a = 8a - 8ar^6$$

$$8ar^6 = a$$

$$8r^6 = 1$$

$$r^6 = \frac{1}{8}$$

$$r^6 = \left(\frac{1}{2}\right)^3$$

$$r^2 = \frac{1}{2}$$

$$r = \frac{1}{\pm\sqrt{2}}$$

$\therefore k = 2$

(4)

(Total for question = 4 marks)

(**Q10 9MA0/02, Specimen papers**)

Q35.

(a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \dots$$

where k is a rational constant to be found.

$\sqrt{4-x} \Rightarrow \sqrt{4\left(1-\frac{x}{4}\right)} \Rightarrow \sqrt{4}\left(\sqrt{1-\frac{x}{4}}\right) \Rightarrow 2\sqrt{1-\frac{x}{4}} \Rightarrow 2\left(1-\frac{x}{4}\right)^{1/2}$

$2\left(1-\frac{x}{4}\right)^{1/2} = 2\left[1 + \left(\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)^2\left(\frac{1}{2}\right) + \dots\right]$

$2\left[1 - \frac{x}{8} - \frac{x^2}{128} + \dots\right]$

$2 - \frac{x}{4} - \frac{x^2}{64} + \dots$

$\therefore \quad k = -\frac{1}{64}$

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

Approximation only valid : when $\left|-\frac{x}{4}\right| < 1$

$|x| < 4$

$1 < 4$ thus $x = 1$ is valid and can be used.

(1)

(Total for question = 5 marks)

(**Q07 9MA0/02, Specimen papers**)

Q36.

(a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1+\frac{5}{2}x-\frac{5}{8}x^2$

$$\left(\sqrt{1+4x}\right)\left(\frac{1}{\sqrt{1-x}}\right) \Rightarrow (1+4x)^{1/2}(1-x)^{-1/2}$$

$$(1+4x)^{1/2} = 1+\left(\tfrac{1}{2}\right)(4x)+\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)(4x)^2\left(\tfrac{1}{2}\right)+\dots$$

$$(1+4x)^{1/2} = 1+2x-2x^2+\dots$$

$$(1-x)^{-1/2} = 1+\left(-\tfrac{1}{2}\right)(-x)+\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)(-x)^2\left(\tfrac{1}{2}\right)+\dots$$

$$(1-x)^{-1/2} = 1+\frac{x}{2}+\frac{3x^2}{8}+\dots$$

$$\left(1+2x-2x^2+\dots\right)\left(1+\frac{x}{2}+\frac{3x^2}{8}+\dots\right)$$

$$1+\frac{x}{2}+\frac{3}{8}x^2+2x+x^2-2x^2+\dots$$

$$1+\frac{5}{2}x-\frac{5}{8}x^2+\dots$$

(6)

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \frac{1}{2}$

Approximation is valid if $|4x| < 1$ $\quad |-x| < 1$

$\underbrace{|x| < \frac{1}{4}}_{\text{more restrictive}}$ $\quad |x| < 1$

Hence condition for validity is only $|x| < \frac{1}{4}$

$\frac{1}{2} > \frac{1}{4}$ hence not valid.

(1)

(c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form.

$$\sqrt{\frac{1+\frac{4}{11}}{1-\frac{1}{11}}} = \sqrt{\frac{3}{2}} = \sqrt{1.5}$$

$$\sqrt{6} = \sqrt{4 \times 1.5} = \sqrt{4} \times \sqrt{1.5} = 2 \times \sqrt{1.5}$$

$$\therefore \sqrt{6} \approx 2 \times \left[1 + \frac{5}{2}\left(\frac{1}{11}\right) - \frac{5}{8}\left(\frac{1}{11}\right)^2\right]$$

$$\sqrt{6} \approx \frac{1183}{484}$$

(3)

(Total for question = 10 marks)

(**Q11 9MA0/01, June 2018**)