Name: ____

Trigonometry Mark Scheme

Date:

Time: 270

Total marks available: 270

Total marks achieved: ______

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Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Uses the common ratio $\frac{5+2\sin\theta}{12\cos\theta}=\frac{6\tan\theta}{5+2\sin\theta}$ o.e.	M1	3.1a
	Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\times12\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(a) Alt	**(a) Alternative example:**		
	Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ $\Rightarrow 12\cos\theta\left(\frac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$	M1	3.1a
	Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(b)	$4\sin^2\theta-52\sin\theta+25=0\Rightarrow \sin\theta=\frac{1}{2}\left(,\frac{25}{2}\right)$	M1	1.1b
	$\theta=\frac{5\pi}{6}$	A1	1.2
		(2)	

(c)	Attempts a value for either a or r e.g. $a=12\cos\theta=\ 12\times-\frac{\sqrt{3}}{2}$ or $r=\frac{5+2\sin\theta}{12\cos\theta}=\frac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$	M1	3.1a
	$"a"=-6\sqrt{3}$ and $"r"=-\frac{1}{\sqrt{3}}$ o.e.	A1	1.1b
	Uses $S_\infty=\frac{a}{1-r}=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$	dM1	2.1
	Rationalises denominator $S_\infty=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\frac{-18}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$	ddM1	1.1b
	$(S_\infty=)9(1-\sqrt{3})$	A1	2.1
		(5)	
			(10 marks)
Notes:			

(a)

M1: For the key step in using the ratio of $\frac{a_2}{a_1}=\frac{a_3}{a_2}$

dM1: Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Alternative:

M1: Expresses the 2nd and 3rd terms in terms of the first term and the common ratio and eliminates "r"

dM1: Multiplies up and uses $\tan\theta \times \cos\theta = \sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

> Other approaches may be seen in (a) and can be marked in a similar way e.g. M1 for correctly obtaining an equation in θ using the GP, M1 for applying $\tan\theta \times \cos\theta = \sin\theta$ or equivalent and eliminating fractions, A1 as above
>
> Example: $u_2 = \dfrac{u_1 \times u_3}{u_2} \Rightarrow 5 + 2\sin\theta = \dfrac{12\cos\theta \times 6\tan\theta}{5 + 2\sin\theta}$ **M1**
>
> $\Rightarrow (5 + 2\sin\theta)^2 = 72\sin\theta$ **dM1**
>
> $25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$
>
> $\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0$ * **A1**

(b)

M1: Attempts to solve $4\sin^2\theta - 52\sin\theta + 25 = 0$. Must be clear they have found $\sin\theta$ and not e.g. just x from $4x^2 - 52x + 25 = 0$. Working does not need to be seen but see general guidance for solving a 3TQ if necessary. Note that the $\dfrac{25}{2}$ does not need to be seen.

A1: $\theta = \dfrac{5\pi}{6}$ and no other values unless they are rejected or the $\dfrac{5\pi}{6}$ clearly selected here and not in (c)

A minimum requirement in (b) is e.g. $\sin\theta = \dfrac{1}{2}$, $\theta = \dfrac{5\pi}{6}$

Do **not** allow 150° for $\dfrac{5\pi}{6}$

(c) Allow full marks in (c) if e.g. $\theta = \dfrac{\pi}{6}$ is their answer to (b) but $\theta = \dfrac{5\pi}{6}$ is used here.

or if e.g. $\theta = \dfrac{5\pi}{6}$ is their answer to (b) but $\theta = \dfrac{\pi}{6}$ is used here allow the M marks only.

M1: For attempting a value (exact or decimal) for either a or r using **their** θ

E.g. $a = 12\cos\theta = \left(12 \times -\dfrac{\sqrt{3}}{2}\right)$ or $r = \dfrac{5 + 2\sin\theta}{12\cos\theta} = \left(\dfrac{5 + 2 \times \frac{1}{2}}{12 \times -\frac{\sqrt{3}}{2}}\right)$ oe e.g. $r = \dfrac{6\tan\theta}{5 + 2\sin\theta} = \left(\dfrac{6 \times -\frac{1}{\sqrt{3}}}{5 + 2 \times \frac{1}{2}}\right)$

A1: Finds both $a = -6\sqrt{3}$ and $r = -\dfrac{1}{\sqrt{3}}$ which can be left unsimplified but $\sin\theta = \dfrac{1}{2}, \cos\theta = -\dfrac{\sqrt{3}}{2}$ and $\tan\theta = -\dfrac{\sqrt{3}}{3}$ (if required) must have been used.

dM1: Uses both values of "a" and "r" with the equation $S_\infty = \dfrac{a}{1-r} = \dfrac{-6\sqrt{3}}{1 + \frac{1}{\sqrt{3}}}$ to create an expression involving surds where a and r have come from appropriate work and $|r| < 1$

Depends on the first method mark.

ddM1: Rationalises denominator. The denominator must be of the form $p \pm q\sqrt{3}$ oe e.g. $p + \dfrac{q}{\sqrt{3}}$

Depends on both previous method marks.

Note that stating e.g. $\dfrac{k}{p + q\sqrt{3}} \times \dfrac{p - q\sqrt{3}}{p - q\sqrt{3}}$ or $\dfrac{k}{p + \frac{q}{\sqrt{3}}} \times \dfrac{p - \frac{q}{\sqrt{3}}}{p - \frac{q}{\sqrt{3}}}$ is sufficient.

A1: Obtains $\left(S_\infty =\right)9\left(1-\sqrt{3}\right)$

Note that full marks are available in (c) for the use of $\theta = 150°$

Note also that marks may be implied in (c) by e.g.

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\theta}{1-\dfrac{5+2\sin\theta}{12\cos\theta}} = \frac{144\cos^2\theta}{12\cos\theta - 5 - 2\sin\theta} = \frac{144\cos^2\dfrac{5\pi}{6}}{12\cos\dfrac{5\pi}{6} - 5 - 2\sin\dfrac{5\pi}{6}}$$

$$= \frac{108}{-6-6\sqrt{3}} = \frac{108}{-6-6\sqrt{3}} \times \frac{-6+6\sqrt{3}}{-6+6\sqrt{3}} = \frac{-648+648\sqrt{3}}{-72} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1 implied dM1 ddM1 A1

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\dfrac{5\pi}{6}}{1-\dfrac{5+2\sin\dfrac{5\pi}{6}}{12\cos\dfrac{5\pi}{6}}} \quad \text{or e.g.} \quad S_\infty = \frac{a}{1-r} = \frac{12\cos\dfrac{\pi}{6}}{1-\dfrac{5+2\sin\dfrac{\pi}{6}}{12\cos\dfrac{\pi}{6}}}$$

And nothing else

scores M1A0dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\dfrac{5\pi}{6}}{1-\dfrac{5+2\sin\dfrac{5\pi}{6}}{12\cos\dfrac{5\pi}{6}}} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\dfrac{\pi}{6}}{1-\dfrac{5+2\sin\dfrac{\pi}{6}}{12\cos\dfrac{\pi}{6}}} = 9\left(1+\sqrt{3}\right)$$

Scores M1A0dM1ddM0A0

$S_\infty = 9\left(1-\sqrt{3}\right)$ with no working scores no marks

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(**Q15 9MA0/02, June 2022**)

Q2.

Question	Scheme	Marks	AOs
(a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2\left(\frac{\theta}{360}\right)$ {where θ is in degrees} • correct formula is $\pi r^2\left(\frac{40}{360}\right)$	B1	2.3
		(1)	
(b) Way 1	{Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
(b) Way 2	{Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
			(3 marks)

Notes for Question	
(a)	
B1:	Explains that the formula use is only valid when angle AOB is applied in radians. See scheme for examples of suitable explanations.
(b)	Way 1
M1:	Correct application of the sector formula using a correct value for θ in radians
Note:	Allow exact equivalents for θ e.g. $\theta = \frac{40\pi}{180}$ or θ in the range [0.68, 0.71]
A1*:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units
(b)	Way 2
M1:	Correct application of the sector formula in degrees
A1:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units.
Note:	Allow exact equivalents such as $\frac{50}{18}\pi$
Note:	Allow M1 A1 for $500\left(\frac{\pi}{180}\right) = \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}

(**Q03 9MA0/02, June 2019**)

Q3.

Question	Scheme	Marks	AOs
(a)	Angle $AOB = \frac{\pi-\theta}{2}$	B1	2.2a
		(1)	
(b)	Area = $2\times\frac{1}{2}r^2\left(\frac{\pi-\theta}{2}\right)+\frac{1}{2}(2r)^2\theta$	M1	2.1
	$=\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta+2r^2\theta=\frac{3}{2}r^2\theta+\frac{1}{2}r^2\pi=\frac{1}{2}r^2(3\theta+\pi)$*	A1*	1.1b
		(2)	
(c)	Perimeter = $4r+2r\left(\frac{\pi-\theta}{2}\right)+2r\theta$	M1	3.1a
	$=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)

B1: Deduces the correct expression for angle AOB

Note that $\frac{180-\theta}{2}$ scores B0

(b)

M1: Fully correct strategy for the area using their angle from (a) appropriately.

Need to see $2\times\frac{1}{2}r^2\alpha$ or just $r^2\alpha$ where α is their angle in terms of θ from part (a) + $\frac{1}{2}(2r)^2\theta$ with or without the brackets.

A1*: Correct proof. For this mark you can condone the omission of the brackets in $\frac{1}{2}(2r)^2\theta$ as long as they are recovered in subsequent work e.g. when this term becomes $2r^2\theta$

The first term must be seen expanded as e.g. $\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta$ or equivalent

(c)

M1: Fully correct strategy for the perimeter using their angle from (a) appropriately

Need to see $4r+2r\alpha+2r\theta$ where α is their angle from part (a) in terms of θ

A1: Correct simplified expression

Note that some candidates may change the angle to degrees at the start and all marks are available e.g.

(a) $\frac{180-\frac{180\theta}{\pi}}{2}$

(b) $2\left(\frac{180-\frac{180\theta}{\pi}}{2}\right)\times\frac{1}{360}\times\pi r^2+\frac{\theta}{360}\times\frac{180}{\pi}\times\pi(2r)^2=\frac{1}{2}\pi r^2-\frac{1}{2}r^2\theta+2r^2\theta=\frac{1}{2}r^2(3\theta+\pi)$

(c) $4r+2\left(\frac{180-\frac{180\theta}{\pi}}{2}\right)\times\frac{1}{360}\times2\pi r+\frac{180\theta}{\pi}\times\frac{1}{360}\times2\pi(2r)=4r+\pi r+r\theta$

Q4.

Question	Scheme	Marks	AOs
(a)	$OC \times 2.3 = 27.6$	M1	1.1b
	e.g. $OC = \frac{27.6}{2.3} = 12\text{ m}$ *	A1*	2.1
		(2)	
(b)	e.g. $(2AOB =)\ \pi - 2.3$	M1	1.1b
	$\frac{\pi - 2.3}{2} \Rightarrow 0.421$ rad *	A1*	2.1
		(2)	
(c)	Area $OCDE = \frac{1}{2} \times 12^2 \times 2.3$	M1	1.1b
	$= 165.6\ (\text{m}^2)$ (accept awrt 166)	A1	1.1b
	$(OB =)\ \frac{35 - 27.6}{2} + 12 = 15.7\text{ m}$	B1	2.1
	Area of OAB (or OFG) $= \frac{1}{2} \times "15.7" \times 7.5 \times \sin 0.421 \quad (= 24.0...\text{m}^2)$	M1	1.1b
	Total area = $165.6 + 2 \times "24.1"$	dM1	3.1a
	$=$ awrt $214\ (\text{m}^2)$	A1	1.1b
		(6)	
			(10 marks)

Notes

(a)

M1: Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in (may be labelled differently in their equation). Values just need to be embedded in an equation or accept an expression for OC e.g. $\frac{27.6}{2.3}$.

May work in degrees which is acceptable.

Condone an alternative letter being used to denote OC such as r

Alternatively, they use $l = r\theta$ with $r = 12$ and $\theta = 2.3$ and verify that $l = 27.6\text{ m}$

A1*: Achieves an expression for OC before proceeding to $OC = 12$ (m) with no errors seen (condone lack of units)

They must show at least $\frac{27.6}{2.3} \Rightarrow OC = 12$ (m) which can score M1A1*

$r = \frac{27.6}{2.3} = 12$ is M1A1* (condone alternative letters for OC)

BUT e.g. $\frac{27.6}{2.3} = 12$(m) on its own is M1A0*

e.g. $OC \times 2.3 = 27.6 \Rightarrow OC = 12$(m) is M1A0*

In the alternative method they verify $l = 27.6$ and conclude that $OC = 12\text{ m}$

We must see the calculation $12 \times 2.3 = 27.6$ and conclude that $OC = 12$ (m)

e.g. arc $= 12 \times 2.3 = 27.6$ so $OC = 12$(m) is M1A1* whereas $12 \times 2.3 = 27.6$ is M1A0*

Also allow e.g. if $OC = 12$(m) then $12 \times 2.3 = 27.6$ ✓ is M1A1*

If they work in degrees and use rounded values this scores A0* (If they work with e.g. $\frac{414}{\pi}$ to keep the angle exact then A1* can still be scored)

(b)

M1: Attempts to subtract 2.3 from π (which may be implied by an expression for AOB which is not the given answer)

e.g. $\frac{1}{2}(\pi-2.3)$ or $\frac{\pi}{2}-1.15$ score M1

May work in degrees e.g. $180-\text{awrt}132$ is M1

Condone invisible brackets e.g. $\pi-2.3\div2$ can still score M1.

A1*: Achieves 0.421 (rad) with no errors seen (ignore any side working which is not part of their main solution). Look for a correct expression which is awrt 0.421 before proceeding to the answer. Alternatively, they may write

e.g $2AOB=\pi-2.3\ (=0.8415..)\Rightarrow AOB=0.421$

Condone if they do not round their answer at the end to 0.421.

Condone lack of units. Condone poor labelling of other angles and it does not require AOB = to score this mark, but do not accept e.g. $ABO=$

If they work in degrees then withhold this mark if they do not show the conversion back to radians.

e.g. $\frac{\pi-2.3}{2}=0.421$(rad) is M1A1*

e.g. $\frac{180-\text{awrt}131.8}{2}\div\frac{180}{\pi}=0.421$ (rad) is M1A1* (conversion from degrees to radians seen)

e.g. $\pi-2.3\div2=0.421$ (rad) is M1A0* (invisible/lack of brackets)

e.g. $\pi-2.3=\frac{0.842...}{2}=0.421$ M1A0* (incorrect joined statement)

(c)

M1: Attempts to use $A=\frac{1}{2}r^2\theta$ with $r=12$ and $\theta=2.3$ The values embedded in the formula is sufficient for this mark. May be implied by a correct answer or further work. Look out for alternative more complex ways to find the area of the sector. e.g. area of semicircle – area of two sectors with $r=12$ and $\theta=0.421$

A1: awrt 166 (may be implied by later work)

B1: A correct expression or value for the length OB or OF which may be a part of a calculation (may see 15.7 in the equation to find the area of AOB)

M1: Attempts to find the area of at least one of the two congruent triangles using their OB found from $\frac{35-27.6}{2}+12\ (=15.7)$, $OA=7.5$ and $\theta=0.421$ in $\frac{1}{2}\times OA\times OB\times\sin C$ (may work in degrees)

Be aware that omitting sine in the formula may give a value close to the area of the triangle which would be M0. **Condone use of** $\theta=0.4$ **or** $\theta=0.42$ if they have rounded angle AOB.

The values embedded in the expression is sufficient to score the mark or may be implied by the value.

Look out for more complex methods to find the area of one or both of the two congruent triangles

e.g. they may split the congruent triangle into two right angled triangles and add the separate areas.

Other alternatives

e.g. finding the area of the trapezium $ABFG$:

$BF = 2\times 15.7\cos 0.421$

Area of $AOB = \frac{1}{2}\left(\left(\frac{15+2\times 15.7\cos 0.421}{2}\right)\times 15.7\sin 0.421 - \frac{1}{2}\times 15.7^2\times \sin 2.3\right)$ o.e.

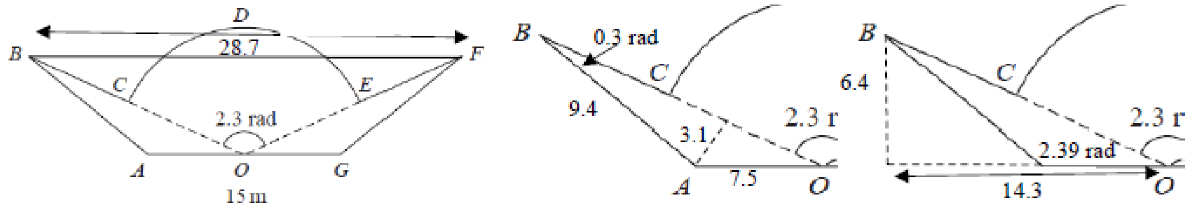
e.g. finding the length AB and either angle OAB or angle ABO:

$AB^2 = 15.7^2 + 7.5^2 - 2\times 15.7\times 7.5\times \cos 0.421 \Rightarrow AB = 9.37...$

$\frac{\sin ABO}{7.5} = \frac{\sin 0.421}{9.37...} \Rightarrow ABO = 0.333...$ or $\frac{\sin OAB}{15.7} = \frac{\sin 0.421}{9.37...} \Rightarrow OAB = 2.4$

Area of ABO $\frac{1}{2}\times 15.7\times 9.37...\times \sin 0.333...$ or $\frac{1}{2}\times 7.5\times 9.37...\times \sin 2.4$

Approximate values are shown below for some of the lengths you may see in calculations:



dM1: Solves the problem by combining appropriate areas together which result in the total area of the concert stage (usually the sum of the areas of the two congruent triangles and the area of the sector).
It is dependent on the previous method marks and the B mark.

A1: awrt 214 (m^2) (condone lack of units). Must follow from a correct method. Isw if they round incorrectly.

(**Q08 9MA0/01, June 2023**)

Q5.

Question	Scheme	Marks	AOs
(a)	Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	M1	1.1b
	$\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t} \quad \left(= 2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t = \frac{2\pi}{3}$ in $\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t} = \left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal $= -\frac{1}{dy/dx} = \left(\frac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$	M1	3.1a
	Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$	A1	1.1b
	Finds $\cos t = \frac{5}{6}, \cancel{-\frac{1}{2}}$	M1	2.4
	Substitutes their $\cos t = \frac{5}{6}$ into $x = 2\cos t$, $y = \sqrt{3}\cos 2t$,	M1	1.1b
	$Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
			(13 marks)

Notes:

(a)

M1: Attempts $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ and achieves a form $k\frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\frac{\sin t\cos t}{\sin t}$

A1: Scored for a correct answer, either $\frac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$

(b)

M1: For substituting $t=\frac{2\pi}{3}$ in their $\frac{dy}{dx}$ which must be in terms of t

M1: Uses the gradient of the normal is the negative reciprocal of the value of $\frac{dy}{dx}$. This may be seen in the equation of l.

B1: States or uses (in their tangent or normal) that $P=\left(-1,-\frac{\sqrt{3}}{2}\right)$

M1: Uses their numerical value of $-1/\frac{dy}{dx}$ with their $\left(-1,-\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P

A1*: This is a proof and all aspects need to be correct. Correct answer only $2x-2\sqrt{3}y-1=0$

(c)

M1: For substituting $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$ to produce an equation in t. Alternatively candidates could use $\cos 2t=2\cos^2 t-1$ to set up an equation of the form $y=Ax^2+B$.

M1: Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic equation in $\cos t$
In the alternative method it is for combining their $y=Ax^2+B$ with $2x-2\sqrt{3}y-1=0$ to get an equation in just one variable

A1: For the correct quadratic equation $12\cos^2 t-4\cos t-5=0$
Alternatively the equations in x and y are $3x^2-2x-5=0$ $12\sqrt{3}y^2+4y-7\sqrt{3}=0$

M1: Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P.

M1: Substitutes their $\cos t=\frac{5}{6}$ or their $t=\arccos\left(\frac{5}{6}\right)$ in $x=2\cos t$ and $y=\sqrt{3}\cos 2t$
If a value of x or y has been found it is for finding the other coordinate.

A1: $Q=\left(\frac{5}{3},\frac{7}{18}\sqrt{3}\right)$. Allow $x=\frac{5}{3}, y=\frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

Q6.

Question	Scheme	Marks	AOs
	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta\sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta\times 3\theta}$ and attempts to simplify	M1	2.1
	$=\frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

M1: Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in the given expression.

See below for description of marking of $\cos 4\theta$

M1: Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$

$\rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta\times 3\theta}$ and attempts to simplify.

Condone missing bracket on the 4θ so $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of θ

Look for an answer of k but condone $k\theta$ following a slip

A1: Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$.

Eg. $\frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta\times 3\theta}=\frac{8\theta^2}{6\theta}=\frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

Alt: $\frac{1-\cos 4\theta}{2\theta\sin 3\theta}=\frac{1-\left(1-2\sin^2 2\theta\right)}{2\theta\sin 3\theta}=\frac{2\sin^2 2\theta}{2\theta\sin 3\theta}=\frac{2\times(2\theta)^2}{2\theta\times 3\theta}=\frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1-2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$

M1 For both of the above and attempts to simplify to a single term.

A1 $\frac{4}{3}$ oe

(**Q01 9MA0/01, June 2018**)

Q7.

Question	Scheme	Marks	AOs
(a)	$\tan\theta + \cot\theta \equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv \dfrac{1}{\frac{1}{2}\sin 2\theta}$	M1	2.1
	$\equiv 2\operatorname{cosec}2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta + \cot\theta = 1 \Rightarrow \sin 2\theta = 2$ AND no real solutions as $-1 \leqslant \sin 2\theta \leqslant 1$	B1	2.4
		(1)	
		(5 marks)	

Notes:
(a)
M1: Writes $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
A1: Achieves a correct intermediate answer of $\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$
M1: Uses the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$
A1*: Completes proof with no errors. This is a given answer.
Note: There are many alternative methods. For example $\tan\theta + \cot\theta \equiv \tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{\tan^2\theta + 1}{\tan\theta} \equiv \dfrac{\sec^2\theta}{\tan\theta} \equiv \dfrac{1}{\cos^2\theta \times \dfrac{\sin\theta}{\cos\theta}} \equiv \dfrac{1}{\cos\theta \times \sin\theta}$ then as the main scheme.
(b)
B1: Scored for sight of $\sin 2\theta = 2$ and a reason as to why this equation has no real solutions. Possible reasons could be $-1 \leqslant \sin 2\theta \leqslant 1$........and therefore $\sin 2\theta \neq 2$ or $\sin 2\theta = 2 \Rightarrow 2\theta = \arcsin 2$ which has no answers as $-1 \leqslant \sin 2\theta \leqslant 1$

(**Q09 9MA0/01, Specimen papers**)

Q8.

Question	Scheme	Marks	AOs
(a)	$D=5+2\sin(30\times6.5)°=\text{awrt } 4.48\text{m}$ with units	B1	3.4
		(1)	
(b)	$3.8=5+2\sin(30t)°\Rightarrow\sin(30t)°=-0.6$	M1 A1	1.1b 1.1b
	$t=10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t=6.5$ into $D=5+2\sin(30t)°$ and stating $D=awrt$ 4.48m. The units must be seen somewhere in (a). So allow when $D=4.482..=4.5$ m

Allow the mark for a correct answer without any working.

(b)

M1: For using $D=3.8$ and proceeding to $\sin(30t)°=k,\quad |k|\le1$

A1: $\sin(30t)°=-0.6$ This may be implied by any correct answer for t such as $t=7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of t for their $\sin(30t)°=k$ after $t=8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)°=-0.6\Rightarrow30t=216.9°$ as well but this gives $t=7.2$)

For the correct $\sin(30t)°=-0.6\Rightarrow30t=323.1\Rightarrow t=\text{awrt } 10.8$

For the incorrect $\sin(30t)°=+0.6\Rightarrow30t=396.9\Rightarrow t=\text{awrt } 13.2$

So award this mark if you see $30t=\text{invsin } their -0.6$ to give the first value of t where $30t > 255$

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

(**Q08 9MA0/01, June 2018**)

Q9.

Question	Scheme	Marks	AOs
(a)	Uses $s = r\theta \Rightarrow 3 = r \times 0.4$	M1	1.2
	$\Rightarrow OD = 7.5$ cm	A1	1.1b
		(2)	
(b)	Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm	M1	3.1a
	Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12 - 7.5)^2 \times (\pi - 0.4)$	M1	1.1b
	$= 27.8\text{cm}^2$	A1ft	1.1b
		(3)	
			(5 marks)

Notes:

(a)

M1: Attempts to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A1: $OD = 7.5$ cm (An answer of 7.5cm implies the use of a correct formula and scores both marks)

(b)

M1: $AOB = \pi - 0.4$ may be implied by the use of AOB = awrt 2.74 or uses radius is (12 – their '7.5')

M1: Follow through on their radius (12 – their OD) and their angle

A1ft: Allow awrt 27.8 cm^2. (Answer 27.75862562). Follow through on their (12 – their '7.5') Note: Do not follow through on a radius that is negative.

(**Q02 9MA0/01, Specimen papers**)

Q10.

Question	Scheme	Marks	AOs
	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = ...$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
			(4 marks)

Notes:

B1: States or uses $\frac{1}{2}r^2\theta = 11$ This may be implied with an embedded found value for θ

B1: States or uses $2r + r\theta = 4r\theta$ or equivalent

M1: Full method to find $r = ...$ This involves combining the equations to eliminate θ or find θ
The initial equations must be of the same "form" (see **) but condone slips when attempting to solve.

It cannot be scored from impossible values for θ Hence only score if $0 < \theta < 2\pi$ FYI $\theta = \frac{2}{3}$ radians

Allow this to be scored from equations such as $..r^2\theta = 11$ and ones that simplify to $..r = ..r\theta$ **

Allow their $2r + r\theta = 4r\theta \Rightarrow \theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $2r + r\theta = 4r\theta \Rightarrow r\theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $\frac{1}{2}r^2\theta = 11 \Rightarrow \theta = \frac{..}{r^2}$ then substitute into their $2r + r\theta = 4r\theta \Rightarrow r = ..$

A1: $r = \sqrt{33}$ only but isw after a correct answer.

The whole question can be attempted using θ in degrees.

B1: States or uses $\frac{\theta}{360} \times \pi r^2 = 11$

B1: States or uses $2r + \frac{\theta}{360} \times 2\pi r = 4 \times \frac{\theta}{360} \times 2\pi r$

(**Q03 9MA0/01, June 2018**)

Q11.

Question	Scheme	Marks	AOs
(a)	$a = 60$	B1	3.1b
	$2 = "60" - b(-20)^2 \Rightarrow b = ...$	M1	3.4
	$H = 60 - 0.145(t - 20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29\cos(9t + 180)° + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
			(7 marks)

Notes
(a) B1: $a = 60$ (may be seen in their final equation of the model or implied by 60 substituted for a in the model) M1: Attempts to find b by substituting in $t = 0$, $H = 2$ and their a and proceeding to a value for b. May be seen as two simultaneous equations formed: $2 = a - b(-20)^2$ and $60 = a - b(20-20)^2$ proceeding to a value for b A1: $H = 60 - 0.145(t-20)^2$ or equivalent such as $H = -\frac{29}{200}t^2 + 5.8t + 2$ or $H = 60 - \frac{29}{200}(t-20)^2$ isw once a correct equation for the model is seen. Must be in terms of H and t. If they just state $a = 60$, $b = 0.145$ then A0 A correct answer with no working seen scores full marks.
(b) B1: 2 cao (condone lack of units) This can be scored even if their model in (a) is incorrect (they may have used symmetry to determine this value)
(c) M1: $(\alpha =)\ 180$ or $(\beta =)\ 31$ Condone $(\alpha =)\ \pi$ A1: $H = 29\cos(9t + 180)° + 31$ or equivalent e.g. $H = -29\cos(9t) + 31$ isw once a correct equation for the model is seen. Must be in terms of H and t. If they just state $\alpha = 180$, $\beta = 31$ then A0. A correct equation with no working seen scores both marks. Does not require the degree symbol.
(d) B1: Score for a reason which makes reference to any of • the alternative model allows repetition (allow phrases e.g. "multiple cycles", "repeated circuits", "cyclical", "periodic", "loops around", "the original model can only go up and down once") • the alternative model after 2 minutes the carriage will be back at the start (e.g."at 2 mins, $H = 2$ ") • the original/quadratic model after 40 seconds (or any time after this) will be negative (e.g. "the height will be negative which cannot happen") • the original model after 2 minutes would not be back at the start Do not allow vague responses on their own e.g. "the original model is a parabola" If calculations are used then they must be correct using a correct model (allow rounded or truncated) Look for a valid reason and ignore reference to anything else as long as it does not contradict

t	0	5	10	15	20	25	30	35	40	45	50	55	60	80	100	120
h	2	27	46	56	60	56	46	27	2	-31	-71	-118	-172	-462	-868	-1390

(**Q13 9MA0/01, June 2023**)

Q12.

Question	Scheme	Marks	AOs
(a)	Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta} = \tan\theta$ It should be $\frac{\sin\theta}{\cos\theta} = \tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°) \neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
		(3 marks)	

Notes:

(a)

B1: Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta} = \tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta} = \tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta = \frac{1}{2}$ not $\tan\theta = 2$'

Accept also statements such as 'it should be $\cot\theta = 2$'

(b)

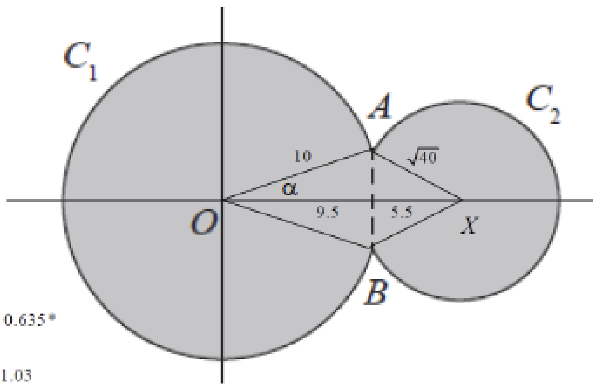
B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta = 2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°) = +ve$ and $2\sin(-26.6°) = -ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x = 5$ squared gives $x^2 = 25$ which has answers ± 5

(**Q02 9MA0/02, Specimen papers**)

Q13.

Question	Scheme	Marks	AOs
(a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos\alpha=\dfrac{\text{"9.5"}}{10}$ to find α then $\times 2$	M1	3.1a
	Angle $AOB = 2\times\arccos\left(\dfrac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635)$ $= 56.48$	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos\beta=\dfrac{15-\text{"9.5"}}{\sqrt{40}} \Rightarrow \beta=...$ (Note AXB =1.03 rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	$= 89.7$	A1	1.1b
		(4)	
			(8 marks)
Notes:			



Angle $AOB = 0.635$*

Angle $AXB = 1.03$

(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.
Look for an attempt to set up an equation in a single variable leading to a value for x or y.

A1: $x=9.5$ (or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle AOB in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100-"9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2+10^2-(\sqrt{39})^2}{2\times10\times10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy. Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)
E

(b)

M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $2\pi 10 + 2\pi\sqrt{40} - 10\times0.635...$ which is fine for this M1

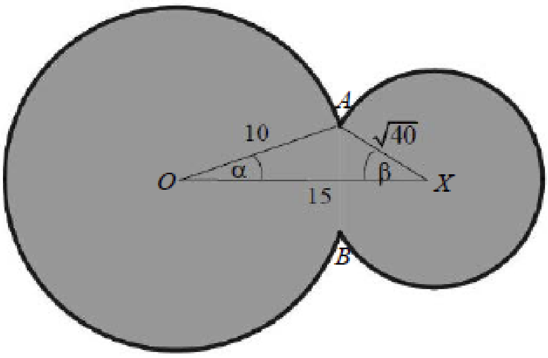
M1: Attempts to use a correct method in order to find angle AXB or AXO in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"}{15-9.5}$ and $\cos AXB = \frac{40+40-(\sqrt{39})^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.
It is dependent upon having scored both M's.

A1: awrt 89.7



(a)
M1: For the key step in attempting to find all lengths in triangle OAX, condoning slips
A1: All three lengths correct
M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

(**Q11 9MA0/01, Oct 2020**)

Q14.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha = \frac{3}{10}$	M1	1.1b
	$\alpha = 16.70°$ so $\sqrt{109}\cos(\theta + 16.70°)$	A1	1.1b
		(3)	
(b)	(i) e.g $H = 11 - 10\cos(80t)° + 3\sin(80t)°$ or $H = 11 - \sqrt{109}\cos(80t + 16.70)°$	B1	3.3
	(ii) $11 + \sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t + "16.70" = 540$	M1	3.4
	$t = \frac{540 - "16.70"}{80} = (6.54)$	M1	1.1b
	t = 6 mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H = 11 - 10\cos(90t)° + 3\sin(90t)°$		3.3
		(1)	
			(9 marks)

Notes:

(a)

B1: $R=\sqrt{109}$ Do not allow decimal equivalents

M1: Allow for $\tan\alpha = \pm\frac{3}{10}$

A1: $\alpha = 16.70°$

(b)(i)

B1: see scheme

(b)(ii)

B1ft: their $11 +$ their $\sqrt{109}$ Allow decimals here.

(c)

M1: Sets $80t + "16.70" = 540$. Follow through on their 16.70

M1: Solves their $80t + "16.70" = 540$ correctly to find t

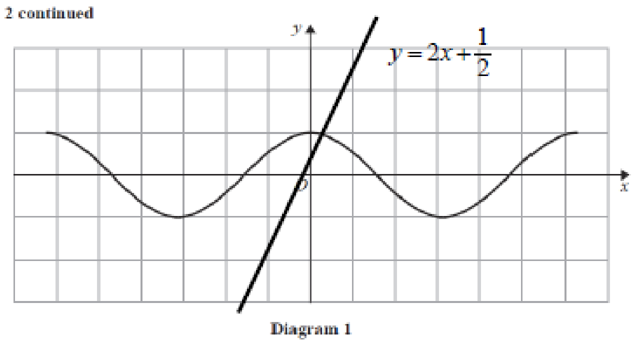
A1: t = 6 mins 32 seconds

(d)

B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

(**Q13 9MA0/02, Specimen papers**)

Q15.

Question	Scheme	Marks	AOs
(a)	 Diagram 1	B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2+\sqrt{5}$	A1	1.1b
		(3)	
			(5 marks)

(a)

B1: Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$ Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

B1: There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion.
The question asks candidates to explain but as a bare minimum allow one "intersection"

Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$ **OR** gradient of ± 2 with one intersection with $\cos x$

(b)

M1: Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

dM1: Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator. Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

A1: Allow $-2+\sqrt{5}$ or awrt 0.236.

Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

(**Q02 9MA0/01, June 2019**)

Q16.

Question	Scheme		Marks	AOs
	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ \ 0 \le t < 2\pi;\ \ C_2: x^2 + y^2 = 66$			
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$	$100\cos^2 t + 32(1-\cos^2 t) = 66$	M1	2.1
			A1	1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = \ldots$	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = \ldots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	

Way 2	$\{\cos^2 t+\sin^2 t=1\Rightarrow\}\ \left(\frac{x}{10}\right)^2+\left(\frac{y}{4\sqrt{2}}\right)^2=1\ \{\Rightarrow 32x^2+100y^2=3200\}$		M1	3.1a
	$\frac{x^2}{100}+\frac{66-x^2}{32}=1$	$\frac{66-y^2}{100}+\frac{y^2}{32}=1$	M1	2.1
			A1	1.1b
	$32x^2+6600-100x^2=3200$ $x^2=50\Rightarrow x=...$	$2112-32y^2+100y^2=3200$ $y^2=16\Rightarrow y=...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=(\text{awrt } 7.07,-4)$		A1	3.2a
			(6)	
Way 3	$\{C_2: x^2+y^2=66\Rightarrow\}\ \ x=\sqrt{66}\cos\alpha,\ y=\sqrt{66}\sin\alpha$ $\{C_1=C_2\ \Rightarrow\}\ \ 10\cos t=\sqrt{66}\cos\alpha,\quad 4\sqrt{2}\sin t=\sqrt{66}\sin\alpha$ $\{\cos^2\alpha+\sin^2\alpha=1\Rightarrow\}\ \left(\frac{10\cos t}{\sqrt{66}}\right)^2+\left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2=1$		M1	3.1a
	then continue with applying the mark scheme for Way 1			
Way 4	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66$		M1	3.1a
	$100\left(\frac{1+\cos 2t}{2}\right)+32\left(\frac{1-\cos 2t}{2}\right)=66$		M1	2.1
			A1	1.1b
	$50+50\cos 2t+16-16\cos 2t=66\ \Rightarrow\ 34\cos 2t+66=66$ $\Rightarrow\ \cos 2t=...$		dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the x-coordinate and value of the y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=(\text{awrt } 7.07,-4)$		A1	3.2a
			(6)	
	Note: Give final A0 for writing $x=5\sqrt{2},\ y=-4$ followed by $S=(-4,5\sqrt{2})$			
				(6 marks)

Notes for Question	
	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t+\cos^2 t\equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\sin t=...$ where $-1\le\sin t\le 1$ or $\cos t=...$ where $-1\le\cos t\le 1$
Note:	Condone 3rd M1 for $\sin^2 t=\frac{1}{2}\Rightarrow\sin t=\frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S=(5\sqrt{2},-4)$ or $S=(\text{awrt } 7.07,-4)$

	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t + \sin^2 t \equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of x only or y only not involving trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $x = ...$ or $y = ...$ their x^2 or their y^2 must be >0 for this mark
M1: **Note:**	See scheme their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$
	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2\alpha + \sin^2\alpha \equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1
	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1: **Note:**	Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t = ...$ where $-1 \le \cos 2t \le 1$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ \ 0 \le t < 2\pi;\ \ C_2: x^2 + y^2 = 66$		
Way 5	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1 A1	2.1 1.1b
	$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \tan t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	

	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t = ...$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

(**Q04 9MA0/02, June 2019**)

Q17.

Question	Scheme	Marks	AOs
	$1-\cos 2\theta \equiv \tan\theta \sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n\in\mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x - 5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\quad -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x - 5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x - 5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x = 0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x - 3\tan x - 5)\tan x = 0$ **or** $(1+\tan^2 x - 3\tan x - 5)(1-\cos 2x) = 0$ **or** $1+\tan^2 x - 5 = 3\tan x$	M1	2.1
	$\tan^2 x - 3\tan x - 4 = 0$	A1	1.1b
	$(\tan x - 4)(\tan x + 1) = 0 \Rightarrow \tan x = \ldots$	M1	1.1b
	$x = -\frac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question	
(a)	**Way 1**
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS $= 2\sin^2\theta$ and RHS $= 2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Notes for Question Continued	
(b)	
B1:	Deduces that the given equation yields a solution $x = 0$
M1:	For using the key step of $\sec^2 x = 1 + \tan^2 x$ and cancels/factorises out $\tan x$ or $(1 - \cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$
Note:	Allow the use of $\pm\sec^2 x = \pm 1 \pm \tan^2 x$ for M1
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$
Note:	E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1
M1:	For a correct method of solving their 3TQ in $\tan x$
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785, awrt 1.326, $-45°$, awrt $75.964°$
A1:	Only $x = -\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$
Note:	**Alternative Method (Alt 1)**

$(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ **or** $(\sec^2 x - 5)(1 - \cos 2x) = 3\tan x(1 - \cos 2x)$			
Deduces $x = 0$		B1	2.2a
$\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$ $1 - 5\cos^2 x = 3\sin x \cos x$ $1 - 5\left(\frac{1 + \cos 2x}{2}\right) = \frac{3}{2}\sin 2x$	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B\cos 2x = \pm C\sin 2x$	M1	2.1
$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ $\{3\sin 2x + 5\cos 2x = -3\}$	$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ o.e.	A1	1.1b
$\sqrt{34}\sin(2x + 1.03) = -3$	Expresses their answer in the form $R\sin(2x + \alpha) = k$; $k \neq 0$ with values for R and α	M1	1.1b
$\sin(2x + 1.03) = -\frac{3}{\sqrt{34}}$			
$x = -\frac{\pi}{4}, 1.326$		A1 A1	1.1b 1.1b

(**Q12 9MA0/02, June 2018**)

Q18.

Question	Scheme	Marks	AOs
(a)	Attempts to use both $\sin(x-60°)=\pm\sin x\cos 60°\pm\cos x\sin 60°$ $\cos(x-30°)=\pm\cos x\cos 30°\pm\sin x\sin 30°$	M1	2.1
	Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$	A1	1.1b
	Either uses $\frac{\sin x}{\cos x}=\tan x$ and attempts to make $\tan x$ the subject E.g. $(2\cos 60°-\sin 30°)\tan x=\cos 30°+2\sin 60°$ Or attempts $\sin 30°$ etc with at least two correct and collects terms in $\sin x$ and $\cos x$ E.g. $\left(2\times\frac{1}{2}-\frac{1}{2}\right)\sin x=\left(2\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\cos x$,	M1	2.1
	Proceeds to given answer showing all key steps E.g. $\frac{1}{2}\tan x=\frac{3\sqrt{3}}{2}\Rightarrow\tan x=3\sqrt{3}$ *	A1*	1.1b
		(4)	
(b)	Deduces that $x=2\theta+60°$	B1	2.2a
	$\tan(2\theta+60°)=3\sqrt{3}\Rightarrow 2\theta+60°=79.1°, 259.1°,.....$	M1	1.1b
	Correct method to find one value of θ E.g $\theta=\frac{79.1°-60°}{2}$	dM1	1.1b
	$\theta=$ awrt $9.6°, 99.6°$ (See note)	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)

M1: Attempts to use both compound angle expansions to set up an equation in $\sin x$ and $\cos x$

The terms must be correct but condone sign errors and a slip on the multiplication of 2

A1: Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$ o.e.

Note that $\cos 60°=\sin 30°$ and $\cos 30°=\sin 60°$

Also allow this mark for candidates who substitute in their trigonometric values "early"

$$2\sin x\times\frac{1}{2}-2\cos x\times\frac{\sqrt{3}}{2}=\cos x\times\frac{\sqrt{3}}{2}+\sin x\times\frac{1}{2} \text{ o.e.}$$

M1: Shows the necessary progress towards showing the given result.

There are three key moves, two of which must be shown for this mark.

- uses $\frac{\sin x}{\cos x}=\tan x$ to form an equation in just $\tan x$.
- uses exact numerical values for $\sin 30°, \sin 60°, \cos 30°, \cos 60°$ with at least two correct
- collects terms in $\sin x$ and $\cos x$ or alternatively in $\tan x$

A1*: Proceeds to the given answer with accurate work showing all necessary lines.

Examples of two proofs showing all necessary lines

E.g. I $2\sin x\cos 60° - 2\cos x\sin 60° = \cos x\cos 30° + \sin x\sin 30°$

$\sin x(2\cos 60° - \sin 30°) = \cos x(\cos 30° + 2\sin 60°)$ 1. collect terms

$(2\cos 60° - \sin 30°)\tan x = \cos 30° + 2\sin 60°$ 2. $\frac{\sin x}{\cos x} = \tan x$ so M1

$\tan x = \frac{\cos 30° + 2\sin 60°}{2\cos 60° - \sin 30°} = \frac{\frac{\sqrt{3}}{2} + \sqrt{3}}{1 - \frac{1}{2}} = 3\sqrt{3}$ 3..uses values and completes proof A1*

E.g II

$2\sin x \times \frac{1}{2} - 2\cos x \times \frac{\sqrt{3}}{2} = \cos x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{1}{2}$ 1.uses values

$\Rightarrow \frac{1}{2}\sin x = \frac{3\sqrt{3}}{2}\cos x$ 2.collects terms so M1

$\Rightarrow \tan x = 3\sqrt{3}$

3. $\frac{\sin x}{\cos x} = \tan x$ completes proof A1*

(b) Hence

B1: Deduces that $x = 2\theta + 60°$ o.e such as $\theta = \frac{x - 60°}{2}$

This is implied for sight of the equation $\tan(2\theta + 60°) = 3\sqrt{3}$

M1: Proceeds from $\tan(2\theta \pm \alpha°) = 3\sqrt{3} \Rightarrow 2\theta \pm \alpha° =$ one of $79.1°, 259.1°, ...$ where $\alpha \neq 0$

One angle for $\arctan(3\sqrt{3})$ **must** be correct in degrees or radians(3sf). FYI radian answers 1.38, 4.52

dM1: Correct method to find one value of θ from their $2\theta \pm \alpha° = 79.1°$ to $\theta = \frac{79.1° \mp \alpha°}{2}$

This is dependent upon one angle being correct, which must be in degrees, for $\arctan(3\sqrt{3})$

$\tan(2\theta + 60°) = 3\sqrt{3} \Rightarrow \theta = 9.6°$ would imply B1 M1 dM1

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range

Otherwise: Via the use of $\cos(2\theta + 30°) = \cos 2\theta\cos 30° - \sin 2\theta\sin 30°$.

$2\sin 2\theta = \cos(2\theta + 30°) \Rightarrow \tan 2\theta = \frac{\sqrt{3}}{5} \Rightarrow \theta = 9.6°, 99.6°$

The order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta = \frac{\sqrt{3}}{5}$ o.e so allow $\tan 2\theta =$ awrt 0.346 or $\tan 2\theta = \frac{\cos 30°}{2 + \sin 30°}$

Or via double angle identities $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ o.e.

M1: Attempts to use the compound angle identities to reach a form $\tan 2\theta = k$ where k is a constant not $3\sqrt{3}$ (or expression in trig terms such as cos 30 etc as seen above)

Or via double angle identities reaches a 3TQ in $\tan\theta$

dM1: Correct order of operations from $\tan 2\theta = k$ leading to $\theta = ...$

Correctly solves their $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ leading to $\theta = ...$

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range.

Note that $\tan(2\theta + 60°) = 3\sqrt{3} \Rightarrow \theta = 9.6°, 99.6°$ is acceptable for full marks

(**Q14 9MA0/01, June 2022**)

Q19.

Question	Scheme	Marks	AOs
(a)	e.g. $6.4^2 = 8^2 + 13^2 - 2(8)(13)\cos ABC$	M1	1.1b
	$(\text{angle } ABC =) \quad \cos^{-1}\left(\dfrac{8^2+13^2-6.4^2}{2\times8\times13}\right) = 0.394 \text{ (radians)*}$	A1*	1.1b
		(2)	
(b)	e.g. $(\text{Area}(ABC) =) \quad \frac{1}{2}(8)^2(0.394) \quad (= \text{awrt } 12.6)$ **or** $(\text{Area}(ABD) =) \quad \frac{1}{2}(8)(13)\sin 0.394 \quad (= \text{awrt } 20.0)$	M1	1.1b
	$(\text{Area}(R) =) \quad \frac{1}{2}(8)(13)\sin 0.394 - \frac{1}{2}(8)^2(0.394) = \ldots$	dM1	3.1a
	$(\text{Area}(R) =)$ awrt 7.34 to 7.37	A1	1.1b
		(3)	
			(5 marks)

Notes

It is acceptable in this question to work in degrees and convert if necessary.
NB Angle ABC in degrees is 22.591…°

(a)

M1: Attempts to use the cosine rule with values correctly placed. May be implied by, e.g.,
$\cos\theta = \frac{8^2+13^2-6.4^2}{2\times8\times13}$ or $\cos^{-1}\left(\frac{6.4^2-8^2-13^2}{-2\times8\times13}\right)$

Condone slips in substitution if a correct cosine formula is seen. The placement of the values should be correct. So, condone e.g. a missing 2 once a correct cosine formula is seen.

Alt 1: attempts the cosine rule and finds angle ADB $(=\text{awrt } 0.5 \text{ or } 29°)$ **or** angle BAD $(=\text{awrt } 2.2 \text{ or } 2.3 \text{ or } 129°)$ **and** uses the sine rule correctly with angle ABC involved.

Alt 2: attempts the cosine rule and finds angle ADB $(=\text{awrt } 0.5 \text{ or } 29°)$ **and** angle BAD $(=\text{awrt } 2.2 \text{ or } 2.3 \text{ or } 129°)$ **and** sums angles ADB, BAD, and ABC to π (or 180°)

Alt 3: using Pythagoras **and** simultaneous equations to find the length of AN or BN (see diagram on the next page) **and** uses e.g. $\cos ABC = \frac{BN}{8}$ or $\sin ABC = \frac{AN}{8}$

A1*: cso Correct proof.
If starting with $6.4^2 = 8^2+13^2-2(8)(13)\cos ABC$ then there must be an intermediate line such as

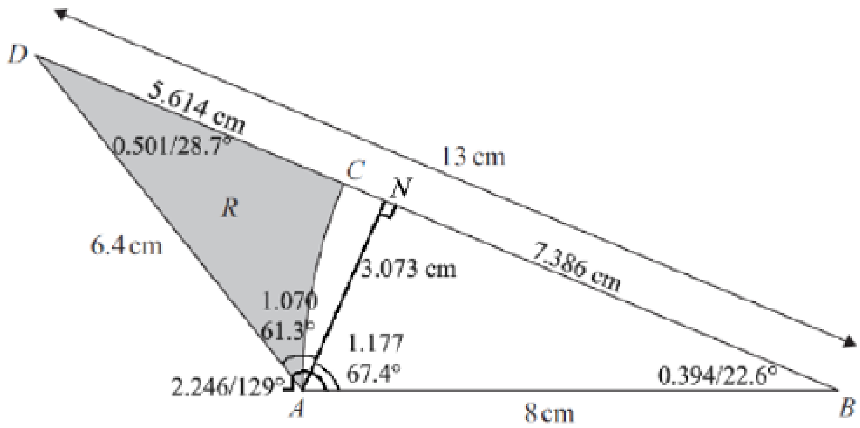
- $\cos ABC = \frac{8^2+13^2-6.4^2}{2\times8\times13}$ or angle $ABC = \cos^{-1}\left(\frac{8^2+13^2-6.4^2}{2\times8\times13}\right)$
- $\cos ABC = \text{awrt } 0.92$ or $\frac{4801}{5200}$
- angle $ABC = \text{awrt } 0.3943$

The minimum required is e.g. $\cos^{-1}\left(\frac{8^2+13^2-6.4^2}{2\times8\times13}\right)$ or $\cos\theta = \frac{8^2+13^2-6.4^2}{2\times8\times13} \to \theta = 0.394$

both of which score M1A1*.
The final value must be 0.394 and not e.g. 0.3943. There should be no obvious incorrect statements in the proof e.g. $\cos(0.92) = 0.394$ and no obvious incorrect work.
Allow the use of, e.g., θ, x, or even e.g. A throughout. Mention of radians is not required.
Those working in degrees and achieve 22.6 can convert to 0.394 without calculation for M1A1
Attempts via verification e.g. $AD^2 = 8^2+13^2-2(8)(13)\cos 0.394 \to AD = \text{awrt } 6.4$ score maximum M1A0*.

(b)	**Note: you may need to check the diagram for working.**
M1:	Attempts the area of the sector **or** the area of the triangle ABD via a **correct** method. May be implied by a full attempt at the area of R. The sector area may be implied by $\frac{1576}{125}$ or by $\frac{0.394}{2\pi}\times\pi\times8^2$ or $\frac{22.6}{360}\times\pi\times8^2$ The angle ABC is given in the question as 0.394 and should be used (or a more accurate value). Do not allow use of a different value for angle ABC (other than the angle in degrees). There are many acceptable alternative approaches to find the area of triangle ABD, e.g. using their angle ADB **or** their angle BAD **or** using right-angled triangles e.g. $\frac{1}{2}(13)(\text{"3.07"})$ **or** $\frac{1}{2}(\text{"7.39"})(\text{"3.07"})+\frac{1}{2}(\text{"5.614"})(\text{"3.07"})$ and could be implied by e.g. 11.3 + 8.6 **Alternatively**, attempts the area of the triangle ACD $\left(\text{e.g.}\ \frac{1}{2}(5)(6.4)\sin\text{"0.501"} = \text{awrt } 7.68\right)$ **or** the area of the segment (in sector ABC) $\left(\frac{1}{2}(8^2)(0.394)-\frac{1}{2}(8^2)\sin 0.394 = \text{awrt } 0.324\right)$ via a **correct** method. The area of the triangle ABC alone is insufficient for this mark. Note that $\sin 0.394$ might be seen as $\sqrt{1-\left(\frac{4801}{5200}\right)^2}$ For values, see the diagram below.
dM1:	Complete and correct method for the area of R. Requires correct attempts at both the area of the sector **and** the area of the triangle ABD **and** subtraction of the two values (or expressions). Allow sector – triangle ABD if this is then recovered by making the area positive. Alternatively, correct attempts at both the area of the triangle ACD **and** the area of the segment (in sector ABC) **and** subtraction of the two values (or expressions). Condone slips in substitution provided a correct formula is seen. May be implied by a correct answer.
A1:	awrt 7.34 to 7.37 no units required but penalise incorrect units. ISW after an acceptable answer is seen.

Helpful Diagram:



Q20.

Question	Scheme	Marks	AOs
	Examples: $4\sin\frac{\theta}{2}\approx 4\left(\frac{\theta}{2}\right),\ 3\cos^2\theta\approx 3\left(1-\frac{\theta^2}{2}\right)^2$ $3\cos^2\theta=3\left(1-\sin^2\theta\right)\approx 3\left(1-\theta^2\right)$ $3\cos^2\theta=3\frac{(\cos 2\theta+1)}{2}\approx\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	M1	1.1a
	Examples: $4\sin\frac{\theta}{2}+3\cos^2\theta\approx 4\left(\frac{\theta}{2}\right)+3\left(1-\frac{\theta^2}{2}\right)^2$ $4\sin\frac{\theta}{2}+3\cos^2\theta=4\left(\frac{\theta}{2}\right)+3\left(1-\sin^2\theta\right)\approx 2\theta+3\left(1-\theta^2\right)$ $4\sin\frac{\theta}{2}+3\cos^2\theta=4\sin\frac{\theta}{2}+3\frac{(\cos 2\theta+1)}{2}\approx 4\left(\frac{\theta}{2}\right)+\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	dM1	1.1b
	$=2\theta+3\left(1-\theta^2+\ldots\right)=3+2\theta-3\theta^2$	A1	2.1
		(3)	
			(3 marks)

Notes

M1: Attempts to use at least one correct approximation **within the given expression**.

Either $\sin\frac{\theta}{2}\approx\frac{\theta}{2}$ or $\cos\theta\approx 1-\frac{\theta^2}{2}$ or e.g. $\sin\theta\approx\theta$ if they write $\cos^2\theta$ as $1-\sin^2\theta$ or e.g. $\cos 2\theta\approx 1-\frac{(2\theta)^2}{2}$ (condone missing brackets) if they write $\cos^2\theta$ as $\frac{1+\cos 2\theta}{2}$.

Allow sign slips only with any identities used but the appropriate approximations must be applied.

dM1: Attempts to use correct approximations **with the given expression** to obtain an expression in terms of θ only. **Depends on the first method mark.**

A1: Correct terms following correct work. Allow the terms in any order and ignore any extra terms if given correct or incorrect.

(**Q04 9MA0/02, Oct 2021**)

Q21.

Question	Scheme	Marks	AOs
	One of $\theta\tan 2\theta = \theta \times 2\theta$ or $1-\cos 3\theta = 1-\left(1-\frac{(3\theta)^2}{2}\right)$ or equivalents.	B1	1.1a
	$\frac{\theta\tan 2\theta}{1-\cos 3\theta} = \frac{\theta \times 2\theta}{1-\left(1-\frac{(3\theta)^2}{2}\right)}$	M1	2.1
	$=\frac{4}{9}$ or exact equivalent.	A1	1.1b
		(3)	
			(3 marks)

Notes
B1: Award this mark for $\theta\tan 2\theta = \theta \times 2\theta$ or $1-\cos 3\theta = 1-\left(1-\frac{(3\theta)^2}{2}\right)$ or equivalents. May be seen when working on numerator or denominator separately or within the fraction. This is a B mark so if awarding for $\cos 3\theta$ do not condone missing brackets e.g. $1-\frac{3\theta^2}{2}$ unless they are recovered or are implied by subsequent work.
M1: Attempts to use both correct small angle approximations in the given expression. For this mark they must have attempted to use $\tan 2\theta = 2\theta$ and $\cos 3\theta = 1-\frac{(3\theta)^2}{2}$ in the given expression but condone poor bracketing e.g. $\frac{\theta\times 2\theta}{1-\left(1-\frac{3\theta^2}{2}\right)}$ or e.g. $\frac{\theta\times 2\theta}{1-1-\frac{3\theta^2}{2}}$ Do not allow e.g. $\frac{\theta\times 2\theta}{1-\frac{(3\theta)^2}{2}}$ as this suggests they are approximating $\frac{\theta\tan 2\theta}{\cos 3\theta}$
A1: Correct value. Do not allow rounded decimals e.g. 0.444 but allow if recurring decimals are clearly indicated e.g. $0.\dot{4}$ Do not allow e.g. $\frac{2}{4.5}$. Ignore any units if given. Isw once a correct answer is seen.

Examples:

$\frac{\theta\times 2\theta}{1-\left(1-\frac{3\theta^2}{2}\right)}=\frac{4}{3}\left(\text{or } -\frac{4}{3}\right)$	scores B1M1A0 (Missing brackets not recovered)
$\frac{\theta\times 2\theta}{1-\frac{(3\theta)^2}{2}}$	scores B1M0A0 (Missing "1 –" in the denominator so M0)
$\frac{\theta\times 2\theta}{1+\left(1-\frac{(3\theta)^2}{2}\right)}$	scores B1M0A0 (Has "1 +" in the denominator so M0)
$\frac{\theta\times\theta}{1-\left(1-\frac{3\theta^2}{2}\right)}=\ldots$	scores B0M0A0 (The B mark could be recovered but M0 because of the incorrect numerator)
$\frac{\theta\times 2\theta}{1-\left(1-\frac{3\theta^2}{2}\right)}=\frac{2\theta^2}{\frac{9\theta^2}{2}}=\frac{2}{9}$	scores B1M1A0 (Missing brackets recovered)
$\frac{\theta\times 2\theta}{1-\left(1-\left(\frac{3\theta}{2}\right)^2\right)}$	Scores B1M0A0 (The denominator suggests an incorrect expansion – unless it was recovered.)

	$\dfrac{\theta \times 2\theta}{\frac{9\theta^2}{2}} = \dfrac{2}{18}$	B1M1A0 (The B1 is awarded for the numerator but can be implied by the denominator. The M1 is implied)
	$\dfrac{\theta \times 2\theta}{1-\left(1-\frac{3\theta^2}{2}\right)} = \dfrac{4}{9}$	B1M1A1 (The correct value implies correct recovery of missing brackets.)

Note that other approaches are possible using identities.
In such cases we will allow <u>correct</u> work leading to an expression that if terms in θ^3 and higher can be ignored will lead to $\frac{4}{9}$
But to score the M mark they must be using correct identities and correct approximations but condone bracketing errors as in the main scheme.

Examples:

$$\frac{\theta \tan 2\theta}{1-\cos 3\theta} = \frac{\theta \times \dfrac{\sin 2\theta}{\cos 2\theta}}{1-\cos 3\theta} = \frac{\theta \times \dfrac{2\theta}{1-\dfrac{(2\theta)^2}{2}}}{1-\left(1-\dfrac{(3\theta)^2}{2}\right)} = \frac{2\theta^2}{1-2\theta^2} \times \frac{2}{9\theta^2} = \frac{4\theta^2}{9\theta^2-18\theta^4}$$

$$= \frac{4\theta^2}{9\theta^2} = \frac{4}{9}$$

Scores B1M1A1

Similarly: $$\frac{\theta \tan 2\theta}{1-\cos 3\theta} = \frac{\theta \times \sin 2\theta}{\cos 2\theta(1-\cos 3\theta)} = \frac{\theta \times 2\theta}{\left(1-\dfrac{(2\theta)^2}{2}\right)\left(1-\left(1-\dfrac{(3\theta)^2}{2}\right)\right)} = \frac{2\theta^2}{1-2\theta^2} \times \frac{2}{9\theta^2} \text{ etc.}$$

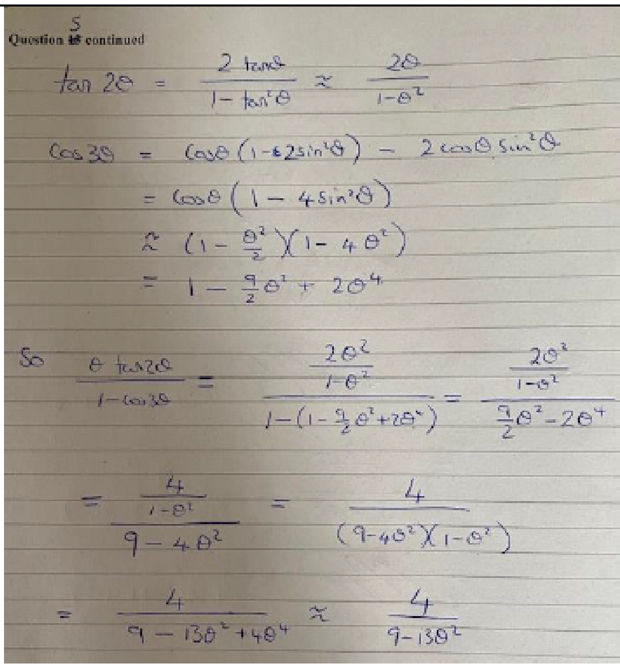
$$\frac{4\theta^2}{9\theta^2-18\theta^4} = \frac{4\theta^2}{9\theta^2} = \frac{4}{9}$$

Scores B1M1A1

$$\frac{\theta \tan 2\theta}{1-\cos 3\theta} = \frac{\theta \times \sin 2\theta}{\cos 2\theta(1-\cos 3\theta)} = \frac{\theta \times 2\theta}{\left(1-\dfrac{(2\theta)^2}{2}\right)\left(1-\left(1-\dfrac{(3\theta)^2}{2}\right)\right)} = \frac{2\theta^2}{1-2\theta^2} \times \frac{2}{9\theta^2}$$

$$= \frac{4\theta^2}{9\theta^2-18\theta^4} = \frac{4}{9-18\theta^2} = \frac{4}{9}$$

Scores B1M1A0
(They cannot just assume the term in θ^2 is 0 unless they provide a convincing limiting argument e.g. $\lim_{\theta \to 0} \dfrac{4}{9-18\theta^2} = \dfrac{4}{9}$ or equivalent)



Question 5 continued

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \approx \frac{2\theta}{1-\theta^2}$$

$$\cos 3\theta = \cos\theta(1-2\sin^2\theta) - 2\cos\theta\sin^2\theta$$

$$= \cos\theta(1-4\sin^2\theta)$$

$$\approx \left(1-\frac{\theta^2}{2}\right)(1-4\theta^2)$$

$$= 1-\frac{9}{2}\theta^2+2\theta^4$$

So

$$\frac{\theta\tan 2\theta}{1-\cos 3\theta} = \frac{\frac{2\theta^2}{1-\theta^2}}{1-\left(1-\frac{9}{2}\theta^2+2\theta^4\right)} = \frac{\frac{2\theta^2}{1-\theta^2}}{\frac{9}{2}\theta^2-2\theta^4}$$

$$= \frac{\frac{4}{1-\theta^2}}{9-4\theta^2} = \frac{4}{(9-4\theta^2)(1-\theta^2)}$$

$$= \frac{4}{9-13\theta^2+4\theta^4} \approx \frac{4}{9-13\theta^2}$$

Scores B1M1A0

$$\frac{\theta\tan 2\theta}{1-\cos 3\theta} = \frac{\theta\times\frac{2\tan\theta}{1-\tan^2\theta}}{1-\left(4\cos^3\theta-3\cos\theta\right)} = \frac{\theta\times\frac{2\theta}{1-\theta^2}}{1-\left(4\left(1-\frac{\theta^2}{2}\right)^3-3\left(1-\frac{\theta^2}{2}\right)\right)}$$

Scores B1M1A0

Note that attempts to use expansions in higher powers of θ should be sent to review.

(**Q05 9MA0/02, June 2024**)

Q22.

Question	Scheme	Marks	AOs
(a)	Attempts $\cos^2(2x) \approx \left(1-\frac{(2x)^2}{2}\right)^2 = ...(1-4x^2+4x^4)$	M1	1.1b
	$1-\cos^2(2x) \approx 1-(1-4x^2+4x^4) = 4x^2-4x^4$ *	A1*	1.1b
		(2)	
(b)	$\frac{1-\cos^2(2x)}{\sin\left(\frac{x}{3}\right)\tan\left(\frac{x}{2}\right)} \approx \frac{4x^2-4x^4}{\left(\frac{x}{3}\right)\left(\frac{x}{2}\right)}$	M1	1.1b
	$= \frac{6}{x^2}(4x^2-4x^4) = 24-24x^2$	A1	2.1
		(2)	
(c)	24	B1ft	2.2a
	If x is (very) small, then any terms in x^2 are negligible.	dB1ft	2.4
		(2)	
			(6 marks)

Notes
(a) M1: Attempts to use $\cos\theta \approx 1-\frac{\theta^2}{2}$ with θ, 2θ, x or $2x$ and squares the resulting expression. Condone poor squaring, e.g., $(1-2x^2)^2 = 1-4x^4$ or missing brackets $\left(1-\frac{2x^2}{2}\right)^2 = (1-x^2)^2 = ...$ but not e.g. $\cos 2x \approx 2\left(1-\frac{x^2}{2}\right)$ May be implied by e.g. $1-(1-2x^2)^2 = 2x^2(2-2x^2)$ from difference of two squares. A1*: Correct proof with an intermediate line such as $1-(1-4x^2+4x^4)$ Do not be concerned if the LHS does not appear and ignore any spurious = 0 in their work. There should be no obvious incorrect statements in the proof e.g. $1-(1+4x^2-4x^4)$ Do not condone incorrect work including invisible brackets such as $1-1-4x^2+4x^4$ or $\left(1-\frac{2x^2}{2}\right)$ but condone a missing trailing bracket e.g. $1-(1-4x^2+4x^4$ Condone an attempt that starts in stages, which may not deal with the full expression e.g., $1-\left(1-\frac{4x^2}{2}\right) \rightarrow 1-(1-2x^2)^2 = ...$ has a missing square on the first bracket but is recovered in the next stage/step. The final line should be in terms of x but condone a slip to e.g. θ in the workings of the proof. If they complete the proof using θ they must revert to x to score this mark. **Special Cases:** In each of these cases below, please send to review. You may see attempts that use e.g. • $\cos 2x = \pm 2\cos^2 x \pm 1$ or $\cos^2 2x = \pm\frac{1}{2} \pm \frac{1}{2}\cos 4x$ • $\cos 2x = \pm 1 \pm 2\sin^2 x$ followed by the small angle approximation for $\sin x$ • Maclaurin expansions

(b)	
M1:	Uses the **given answer** to part (a) and writes $\sin\left(\frac{x}{3}\right)$ **and** $\tan\left(\frac{x}{2}\right)$ using correct small angle approximations. Only allow misreads that are clearly misreads e.g. they cannot replace $\tan\left(\frac{x}{2}\right)$ with $\frac{x}{3}$ unless $\tan\left(\frac{x}{3}\right)$ is seen first. In such cases they will lose the A mark in (b) but both of the marks in (c) are available. Condone mixed variables for this mark.
A1:	$24-24x^2$ but condone e.g. $24+-24x^2$ or e.g. $a=24$ and $b=-24$ Condone recovery of missing/invisible brackets but the work must otherwise be correct. Do not condone mixed variables being recovered unless explicitly replaced with x before cancelling x^2. No marks are scored in (b) for using the Maclaurin expansions for $\sin\left(\frac{x}{3}\right)$ and/or $\tan\left(\frac{x}{2}\right)$
(c)	
B1ft:	24 but this must follow from the non-zero constant term in their answer to (b). Do not allow e.g. $x = 24$ Allow follow through on their non-zero constant term from a polynomial in x.
dB1ft:	Suitable reason given but it should refer to their x^2 term in some way (and any additional terms if they have any). Ignore spurious remarks e.g. "if $x < 1$" unless contradictory. Dependent on the previous B1ft mark. Some acceptable examples: • $"24"x^2 \to 0$ or $x^2 \to 0$ • "We can ignore the x^2 term" • "x^2 is much smaller than 24" • As $x \to 0$ their $a + bx^2 \to a$ (condone as $x \to 0$, bx^2 "becomes" 0) • $\lim_{x\to 0} a+bx^2 = \left(a+b(0)^2\right) = a$ • Since x is very small, $24 - 24(0)^2 = 24$ (and condone if the squared is missing). They cannot *just* substitute in 0 or a very small value for x to score the mark for the reason. A reason such as "the answer rounds to 24" is not acceptable. There must be some justification, either "as $x \to 0$" or "since x is very small" so e.g. "$24 - 24(0)^2 = 24$ on its own scores B1ft dB0ft This mark may be scored from any polynomial in x that includes at least a constant term and an x^2 term. They cannot go back to e.g. $4x^4$ or $\dfrac{4x^4}{\frac{1}{6}x^2}$ and say this is negligible or $\to 0$ as they haven't dealt with the x in the denominator.

(**Q06 9MA0/02, June 2025**)

Q23.

Question	Scheme	Marks	AOs
(a)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$ *	A1*	2.1
		(3)	
(b)	$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$ $\Rightarrow \cos x \cot x = \cos x \cot(3x - 50°)$		
	$\cot x = \cot(3x-50°) \Rightarrow x = 3x - 50°$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $\cot x = \cot(3x-50°) \Rightarrow x + 180° = 3x - 50°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $x = 90°$	B1	2.2a
		(5)	
			(8 marks)
Notes:			

(a) Condone a full proof in x (or other variable) instead of θ's here

B1: States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ Do not accept $\operatorname{cosec}\theta = \dfrac{1}{\sin}$ with the θ missing

M1: For the key step in forming a single fraction/common denominator

E.g. $\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$. Allow if written separately $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$

Condone missing variables for this M mark

A1*: Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

(b) Condone θ's instead of x's here

M1: Uses part (a), cancels or factorises out the $\cos x$ term, to establish that one solution is found when $x = 3x - 50°$.

You may see solutions where $\cot A - \cot B = 0 \Rightarrow \cot(A-B) = 0$ or $\tan A - \tan B = 0 \Rightarrow \tan(A-B) = 0$.

As long as they don't state $\cot A - \cot B = \cot(A-B)$ or $\tan A - \tan B = \tan(A-B)$ this is acceptable

A1: $x = 25°$

M1: For the key step in realising that $\cot x$ has a period of 180° and a second solution can be found by solving $x + 180° = 3x - 50°$. The sight of $x = 115°$ can imply this mark provided the step $x = 3x - 50°$ has been seen. Using reciprocal functions it is for realising that $\tan x$ has a period of 180°

A1: $x = 115°$ Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

B1: Deduces that a solution can be found from $\cos x = 0 \Rightarrow x = 90°$. Ignore additional values here.

Solutions with limited working. The question demands that candidates show all stages of working.

SC: $\cos x \cot x = \cos x \cot(3x-50°) \Rightarrow \cot x = \cot(3x-50°) \Rightarrow x = 25°, 115°$

They have shown some working so can score B1, B1 marked on epen as 11000

Alt 1- Right hand side to left hand side

Question	Scheme	Marks	AOs
(a)	States or uses $\cot\theta = \frac{\cos\theta}{\sin\theta}$	B1	1.2
	$\cos\theta\cot\theta = \frac{\cos^2\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \frac{1}{\sin\theta} - \sin\theta = \text{cosec}\,\theta - \sin\theta$ *	A1*	2.1
		(3)	

Alt 2- Works on both sides

Question	Scheme	Marks	AOs
(a)	States or uses $\cot\theta = \frac{\cos\theta}{\sin\theta}$ or $\text{cosec}\,\theta = \frac{1}{\sin\theta}$	B1	1.2
	$LHS = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$ $RHS = \cos\theta\cot\theta = \frac{\cos^2\theta}{\sin\theta}$	M1	2.1
	States a conclusion E.g. "HENCE TRUE", "QED" or $\text{cosec}\,\theta - \sin\theta \equiv \cos\theta\cot\theta$ o.e. (condone = for ≡)	A1*	2.1
		(3)	

Alt (b)

Question	Scheme	Marks	AOs
	$\cot x = \cot(3x-50°) \Rightarrow \frac{\cos x}{\sin x} = \frac{\cos(3x-50°)}{\sin(3x-50°)}$ $\sin(3x-50°)\cos x - \cos(3x-50°)\sin x = 0$ $\sin((3x-50°)-x) = 0$ $2x-50° = 0$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $2x-50° = 180°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $\cos x = 0 \Rightarrow x = 90°$	B1	2.2a
		(5)	

(**Q12 9MA0/01, Oct 2020**)

Q24.

Question	Scheme	Marks	AOs
(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1a
	$\Rightarrow \theta = 5.26°$	A1ft	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

M1: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x = \frac{1}{4}, -\frac{1}{3}$

M1: Obtains two correct values for their $\sin x = k$

A1: All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

(b)

M1: For setting $2\theta - 30° =$ their '$-19.47°$'

A1ft: $\theta = 5.26°$ but allow a follow through on their '$-19.47°$'

(**Q12 9MA0/02, Specimen papers**)

Q25.

Question	Scheme	Marks	AOs
	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1-\sin^2 x) = 1$ | $16(1-\cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}\ \left\{= \frac{2\pm\sqrt{3}}{4}\text{ or } 0.933..., 0.066...\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	

(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta-\alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$		M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$	$(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$	$\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$	e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
	Note: Working in radians does not affect any of the first 4 marks			
			(5)	
				(9 marks)

Question	Scheme		Marks	AOs
	(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$			
(ii) Alt 1	Complete strategy, i.e. • Attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta + \sin^2\theta = 1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$		M1	3.1a
	e.g. $25\sin^2\theta = 4 + 20\cos\theta + 25\cos^2\theta$ $\Rightarrow 25(1-\cos^2\theta) = 4 + 20\cos\theta + 25\cos^2\theta$ **or** e.g. $25\sin^2\theta - 20\sin\theta + 4 = 25\cos^2\theta$ $\Rightarrow 25\sin^2\theta - 20\sin\theta + 4 = 25(1-\sin^2\theta)$		M1	1.1b
	$50\cos^2\theta + 20\cos\theta - 21 = 0$	$50\sin^2\theta - 20\sin\theta - 21 = 0$		
	$\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e.	$\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e.	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arccos\left(\frac{-2+\sqrt{46}}{10}\right)$	e.g. $\theta = \arcsin\left(\frac{2+\sqrt{46}}{10}\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
			(5)	

Notes for Question

(i)	
B1:	For recalling that $\sec x = \frac{1}{\cos x}$
M1:	Correct strategy of • **Way 1:** applying $\sin 2x = 2\sin x\cos x$ and proceeding to $\sin 2x = k,\ \lvert k\rvert \le 1,\ k \ne 0$ • **Way 2:** squaring both sides, applying $\cos^2 x + \sin^2 x = 1$ and solving a quadratic equation in either $\sin^2 x$ or $\cos^2 x$ to give $\sin^2 x = k$ or $\cos^2 x = k,\ \lvert k\rvert \le 1,\ k \ne 0$
dM1:	Uses the correct order of operations to find at least one value for x in either radians or degrees
A1:	Clear reasoning to achieve both $x = \frac{\pi}{12}, \frac{5\pi}{12}$ and no other values in the range $0 \le x < \frac{\pi}{2}$
Note:	Give dM1 for $\sin 2x = \frac{1}{2} \Rightarrow$ any of $\frac{\pi}{12}, \frac{5\pi}{12}, 15°, 75°$, awrt 0.26 or awrt 1.3
Note:	Give special case, SC B1M0M0A0 for writing down any of $\frac{\pi}{12}, \frac{5\pi}{12}, 15°$ or $75°$ with no working

Notes for Question Continued	
(ii)	
M1:	See scheme
Note:	**Alternative strategy:** Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\cos(\theta+\alpha) = -2$, finds both R and α, and proceeds to $\cos(\theta+\alpha) = k$, $\lvert k\rvert < 1$, $k \neq 0$
M1:	Either • uses $R\sin(\theta-\alpha)$ to find the values of both R and α • attempts to apply $(5\sin\theta - 5\cos\theta)^2 = 2^2$, uses $\cos^2\theta + \sin^2\theta = 1$ and proceeds to find an equation of the form $\pm\lambda \pm \mu\sin 2\theta = \pm\beta$ or $\pm\mu\sin 2\theta = \pm\beta;\ \mu \neq 0$ • attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ and uses $\cos^2\theta + \sin^2\theta = 1$ to form an equation in $\cos\theta$ only or $\sin\theta$ only
A1:	For $\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$, o.e., $\cos(\theta + 45°) = -\frac{2}{\sqrt{50}}$, o.e. or $\sin 2\theta = \frac{21}{25}$, o.e. or $\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e. or $\cos\theta =$ awrt 0.48, awrt -0.88 or $\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e., or $\sin\theta =$ awrt 0.88, awrt -0.48
Note:	$\sin(\theta - 45°)$, $\cos(\theta + 45°)$, $\sin 2\theta$ must be made the subject for A1
dM1:	**dependent on the first M mark** Uses the correct order of operations to find at least one value for x in either degrees or radians
Note:	dM1 can also be given for $\theta = 180° - \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ or $\theta = \frac{1}{2}\left(180° - \arcsin\left(\frac{21}{25}\right)\right)$
A1:	Clear reasoning to achieve both $\theta =$ awrt 61.4°, awrt 208.6° and no other values in the range $0 \leq \theta < 360°$
Note:	Give M0M0A0M0A0 for writing down any of $\theta =$ awrt 61.4°, awrt 208.6° with no working
Note:	<u>**Alternative solutions:**</u> (to be marked in the same way as Alt 1): • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5\tan\theta - 5 = 2\sec\theta \Rightarrow (5\tan\theta - 5)^2 = (2\sec\theta)^2$ $\Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4\sec^2\theta \Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4(1+\tan^2\theta)$ $\Rightarrow 21\tan^2\theta - 50\tan\theta + 21 = 0 \Rightarrow \tan\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5 - 5\cot\theta = 2\operatorname{cosec}\theta \Rightarrow (5 - 5\cot\theta)^2 = (2\operatorname{cosec}\theta)^2$ $\Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4\operatorname{cosec}^2\theta \Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4(1+\cot^2\theta)$ $\Rightarrow 21\cot^2\theta - 50\cot\theta + 21 = 0 \Rightarrow \cot\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only

(**Q07 9MA0/02, June 2018**)

Q26.

Question	Scheme	Marks	AOs
	$\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) Way 1	$\{\text{LHS}=\}\ \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$=\dfrac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}\ \left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS}=\}\ \dfrac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\sin\theta}+\dfrac{\sin 2\theta\cos\theta+\cos 2\theta\sin\theta}{\cos\theta}$		
	$=\dfrac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$=\dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}\ \left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS}=\}\ \dfrac{2\cos 2\theta}{\sin 2\theta}=\dfrac{2\cos(3\theta-\theta)}{\sin 2\theta}=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$=\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	

(b) Way 1	$\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=4\Rightarrow\right\}\ 2\cot 2\theta=4\Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right)=4$	M1	1.1b
	Rearranges to give $\tan 2\theta=k;\ k\neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90^\circ<\theta<180^\circ,\ \tan 2\theta=\dfrac{1}{2}\Rightarrow\right\}$		
	Only one solution of $\theta=103.3^\circ$ (1 dp) or awrt 103.3°	A1	2.2a
		(3)	
(b) Way 2	$\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=4\Rightarrow\right\}\ 2\cot 2\theta=4\Rightarrow\dfrac{2}{\tan 2\theta}=4$	M1	1.1b
	$\dfrac{2}{\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)}=4\Rightarrow 2(1-\tan^2\theta)=8\tan\theta$ $\Rightarrow\ \tan^2\theta+4\tan\theta-1=0\Rightarrow\tan\theta=\dfrac{-4\pm\sqrt{(4)^2-4(1)(-1)}}{2(1)}$ $\{\Rightarrow\tan\theta=-2\pm\sqrt{5}\}\Rightarrow\tan\theta=k;\ k\neq 0\Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90^\circ<\theta<180^\circ,\ \tan\theta=-2-\sqrt{5}\Rightarrow\}$		
	Only one solution of $\theta=103.3^\circ$ (1 dp) or awrt 103.3°	A1	2.2a
		(3)	
			(7 marks)

Notes for Question	
(a)	**Way 1 and Way 2**
M1:	Correct valid method forming a common denominator of $\sin\theta\cos\theta$ i.e. correct process of $\dfrac{(...)\cos\theta+(...)\sin\theta}{\cos\theta\sin\theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta-\theta)$ or $\cos 2\theta$
dM1:	**dependent on the previous M mark** Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$
A1*	Correct proof
Note:	Writing $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta} + \dfrac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$
Note:	Allow 2nd M1 for stating a correct $\sin 2\theta = 2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$
(a)	**Way 3**
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$
dM1:	**dependent on the previous M mark** Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1st M1 1st A1 (together) for any of LHS $\rightarrow \dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ or LHS $\rightarrow \dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or LHS $\rightarrow \cos 2\theta(\cot\theta+\tan\theta)$ or LHS $\rightarrow \cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ (i.e. where $\cos 2\theta$ has been factorised out)

Note:	Allow 1st M1 1st A1 for progressing as far as LHS $= ... = \cot x - \tan x$
Note:	The following is a correct alternative solution $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ $= \dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *
Note:	E.g. going from $\dfrac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ with no intermediate working is 1st A0

Notes for Question Continued	
(b)	**Way 1**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Rearranges to give $\tan 2\theta = k,\ k \neq 0$, and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
Note:	1st M1 can be implied by seeing $\tan 2\theta = \frac{1}{2}$
Note:	Condone 2nd M1 for applying $\frac{1}{2}\arctan\left(\frac{1}{2}\right)$ $\{= 13.28...\}$
(b)	**Way 2**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Applies $\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0$, and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$
Note:	Give M1 dM1 A1 for no working leading to $\theta =$ awrt $103.3°$ and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta =$ awrt $103.3°$ and other solutions which can be either outside or inside the range $90° < \theta < 180°$

(**Q12 9MA0/02, June 2019**)

Q27.

Question	Scheme	Marks	AOs
(a)	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\cos 2\theta+\sin 2\theta}$ or $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\cos 2\theta+\sin 2\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	M1	2.1
	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	A1	1.1b
	$=\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\frac{2\sin\theta(\sin\theta+\cos\theta)}{2\cos\theta(\cos\theta+\sin\theta)}$	dM1	2.1
	$=\frac{\sin\theta}{\cos\theta}=\tan\theta$ *	A1*	1.1b
		(4)	
(b)	$\frac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x}=3\sin 2x \Rightarrow \tan 2x=3\sin 2x$ o.e	M1	3.1a
	$\Rightarrow \sin 2x-3\sin 2x\cos 2x=0$ $\Rightarrow \sin 2x(1-3\cos 2x)=0$ $\Rightarrow (\sin 2x=0,)\ \cos 2x=\frac{1}{3}$	A1	1.1b
	$x=90°$, awrt $35.3°$, awrt $144.7°$	A1 A1	1.1b 2.1
		(4)	
			(8 marks)

Notes

(a)

M1: Attempts to use a correct double angle formulae for both $\sin 2\theta$ and $\cos 2\theta$ (seen once).
The application of the formula for $\cos 2\theta$ must be the one that cancels out the "1"
So look for $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator or $\cos 2\theta = 2\cos^2\theta - 1$ in the denominator
Note that $\cos 2\theta = \cos^2\theta - \sin^2\theta$ may be used as well as using $\cos^2\theta + \sin^2\theta = 1$

A1: $\dfrac{1-(1-2\sin^2\theta)+2\sin\theta\cos\theta}{1+(2\cos^2\theta-1)+2\sin\theta\cos\theta}$ or $\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$

dM1: Factorises numerator and denominator in order to demonstrate cancelling of $(\sin\theta + \cos\theta)$

A1*: Fully correct proof with no errors.

You must see an intermediate line of $\dfrac{2\sin\theta(\sin\theta+\cos\theta)}{2\cos\theta(\cos\theta+\sin\theta)}$ or $\dfrac{\sin\theta}{\cos\theta}$ or even $\dfrac{2\sin\theta}{2\cos\theta}$

Withhold this mark if you see, within the body of the proof,

- notational errors. E.g. $\cos 2\theta = 1 - 2\sin^2$ or $\cos\theta^2$ for $\cos^2\theta$
- mixed variables. E.g. $\cos 2\theta = 2\cos^2 x - 1$

(b)

M1: Makes the connection with part (a) and writes the lhs as $\tan 2x$. Condone $x \leftrightarrow \theta$ $\tan 2\theta = 3\sin 2\theta$

A1: Obtains $\cos 2x = \frac{1}{3}$ o.e. with $x \leftrightarrow \theta$. You may see $\sin^2 x = \frac{1}{3}$ or $\cos^2 x = \frac{2}{3}$ after use of double angle formulae.

A1: Two "correct" values. Condone accuracy of awrt $90°$, $35°$, $145°$
Also condone radian values here. Look for 2 of awrt 0.62, 1.57, 2.53

A1: All correct (allow awrt) and no other values in range. Condone $x \leftrightarrow \theta$ if used consistently

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Answers without working in (b): Just answers and no working score 0 marks.
If the first line is written out, i.e. $\tan 2x = 3\sin 2x$ followed by all three correct answers score 1100 .

(**Q10 9MA0/01, Oct 2021**)

Q28.

Question	Scheme	Marks	AOs
(a)	e.g. $2\frac{\sin\theta}{\cos\theta}(8\cos\theta+23(\underline{1-\cos^2\theta}))=8\times\underline{2\sin\theta\cos\theta\sec^2\theta}$	B1	1.2
	$2\tan\theta(8\cos\theta+23\sin^2\theta)=8\sin 2\theta\sec^2\theta$ $\Rightarrow 2\sin\theta\cos\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin 2\theta$ $\sin 2\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin 2\theta$ $\sin 2\theta(23\cos^2\theta-8\cos\theta-15)=0$	M1A1	2.1 2.2a
		(3)	
(b)	$\sin 2x(23\cos^2 x-8\cos x-15)=0$		
	$\sin 2x=0\Rightarrow x=360°$ or $540°$	B1	2.2a
	$23\cos^2 x-8\cos x-15\Rightarrow \cos x=-\frac{15}{23}$	M1	1.1b
	$\cos x=-\frac{15}{23}\Rightarrow x=...$	dM1	1.1b
	$x=360°, 540°$ and awrt $491°$ only	A1	2.3
		(4)	
			(7 marks)

Notes

(a) Allow use of e.g. x but the final mark requires the equation to be in terms of θ

B1(M1 on EPEN): For recalling and using at least one correct trigonometric identity in the given equation.

e.g. one of: $\sin^2\theta+\cos^2\theta=1$, $1+\tan^2\theta=\sec^2\theta$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\sin 2\theta=2\sin\theta\cos\theta$

This may be seen explicitly or may be implied by their working by e.g. $\tan\theta\cos\theta=\sin\theta$ or they might multiply both sides by $\cos^2\theta$ leaving $8\sin 2\theta$ on the rhs implying $1+\tan^2\theta=\sec^2\theta$

M1: For manipulating the equation using trigonometric identities (condoning sign slips only in the identities and arithmetic slips) to obtain an expression of the form:

$A\sin 2\theta\cos^2\theta+B\sin 2\theta\cos\theta+C\sin 2\theta\ (=0)$ or $\sin 2\theta\left(A\cos^2\theta+B\cos\theta+C\right)\ (=0)$ with $A, B, C\neq 0$

A1: $\sin 2\theta(23\cos^2\theta-8\cos\theta-15)=0$ oe e.g. $\sin 2\theta(-23\cos^2\theta+8\cos\theta+15)=0$ cao

Note that this is not a given answer so condone notational slips e.g. $\cos\theta^2$ for $\cos^2\theta$ provided the intention is clear but the final equation must have no notational errors.

Note that the "= 0" is not required for the M1 but is required for the A1

Note: some candidates arrive at the correct final answer fortuitously following errors in their work.

(b) Allow all marks in (b) to score if the correct equation is obtained fortuitously in part (a)
Also allow use of θ instead of x throughout in part (b). Correct answers, no working scores max 1000

B1: For one of $x=360(°)$ or $x=540(°)$ Condone $x=2\pi$ or $x=3\pi$ for this mark.

The degrees symbol is not required. This may come from $\cos x=1$

M1: Attempts to solve their 3TQ from part (a) or a "made up" 3TQ (which may only be seen in (b)) leading to a value for $\cos x$. The general guidance for solving a 3 term quadratic equation can be applied. Allow solution(s) from a calculator which may be implied by at least one correct value for their 3TQ. Must be a value for $\cos x$ and not e.g. x.

dM1: Attempts to find one of their angles in the range $360<x<540$ (but not 450) for their $\cos x=k$ where $|k|<1$ May be implied by their value(s) but must be in degrees.

Requires them to state a value for cos$x$. Must be checked (you can check cos(their x) = their k (1sf))

A1: $x=360°, 540°$ and awrt $491°$ only with no other values in range (including 450).

The degrees symbol is not required. awrt 491 must come from $\cos x=-\frac{15}{23}$

(**Q14 9MA0/02, June 2023**)

Q29.

Question	Scheme	Marks	AOs
(a) Way 1	$\left(\frac{1}{\text{cosec}\,\theta-1}+\frac{1}{\text{cosec}\,\theta+1}\equiv\right)\frac{\text{cosec}\,\theta+1+\text{cosec}\,\theta-1}{(\text{cosec}\,\theta-1)(\text{cosec}\,\theta+1)}$	B1	1.1b
	$\equiv\frac{2\text{cosec}\,\theta}{\text{cosec}^2\theta-1}\equiv\frac{2\text{cosec}\,\theta}{\cot^2\theta}$ or e.g. $\equiv\frac{2\sin\theta}{1-\sin^2\theta}\equiv\frac{2\sin\theta}{\cos^2\theta}$	M1	1.1b
	$\frac{2\text{cosec}\,\theta}{\cot^2\theta}\equiv\frac{2}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta}\equiv 2\tan\theta\sec\theta$ * or $\frac{2\sin\theta}{\cos^2\theta}\equiv 2\tan\theta\sec\theta$ *	A1*	2.1
		(3)	
(a) Way 2	**"Meets in the middle"**		
	$\left(\text{LHS}=\frac{1}{\text{cosec}\,\theta-1}+\frac{1}{\text{cosec}\,\theta+1}\equiv\right)\frac{\text{cosec}\,\theta+1+\text{cosec}\,\theta-1}{(\text{cosec}\,\theta-1)(\text{cosec}\,\theta+1)}$	B1	1.1b
	$\equiv\frac{2\text{cosec}\,\theta}{\text{cosec}^2\theta-1}\equiv\frac{2\text{cosec}\,\theta}{\cot^2\theta}$	M1	1.1b
	$\text{RHS}=2\tan\theta\sec\theta\equiv\frac{2\sin\theta}{\cos^2\theta}\equiv\frac{2\sin^2\theta}{\sin\theta\cos^2\theta}$ $\equiv\frac{2}{\sin\theta}\times\frac{\sin^2\theta}{\cos^2\theta}\equiv\frac{2\text{cosec}\,\theta}{\cot^2\theta}=\text{LHS}$ or e.g. QED or e.g. Proven	A1*	2.1
		(3)	

Part (a) Notes

(a) **Condone a complete proof <u>entirely</u> in x (or another variable) instead of θ**
Condone "=" for "≡"

Note that we are marking this as **B1M1A1** not **M1M1A1**

B1: Adds the fractions to obtain a **correct** single fraction (not fractions over fractions) in any form. Condone missing brackets when they combine their fractions as long as they are recovered to give a correct fraction.

This can be done in a variety of ways but when combined, the fraction must be **correct** e.g.

$$\frac{\text{cosec}\,\theta+1+\text{cosec}\,\theta-1}{(\text{cosec}\,\theta-1)(\text{cosec}\,\theta+1)} \text{ or } \frac{2\text{cosec}\,\theta}{(\text{cosec}\,\theta-1)(\text{cosec}\,\theta+1)} \text{ or } \frac{2\text{cosec}\,\theta}{\text{cosec}^2\theta-1}$$

$$\text{or e.g. } \left(\frac{1}{\frac{1}{\sin\theta}-1}+\frac{1}{\frac{1}{\sin\theta}+1}=\frac{\sin\theta}{1-\sin\theta}+\frac{\sin\theta}{1+\sin\theta}\right)=\frac{2\sin\theta}{1-\sin^2\theta} \text{ etc.}$$

M1: Uses a **correct** Pythagorean identity anywhere in their attempt e.g. $\text{cosec}^2\theta-1=\cot^2\theta$, $\sin^2\theta+\cos^2\theta=1$ etc. or equivalent

A1*: Correct work with all necessary steps shown leading to the given answer. See scheme for the necessary steps. They need to proceed via sine and cosine to the given answer. There should be **no notational or bracketing errors and no mixed or missing variables**. E.g. we would consider $\cos^2\theta$ written as $\cos\theta^2$ a notational error.
Condone reaching $2\sec\theta\tan\theta$ *

<u>Way 2 (Meet in the middle)</u>

B1: See Way 1

M1: See Way 1

A1*: Correct work on the RHS with all necessary steps shown leading to showing the equivalence with the LHS. See scheme for the necessary steps. There should be no notational or bracketing errors and no mixed or missing variables.
For this approach there must be a (minimal) conclusion e.g. "= LHS", "QED", "Hence proven" etc.

It is possible to start with the rhs e.g.:

$$2\tan\theta\sec\theta = 2\frac{\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta}$$
$$=\frac{2\sin\theta}{\cos^2\theta}=\frac{2\operatorname{cosec}\theta}{\cot^2\theta}$$
$$=\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}=\frac{2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}$$
$$=\frac{\operatorname{cosec}\theta+1+\operatorname{cosec}\theta-1}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}$$
$$=\frac{1}{\operatorname{cosec}\theta-1}+\frac{1}{\operatorname{cosec}\theta+1}$$

B1: Correctly reaches $2\tan\theta\sec\theta=\frac{2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}$

M1: Uses a correct Pythagorean identity anywhere in their attempt e.g. $\operatorname{cosec}^2\theta-1=\cot^2\theta$, $\sin^2\theta+\cos^2\theta=1$ etc. or equivalent

A1*: Correct work with all necessary steps shown leading to the lhs. See scheme for the necessary steps. There should be no notational or bracketing errors and no mixed or missing variables.

(b)	$2\tan 2x \sec 2x = \cot 2x \sec 2x$	B1	2.2a
	$2\tan 2x \sec 2x - \cot 2x \sec 2x = 0$ $\Rightarrow \sec 2x(2\tan 2x - \cot 2x) = 0$ $2\tan 2x - \cot 2x = 0 \Rightarrow 2\tan 2x = \cot 2x \Rightarrow \tan^2 2x = \frac{1}{2}$ **or** $2\tan 2x - \cot 2x = 0 \Rightarrow 2\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\sin 2x} \Rightarrow 2\sin^2 2x = \cos^2 2x$ $\Rightarrow 2\sin^2 2x = 1 - \sin^2 2x \Rightarrow \sin^2 2x = \frac{1}{3}$ **or** $2(1-\cos^2 2x) = \cos^2 2x \Rightarrow \cos^2 2x = \frac{2}{3}$ **or** $2\tan 2x = \cot 2x \Rightarrow \frac{4\tan x}{1-\tan^2 x} = \frac{1-\tan^2 x}{2\tan x}$ $\Rightarrow \tan^4 x - 10\tan^2 x + 1 = 0$ **or** $2\tan 2x - \cot 2x = 0 \Rightarrow 2\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\sin 2x} \Rightarrow 2\sin^2 2x = \cos^2 2x$ $2\sin^2 2x = \cos^2 2x \Rightarrow 1 - \cos 4x = \frac{1}{2}(\cos 4x + 1) \Rightarrow \cos 4x = \frac{1}{3}$	M1	2.1
	$2x = \tan^{-1}\frac{1}{\sqrt{2}} = K \Rightarrow x = \frac{K}{2}$ **or** $2x = \sin^{-1}\frac{1}{\sqrt{3}} = K \Rightarrow x = \frac{K}{2}$ **or** $2x = \cos^{-1}\sqrt{\frac{2}{3}} = K \Rightarrow x = \frac{K}{2}$ **or** $\tan^2 x = 5 \pm 2\sqrt{6} \Rightarrow x = \tan^{-1}\left(\sqrt{5 \pm 2\sqrt{6}}\right)$ **or** $\cos 4x = \frac{1}{3} \Rightarrow 4x = \cos^{-1}\left(\frac{1}{3}\right) = K \Rightarrow x = \frac{1}{4}K$	M1	1.1b
	$x = 17.6°,\ 72.4°$	A1	1.1b
		(4)	
			(7 marks)

(b) Notes

(b) Note that attempts solve an equation of the form:

$2\tan x\sec x=\cot 2x\sec 2x$ or e.g. $2\tan\theta\sec\theta=\cot 2\theta\sec 2\theta$ or e.g. $2\tan\theta\sec\theta=\cot 2x\sec 2x$

Will generally score no marks in part (b)

Condone the use of θ instead of x here.

B1: Deduces the correct equation using the result from part (a)

M1: Factors out or cancels the $\sec 2x$ to obtain $...\tan 2x\pm...\cot 2x=0$ oe e.g. $...\tan 2x=\pm...\cot 2x$ leading to an equation of the form:

$\tan^2 2x=\alpha$ oe e.g. $\cot^2 2x=\frac{1}{\alpha}$ where $\alpha>0$

Or uses $\tan 2x=\frac{\sin 2x}{\cos 2x}$ and $\cot 2x=\frac{\cos 2x}{\sin 2x}$ and then $\sin^2 2x=\pm1\pm\cos^2 2x$ or $\cos^2 2x=\pm1\pm\sin^2 2x$ to obtain an equation of the form $\sin^2 2x=\beta$ or $\cos^2 2x=\gamma$ oe e.g. $\text{cosec}^2 2x=\frac{1}{\beta}$ or $\sec^2 2x=\frac{1}{\gamma}$ where $0<\beta<1$ or $0<\gamma<1$

Or uses $\tan 2x=\frac{2\tan x}{1-\tan^2 x}$ and $\cot 2x=\frac{1-\tan^2 x}{2\tan x}$ to obtain a 3TQ in $\tan^2 x$ (or possibly in $\sec^2 x$)

Or uses $\tan 2x=\frac{\sin 2x}{\cos 2x}$ and $\cot 2x=\frac{\cos 2x}{\sin 2x}$ and then $2\sin^2 2x=\pm1\pm\cos 4x$ and $2\cos^2 2x=\pm1\pm\cos 4x$ to obtain an equation of the form $\cos 4x=k,\quad 0<k<1$

M1: Correct order of operations from $\tan^2 2x=\alpha$ or $\sin^2 2x=\beta$ or $\cos^2 2x=\gamma$ or $\cos 4x=k$ or equivalents e.g. $\text{cosec}^2 2x=\frac{1}{\beta}$ where $\alpha>0$ or $0<\beta<1$ or $0<\gamma<1$ or $0<k<1$ leading to at least one value for x e.g. square roots, finds inverse tan/sin/cos/cosec and divides by 2 or inverse cos and divides by 4

or from $\tan^2 x=k,\ k>0$ which follows their equation (may need to check) and then finds $x=\tan^{-1}\sqrt{k}$

You may need to check their value(s) (in degrees or radians) to see if the correct order of operations has been used. May be implied by e.g. 17.6° or 17.7° provided no incorrect work is seen.

A1: Correct values. Allow awrt 17.6° and awrt 72.4°. The degrees symbol is not required. Ignore any values outside the range (correct or incorrect) but if there are extra angles in range score A0. Answers in radians score A0.

Note that some candidates may convert to $\sin x$ or $\cos x$ and then solve:

E.g.

$$\tan^2 2x=\frac{1}{2}\Rightarrow\frac{\sin^2 2x}{\cos^2 2x}=\frac{1}{2}\Rightarrow\frac{4\sin^2 x\cos^2 x}{\left(2\cos^2 x-1\right)^2}\to 12\cos^4 x-12\cos^2 x+1=0$$

$$\text{or }\frac{4\sin^2 x\cos^2 x}{\left(1-2\sin^2 x\right)^2}\to 12\sin^4 x-12\sin^2 x+1=0$$

$$\Rightarrow\cos^2 x/\sin^2 x=\frac{3\pm\sqrt{6}}{6}\Rightarrow\cos x/\sin x=\pm\sqrt{\frac{3\pm\sqrt{6}}{6}}\Rightarrow x=17.6°,\ 72.4°$$

These can be marked in a similar way.

Alternative not using part (a):

$$\frac{1}{\operatorname{cosec}2x-1}+\frac{1}{\operatorname{cosec}2x+1}=\cot 2x\sec 2x$$

$$\Rightarrow \frac{2\operatorname{cosec}2x}{\operatorname{cosec}^2 2x-1}=\cot 2x\sec 2x$$

$$\Rightarrow \frac{2\operatorname{cosec}2x}{\cot^2 2x}=\frac{\cos 2x}{\sin 2x}\times\frac{1}{\cos 2x}=\operatorname{cosec}2x$$

$$\Rightarrow 2\tan^2 2x=1 \Rightarrow \tan^2 2x=\frac{1}{2}$$

Score as:

M1: For correct work leading to one of the forms in the main scheme e.g.

$$\tan^2 2x=\alpha \text{ oe e.g. } \cot^2 2x=\frac{1}{\alpha} \text{ where } \alpha>0$$

or

$$\sin^2 2x=\beta \text{ or } \cos^2 2x=\gamma \text{ oe e.g. } \operatorname{cosec}^2 2x=\frac{1}{\beta} \text{ or } \sec^2 2x=\frac{1}{\gamma}$$

where $0<\beta<1$ or $0<\gamma<1$

B1: Any correct equation e.g. $\tan^2 2x=\frac{1}{2}$, $\cot^2 2x=2$, $\sin^2 2x=\frac{1}{3}$ etc.

Then **M1A1** as main scheme

(**Q08 9MA0/02, June 2024**)

Q30.

Question	Scheme	Marks	AOs
(a)	Attempts to use $\sin(x+30°)=\sin x\cos 30°\pm\cos x\sin 30°$ $\cos(x+30°)=\cos x\cos 30°\mp\sin x\sin 30°$ $\Rightarrow \pm\sin x\cos 30°\pm\cos x\sin 30°\pm\sqrt{3}(\pm\cos x\cos 30°\pm\sin x\sin 30°)$	M1	2.1
	Correct expression $\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}(\cos x\cos 30°-\sin x\sin 30°)$	A1	1.1b
	States or implies that $\sin 30°=\frac{1}{2}$ and $\cos 30°=\frac{\sqrt{3}}{2}$ $=\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)=2\cos x$ *	A1*	2.1
		(3)	
(b)	$2\cos\theta=3\sin 2\theta\Rightarrow 2\cos\theta=6\sin\theta\cos\theta$	M1	2.1
	$\sin\theta=\frac{1}{3}$	A1	1.1b
	$\theta=\arcsin\frac{1}{3}\Rightarrow\theta=\ldots$	dM1	1.1b
	$(\theta=)$ awrt $19.5°$, awrt $160.5°$, $90°$	A1	2.2a
		(4)	
			(7 marks)

Notes:

(a) **Condone a complete proof entirely in θ (or another variable) instead of x.**
Do not be concerned with the omission of degrees.

M1: Attempts to use both compound angle expansions to set up an expression in $\sin x$ and $\cos x$
i.e. $\pm\sin x\cos 30°\pm\cos x\sin 30°\pm\sqrt{3}\left(\pm\cos x\cos 30°\pm\sin x\sin 30°\right)$
The terms must be correct but condone sign errors and a slip on the multiplication of the $\sqrt{3}$ if they attempt to multiply out the brackets. (The $\sqrt{3}$ may be omitted entirely)
This mark may be implied by further work
e.g. $\pm\frac{\sqrt{3}}{2}\sin x\pm\frac{1}{2}\cos x\pm\sqrt{3}\left(\pm\frac{\sqrt{3}}{2}\cos x\pm\frac{1}{2}\sin x\right)$

A1: Correct expression $\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}\left(\cos x\cos 30°-\sin x\sin 30°\right)$ o.e.
May be implied by
$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)$ (or implied if multiplied out)

A1*: Proceeds to the given answer with **no errors seen including invisible brackets** (condone a missing trailing bracket). We must see the exact numerical values used for $\sin 30°$ and $\cos 30°$ before proceeding to the given answer.
Minimum required
$\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x+\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)=2\cos x$ which scores M1A1A1*
$\sin x\cos 30°+\cos x\sin 30°+\sqrt{3}\left(\cos x\cos 30°-\sin x\sin 30°\right)=2\cos x$ scores M1A1A0*
There should not be any notational or bracketing errors and no mixed or missing variables.

Alternative for part (a)

M1: Writes the left hand side of the equation as
$\frac{1}{2}\sin\left(x+30°\right)+\frac{\sqrt{3}}{2}\cos\left(x+30°\right)=\sin 30°\sin\left(x+30°\right)+\cos 30°\cos\left(x+30°\right)$

A1: Correct expression for $\sin 30°\sin\left(x+30°\right)+\cos 30°\cos\left(x+30°\right)=\cos x$

A1*: Proceeds to the given answer $\sin\left(x+30°\right)+\sqrt{3}\cos\left(x+30°\right)=2\cos\left(x+30°-30°\right)=2\cos x$
with **no errors seen including invisible brackets** (condone a missing trailing bracket).
There should not be any notational or bracketing errors and no mixed or missing variables.

Alternative part (a) – using the R-alpha method

M1: States e.g. $R\cos(x+30\pm\alpha)=\sqrt{3}\cos(x+30)\mp\sin(x+30)$

and attempts to find either R or α correctly. (may be implied)

A1: Achieves $2\cos(x+30-30)$

A1*: Achieves $2\cos x$ with no errors seen and both stages of working shown e.g.

- States $\left(R\cos(x+30\pm\alpha)=\right)\sqrt{3}\cos(x+30)\cos\alpha\mp\sin(x+30)\sin\alpha$ oe
- Shows $R=\sqrt{1+3}=2$ and $\tan\alpha=\left(\frac{1}{\sqrt{3}}\right)$ o.e. e.g. using cosine or sine

---

(b) Condone the use of x for θ and mixed variables

M1: Sets $2\cos\theta=3\sin 2\theta$ and proceeds to $A\cos\theta=B\sin\theta\cos\theta$. (allow $A=B$)
May be implied by $\sin\theta=k$ $(k\neq 0,1)$

A1: $\sin\theta=\frac{1}{3}$

dM1: Finds at least one of their values of θ for their $\sin\theta=k$ $(k\neq 0,1)$ It is dependent on the previous method mark. You may need to check their value(s) (in degrees or radians) but may be implied by e.g. awrt $19°$ (or awrt $20°$) or awrt $161°$ (or awrt $160°$)
(awrt 0.34 or awrt 2.8 in radians)

A1: Deduces that $(\theta=)$ $90°$ as well as giving $(\theta=)$ awrt $19.5°$, awrt $160.5°$ with no other values in the given range (ignore any found outside of the range)
The degree symbol is not required. (Note the angles are 19.4712206…and 160.528779…)
Answers in radians score A0.

(**Q14 9MA0/01, June 2025**)

Q31.

Question	Scheme	Marks	AOs
(i)	$(n =)\ 3$	B1	2.2a
		(1)	
(ii)	30 solutions **or** e.g. each interval of length π has 6 solutions	B1	2.2a
	e.g., Each interval of length π has 6 solutions, and there are 5 intervals of length π, so $5\times6=30$ solutions.	dB1	2.4
		(2)	
		(3 marks)	

Notes

(i)

B1: cao $(n =)\ 3$

(ii)

B1: Either:

- Deduces 30 solutions
- **or** deduces the correct **total** number of solutions for $\sin(nx) = k$ **and** $\sin(nx) = -k$ (or $\sin^2(nx) = k^2$) in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$
- **or** deduces the correct number of solutions in 0 to 5π for either $\sin(nx) = k$ or $\sin(nx) = -k$
- **or** deduces the correct **total** number of solutions for $\sin(x) = k$ **and** $\sin(x) = -k$ in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$

dB1: Requires 30 (solutions) and correct justification comprising all the elements in **one** of the bullet points below. See the tables on the next page for the correct values.

- Deduces the correct **total** number of solutions for $\sin(nx) = k$ **and** $\sin(nx) = -k$ (or $\sin^2(nx) = k^2$) in any relevant interval $(0 \text{ to } \pi, 2\pi \text{ or } 4\pi)$ **and** scales the interval to 0 to 5π
- **or** deduces the correct number of solutions in 0 to 5π for $\sin(nx) = k$ **and** for $\sin(nx) = -k$ and then adds
- **or** finds the **total** correct number of solutions for $\sin(x) = k$ **and** $\sin(x) = -k$ in any relevant interval $(0 \text{ to } \pi, 2\pi, 4\pi \text{ or } 5\pi)$ **and** then scales the interval to 0 to 5π **and** multiplies by 3

Just stating e.g. $2\times6\times2.5$ is insufficient for the dB1 mark without further justification.
There are many acceptable variations, and some examples are below and on the next page.

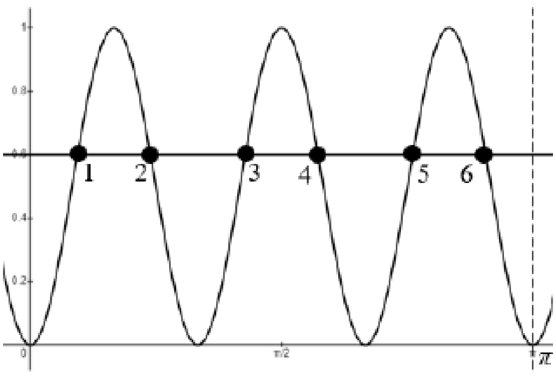
Some examples:

- 6 solutions in each interval of length π (B1) **and** 5 intervals of length π so 30 (dB1)
- 12 solutions in each interval of length 2π (B1) **and** 2.5 intervals of length 2π so 30 (dB1)
- 2 solutions for $\sin x = k$ up to 2π, so for $\sin^2 x$ there are 4 per 2π (B1)

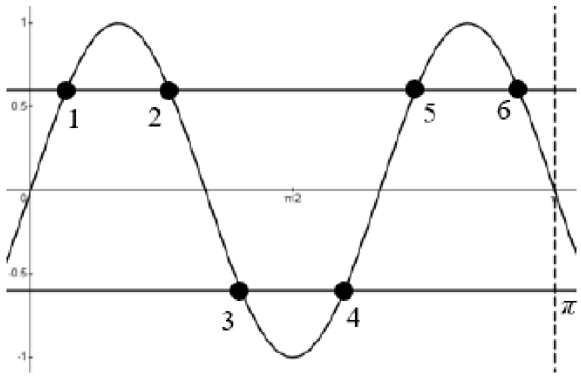
  and since $0 \leqslant x < 5\pi$ then $0 \leqslant 3x < 15\pi$ so $\frac{15}{2} \times 4 = 30$ (dB1)
- Double the 6 solutions since $\sin(nx) = \pm k$ (or $\sin^2(nx) = k^2$) so 12 (B1) **and** so 24 in 4π, hence 30 in 5π (dB1)
- 16 solutions for $\sin(nx) = k$ (B1) **and** 14 (solutions) for $\sin(nx) = -k \rightarrow 30$ (dB1)
- $\sin^2(3x) = \frac{1-\cos(6x)}{2}$ and as $\cos x = k$ has 5 solutions in 0 to 5π (B1) **and** so $\cos 6x$ has $5 \times 6 = 30$ (dB1)

Graphical approaches may be used but these must be convincing, clearly showing the correct number of solutions in one of the relevant intervals. Be lenient with the shape of the curve.
e.g. 30 solutions in $0 \leq x < 5\pi$ [with 6 solutions on, for example, either sketch below]

$y = \sin^2(3x)$ $0 \leq x < \pi$



$y = \sin(3x)$ $0 \leq x < \pi$



Note that a suitable graph of $\sin x$ showing, e.g. 10 solutions up to 5π, followed by 30 (calculation of $\times 3$ clearly implied) would also be eligible to score the dB1.

The following are examples of an incorrect justification but scores B1dB0 for reaching 30:

- $\sin(nx) = k$ has 3×6 solutions in the interval 0 to 5π, $\sin(nx) = -k$ has 3×4 solutions (in the interval 0 to 5π) so 18 + 12 = 30 solutions in total.
- $\sin(nx) = k$ has 15 solutions in the interval 0 to 5π, so for $\sin^2(nx)$ has 15×2 = 30 solutions in total.

For reference, the tables below show the total number of solutions for each branch of $\sin^2(nx) = k^2$ with $n = 3$ and the bold values score the first B1 **provided they are in the correct interval**.

	$0 \le x < \pi$	$0 \le x < 2\pi$	$0 \le x < 4\pi$	$0 \le x < 5\pi$
$\sin(nx) = k$	4	6	12	**16**
$\sin(nx) = -k$	2	6	12	**14**
Total solutions	**6**	**12**	**24**	**30**

and $n = 1$ for those that deal with nx ($3x$) last.

	$0 \le x < \pi$	$0 \le x < 2\pi$	$0 \le x < 4\pi$	$0 \le x < 5\pi$
$\sin(x) = k$	2	2	4	6
$\sin(x) = -k$	0	2	4	4
Total solutions	**2**	**4**	**8**	**10**

(**Q13 9MA0/02, June 2025**)

Q32.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha=\frac{1}{2}$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=0.464$	A1	1.1b
		(3)	
(b)(i)	$3+2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t+0.464)=1\Rightarrow 0.5t+0.464=2\pi$ $\Rightarrow t=...$	M1	3.4
	$t=11.6$	A1	1.1b
		(3)	
(c)	$3+2\sqrt{5}\cos(0.5t+0.464)=0$ $\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}\Rightarrow 0.5t+0.464=\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t=2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)-0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977...-0.464)-2(2.306...-0.464)$	dM1	3.1b
	$=3.34$	A1	1.1b
		(4)	
(d)	e.g. the "3" would need to vary	B1	3.5c
		(1)	
			(11 marks)

Notes

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{1}{2}$ or $\sin\alpha=\pm\frac{1}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

It is implied by either awrt 0.464 (radians) or awrt 26.6 (degrees)

A1: $\alpha=$awrt 0.464

(b)(i)

B1ft: For $(3+2\sqrt{5})$ m or awrt 7.47 m and remember to isw. Condone lack of units.

Follow through on their R value so allow $3+2\times$Their R. (Allow in decimals with at least 3sf accuracy)

(b)(ii)

M1: Uses $0.5t\pm"0.464"=2\pi$ to obtain a value for t

Follow through on their 0.464 but this angle must be in radians.

It is possible in degrees but only using $0.5t\pm"26.6"=360$

A1: Awrt 11.6

Alternative for (b):

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t) \Rightarrow \frac{dH}{dt} = -2\sin(0.5t) - \cos(0.5t) = 0$$

$$\Rightarrow \tan(0.5t) = -\frac{1}{2} \Rightarrow 0.5t = 2.677..., 5.819... \Rightarrow t = 5.36, 11.6$$

$$t = 11.6 \Rightarrow H = 7.47$$

Score as follows:

M1: For a complete method:

Attempts $\frac{dH}{dt}$ and attempts to solve $\frac{dH}{dt} = 0$ for t

A1: For t = awrt 11.6

B1ft: For awrt 7.47 or 3 + 2×Their R

(c)

M1: Uses the model and sets $3 + 2"\sqrt{5}"\cos(...) = 0$ and proceeds to $\cos(...) = k$ where $|k| < 1$.

Allow e.g. $3 + 2"\sqrt{5}"\cos(...) < 0$

dM1: Solves $\cos(0.5t \pm "0.464") = k$ where $|k| < 1$ to obtain at least one value for t

This requires e.g. $2\left(\pi + \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$ or e.g. $2\left(\pi - \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$

Depends on the previous method mark.

dM1: A fully correct strategy to find the required duration. E.g. finds 2 consecutive values of t when $H = 0$ and subtracts. Alternatively finds t when H is minimum and uses the times found correctly to find the required duration.

Depends on the previous method mark.

<u>Examples:</u>

Second time at water level – first time at water level:

$$2\left(\pi + \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) = 7.02685... - 3.68492...$$

2×(first time at minimum point – first time at water level):

$$2\left(2\left(\pi - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right)\right) = 2(5.35589... - 3.68492...)$$

Note that both of these examples equate to $4\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ **which is not immediately obvious but may be seen as an overall method.**

There may be other methods – if you are not sure if they deserve credit send to review.

A1: Correct value. Must be 3.34 (**not** awrt).

<u>Special Cases in (c):</u>

Note that if candidates have an incorrect α and have e.g. $3 + 2\sqrt{5}\cos(0.5t - 0.464)$, this has no impact on the final answer. So for candidates using $3 + 2\sqrt{5}\cos(0.5t \pm \alpha)$ in (c) allow all the marks including the A mark as a correct method should always lead to 3.34

<u>Some values to look for:</u>

$$0.5t \pm "0.464" = \pm 2.306, \pm 3.977, \pm 8.598, \pm 10.26$$

(d)

B1: Correct refinement e.g. As in scheme. If they suggest a specific function to replace the "3" then it must be sensible e.g. a trigonometric function rather than e.g. a quadratic/linear one.

Q33.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{2^2+8^2}=\sqrt{68}=2\sqrt{17}$	B1	1.1b
	$2\cos\theta+8\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ $2=R\cos\alpha \quad 8=R\sin\alpha$ $\tan\alpha=\frac{8}{2}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=$ awrt 1.326	A1	2.2a
		(3)	
(b)(i)	$4.5\times"2\sqrt{17}"$	M1	1.1b
	$9\sqrt{17}$	A1	2.2a
(ii)	awrt 1.33	B1ft	2.2a
		(3)	
		(6 marks)	

Notes

(a)

B1: $R=2\sqrt{17}$ or $\sqrt{68}$.

$\pm2\sqrt{17}$ or $\pm\sqrt{68}$ score B0

(Condone if this comes from e.g., $8=R\cos\alpha \quad 2=R\sin\alpha$)

Decimal answers score B0 unless the exact value is seen then apply isw.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{8}{2}$, $\cos\alpha=\pm\frac{2}{"\sqrt{68}"}$, $\sin\alpha=\pm\frac{8}{"\sqrt{68}"}$

May be implied by awrt 1.33 radians or 76 degrees

A1: awrt 1.326 for α. Apply isw if this is then subsequently rounded to e.g. 1.33

(b)(i)

M1: For a value of $\pm4.5\times$ their R or allow $\pm4.5R$ (with the letter R)

But not embedded in an expression e.g. $9\sqrt{17}\cos(\theta-\alpha)$ unless extracted later.

Note that the sum may be found as $9\cos x+36\sin x$ with the maximum then found using calculus

e.g. $S=9\cos x+36\sin x\Rightarrow\frac{dS}{dx}=-9\sin x+36\cos x=0\Rightarrow\tan x=4\Rightarrow\sin x=\frac{4}{\sqrt{17}}$, $\cos x=\frac{1}{\sqrt{17}}$

$\Rightarrow 9\cos x+36\sin x=9\sqrt{17}$. This will score M1 once they reach $\pm4.5\times$ their R

May be implied by $9\sqrt{17}$ or awrt 37.1 (which may come from a graphical method)

May also see e.g. $\text{Max}(9\cos x+36\sin x)=\sqrt{9^2+36^2}=...$

A1: $9\sqrt{17}$ or exact equivalent e.g. $\sqrt{1377}$, $4.5\sqrt{68}$, $4.5(2\sqrt{17})$ and apply isw once a correct answer is seen

(ii)

B1ft: awrt 1.33 (or follow through on their α even if in degrees (76), no matter how accurate)

(**Q08 9MA0/02, June 2023**)

Q34.

Question	Scheme	Marks	AOs
(a)	$5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9\times\frac{\sin\theta}{\cos\theta}$ $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe	A1	1.1b
	Correct order of operations For example $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\frac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$	A1	1.1b
	All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$	A1	1.1b
	$\theta = 0°, \pm 180°$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25° = -18.4°$	M1	1.1b
	$x = 6.6°$	A1ft	2.2a
		(2)	
		(8 marks)	

(a)

M1: Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = ...\sin\theta\cos\theta$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

A1: Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to form a correct simplified equation in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is for a correct equation in $\sin\theta$ or $\cos\theta$

dM1: Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to find at least one value for θ (Eg. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \frac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

A1: Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

A1: All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

B1: $\theta = 0°, \pm 180°$ This can be scored independent of method.

(b)

M1: Attempts to solve $x - 25° = "\theta"$ where θ is a solution of their part (a)

A1ft: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their θ must be chosen

Q35.

Question	Scheme	Marks	AOs
(a)	$K = 500$	B1	1.1b
	$\tan\alpha = \dfrac{480}{140} \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha =$ awrt $73.74°$ or $500\cos(\theta + 73.74)°$	A1	1.1b
		(3)	
(b)(i)	$R = 1000 + 500\cos(30t + 73.74)°$ or $R = 1000 + 140\cos(30t)° - 480\sin(30t)°$	B1ft	3.3
(b)(ii)	$\{R_{min} =\}\,500$	B1ft	3.4
		(2)	
(c)	$t = 3.5 \Rightarrow R = "1000" + "500"\cos(30(3.5) + "73.74")° = ...$	M1	3.4
	$R =$ awrt $500.1...$ so the model is reliable	A1	3.5a
		(2)	
(d)	$\sin(30t + 70)° = -1 \Rightarrow 30t + 70 = 270 \Rightarrow 30t = ...$ (or $t = ...$)	M1	3.4
	$30t = 200$ $\left(\text{or } t = \dfrac{20}{3}\right)$	A1	1.1b
	$R = "1000" + "500"\cos\left(30\left("\dfrac{20}{3}"\right) + "73.74"\right)°$ or $R = "1000" + 140\cos("200")° - 480\sin("200")°$	dM1	3.4
	$R = 1032$ (or 1033)	A1	1.1b
		(4)	
			(11 marks)

Notes

Note: Candidates working in radians are able to score all the M and B marks in this question.
Condone the absence of the degrees symbol throughout the whole question.

(a)

B1: Correct value for K. Condone $R = 500$

M1: Award for $\tan\alpha = \pm\dfrac{480}{140} \Rightarrow \alpha = ...$, $\tan\alpha = \pm\dfrac{140}{480} \Rightarrow \alpha = ...$, $\sin\alpha = \pm\dfrac{480}{"500"} \Rightarrow \alpha = ...$ or $\cos\alpha = \pm\dfrac{140}{"500"} \Rightarrow \alpha = ...$

Note $\alpha =$ awrt 1.3 (rad) implies this mark.

A1: $\alpha =$ awrt $73.74\{°\}$ or correct expression $500\cos(\theta + 73.74)\{°\}$

(b)(i) Note: mark parts (b)(i) and (b)(ii) together.

B1ft: Correct equation of the model in either form including the $R =$ following through on their numerical K ($0 < K \leqslant 750$) and their numerical α.

Allow for e.g. $R = 1500 - "500" + "500"\cos(30t + "73.74")\{°\}$ or for

$R = 1500 - "500" + 140\cos(30t)° - 480\sin(30t)°$ but not e.g. $R = 1500 - K + K\cos(30t + \alpha)°$

$R = 1000 + 140\cos 30t - 480\sin 30t$ (without the brackets) is correct.

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

(b)(ii)

B1ft: 500 or follow through on (their A – their K) or (1500 – 2 × their K) provided it is non-negative and less than 1500. It must be clear this is their answer to (b)(ii) so expect to see e.g. (b) or $R_{\min} =$ or an indication it is the minimum.

(c) Note: if θ is used in place of $30t$ then they must revert back to $30t$ correctly to access the marks.

M1: Substitutes $t = 3.5$ into their model for the number of rabbits (you may need to check if no method is shown)

or substitutes $t = 3.5$ into their $\cos(30t+\alpha)°$

Condone substitution of a value of t in the range $3 \leqslant t \leqslant 4.5$ for this mark.

A1: $R =$ awrt 500.1… **or** 500 (not awrt) following substitution of $t = 3.5$, suggesting that the model is valid/reliable/appropriate/good.

or $\cos(30(3.5)+73.74)\{°\} \approx -1$ suggesting that the model is valid/reliable/appropriate/good.

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

Alt:

M1: Minimum occurs when $A + K\cos(30t+\alpha)° = R_{\min} \Rightarrow \cos(30t+\alpha)° = \lambda$ with $|\lambda| \leqslant 1$ leading to $t = ...$

May just see $\cos(30t+73.74)° = -1 \Rightarrow t = ...$ (or their $\cos(30t+\alpha)° = -1 \Rightarrow t = ...$)

$30t+\alpha = 180 \Rightarrow t = ...$ implies this mark. Condone $30t+\alpha = \pi \Rightarrow t = ...$ for this mark.

A1: $t = 3.54...$ (i.e. the middle of April) so the model is valid/reliable/appropriate/good.

Do not condone incorrect statements, e.g., $t = 3.54...$ i.e. the middle of March so close to middle of April. If using $A + K\cos(30t+\alpha)° = R_{\min}$ then $R_{\min}$ must be = their A – their K

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

Alt 2 using differentiation (Condoned)

M1: Condone finding the minimum using $...\sin(30t+73.74)° = 0 \Rightarrow t = ...$ (or their $\sin(30t+\alpha)° = 0 \Rightarrow t = ...$)

$30t+\alpha = 180 \Rightarrow t = ...$ implies this mark. Condone $30t+\alpha = \pi \Rightarrow t = ...$ for this mark.

A1: $t = 3.54...$ (i.e. the middle of April) suggesting that the model is valid/reliable/appropriate.

Do not condone incorrect statements, e.g., $t = 3.54...$ i.e. the middle of March so close to middle of April.

The complete derivative for $\frac{dR}{dt}$ does not need to be seen.

Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

(d) Note: if θ is used in place of $30t$ then they must revert back to $30t$ correctly to access the marks.

M1: Realises that $\sin(30t+70)° = -1$, reaches $30t+70 = 270$ or -90 and attempts to find t (or $30t$)

Condone attempts using differentiation. The minimum occurs when $\cos(30t+70)° = 0 \Rightarrow 30t+70 = 270 \Rightarrow 30t = ...$ (or $t = ...$). They must use 270 or –90 and **not** 90 to achieve the minimum. Condone $30t+\alpha = \frac{3\pi}{2} \Rightarrow t = ...$ for this mark but not $30t+\alpha = \frac{\pi}{2} \Rightarrow t = ...$

A1: Correct value for $30t$ (or t) Accept rounded or truncated values to at least 3s.f. e.g. 6.66 or 6.67

dM1: Substitutes their value of $t > 0$ (or $30t > 0$) coming from $30t+70 = \mathbf{270}$ into their model for R

A1: Correct number of rabbits. Allow 1032 or 1033 but must be whole numbers and not just 1030. Allow this mark if they have truncated or rounded an otherwise correct α (to 3s.f.)

(**Q12 9MA0/01, June 2024**)

Q36.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha = 2 \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12}+1.107-3=\frac{\pi}{2}\Rightarrow t=$	M1	3.1b
	$t=$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)
Notes:			

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value of α from $\tan\alpha=\pm 2, \tan\alpha=\pm\frac{1}{2}, \sin\alpha=\pm\frac{2}{"R"}$ OR $\cos\alpha=\pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

A1: $\alpha=$ awrt 1.107

(b)

B1ft: Deduces that the maximum temperature is $\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C Remember to isw

Condone a lack of units. Follow through on their value of R so allow $(5+"R")$ °C

(c)

M1: An complete strategy to find t from $\frac{\pi t}{12}\pm 1.107-3=\frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t\pm 63.4-171.9=90$

A1: awrt $t=13.2$

A1: The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

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It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$\frac{d\theta}{dt}=\frac{\pi}{12}\cos\left(\frac{\pi t}{12}-3\right)-\frac{2\pi}{12}\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow\tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ scores M1 A1 A1

$\frac{d\theta}{dt}=\cos\left(\frac{\pi t}{12}-3\right)-2\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow\tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ they can score M1 A0 A1 (SC)

A value of $t=1.23$ implies the minimum value has been found and therefore incorrect method M0.

(**Q06 9MA0/01, Oct 2020**)

Q37.

Question	Scheme	Marks	AOs
(a)	$\cos 3A = \cos(2A + A) = \cos 2A\cos A - \sin 2A\sin A$	M1	3.1a
	$= \left(2\cos^2 A - 1\right)\cos A - \left(2\sin A\cos A\right)\sin A$	dM1	1.1b
	$= \left(2\cos^2 A - 1\right)\cos A - 2\cos A\left(1 - \cos^2 A\right)$	ddM1	2.1
	$= 4\cos^3 A - 3\cos A$ *	A1*	1.1b
		(4)	
(b)	$1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x\left(4\cos^2 x - \cos x - 3\right) = 0$ $\Rightarrow \cos x\left(4\cos x + 3\right)\left(\cos x - 1\right) = 0$ $\Rightarrow \cos x = \ldots$	dM1	3.1a
	Two of $-90°$, 0, $90°$, awrt $139°$	A1	1.1b
	All four of $-90°$, 0, $90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)

Allow a proof in terms of *x* rather than A

M1: Attempts to use the compound angle formula for $\cos(2A + A)$ or $\cos(A + 2A)$

Condone a slip in sign

dM1: Uses correct double angle identities for $\cos 2A$ and $\sin 2A$

$\cos 2A = 2\cos^2 A - 1$ must be used. If either of the other two versions are used expect to see an attempt to replace $\sin^2 A$ by $1 - \cos^2 A$ at a later stage.

Depends on previous mark.

ddM1: Attempts to get all terms in terms of $\cos A$ using correct and appropriate identities.

Depends on both previous marks.

A1*: A completely correct and rigorous proof including correct notation, no mixed variables, missing brackets etc.

Alternative right to left is possible:

$4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right) = \cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right) = \cos A\left(\cos 2A - 2\sin^2 A\right)$

$= \cos A\cos 2A - 2\sin A\cos A\sin A = \cos A\cos 2A - \sin 2A\sin A = \cos(2A + A) = \cos 3A$

Score M1: For $4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right)$

dM1: For $\cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right)$ (Replaces $4\cos^2 A - 1$ by $2\cos^2 A - 1$ and $2\left(1 - \sin^2 A\right)$)

ddM1: Reaches $\cos A\cos 2A - \sin 2A\sin A$

A1: $\cos(2A + A) = \cos 3A$

(b)

M1: For an attempt to produce an equation just in $\cos x$ using both part (a) and the identity $\sin^2 x = 1 - \cos^2 x$

Allow one slip in sign or coefficient when copying the result from part (a)

dM1: Dependent upon the preceding mark. It is for taking the cubic equation in $\cos x$ and making a valid attempt to solve. This could include factorisation or division of a $\cos x$ term followed by an attempt to solve the 3 term quadratic equation in $\cos x$ to reach at least one non zero value for $\cos x$.

May also be scored for solving the cubic equation in $\cos x$ to reach at least one non zero value for $\cos x$.

A1: Two of $-90°$, 0, $90°$, awrt $139°$ **Depends on the <u>first</u> method mark.**

A1: All four of $-90°$, 0, $90°$, awrt $139°$ with no extra solutions offered within the range.

Note that this is an alternative approach for obtaining the cubic equation in (b):

$$1-\cos 3x = \sin^2 x \Rightarrow 1-\cos 3x = \frac{1}{2}(1-\cos 2x)$$

$$\Rightarrow 2-2\cos 3x = 1-\cos 2x$$

$$\Rightarrow 1 = 2\cos 3x - \cos 2x$$

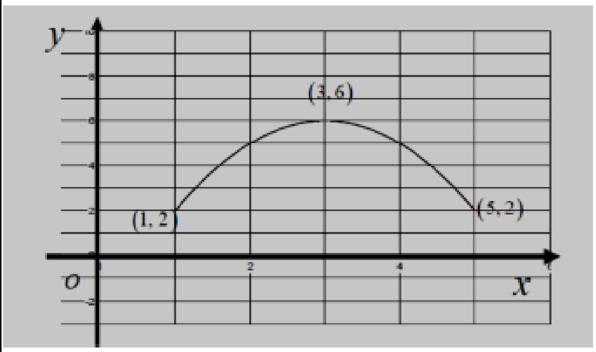
$$\Rightarrow 1 = 2\left(4\cos^3 x - 3\cos x\right) - \left(2\cos^2 x - 1\right)$$

$$\Rightarrow 0 = 4\cos^3 x - 3\cos x - \cos^2 x$$

The M1 will be scored on the penultimate line when they use part (a) and use the correct identity for cos $2x$

(**Q10 9MA0/02, Oct 2020**)

Q38.

Question	Scheme	Marks	AOs
(a)	Attempts to use $\cos 2t = 1-2\sin^2 t \Rightarrow \frac{y-4}{2} = 1-2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y-4 = 2-4\times\frac{(x-3)^2}{4} \Rightarrow y = 6-(x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3+2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	

(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2 - 7x + (k+3) = 0$ Or y $\quad y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow \ 49 - 4\times1\times(k+3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$ or $(7-2k)^2 - 4\times1\times(k^2 - 6k + 3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leqslant k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)

M1: Uses $\cos 2t = 1-2\sin^2 t$ in an attempt to eliminate t

A1*: Proceeds to $y = 6-(x-3)^2$ without any errors

Allow a proof where they start with $y = 6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1,2)$ and $(5,2)$

B1: Any suitable explanation as to why *C* does not include all points of $y = 6-(x-3)^2, \quad x \in \mathbb{R}$

This should include a reference to **the limits on sin or cos with a link to a restriction on *x* or *y*.**

For example

'As $-1 \le \sin t \le 1$ then $1 \leqslant x \leqslant 5$' Condone in words '*x* lies between 1 and 5' and strict inequalities

'As $\sin t \le 1$ then $x \leqslant 5$' Condone in words '*x* is less than 5'

'As $-1 \le \cos(2t) \le 1$ then $2 \leqslant y \leqslant 6$ ' Condone in words '*y* lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2 \leqslant x \leqslant 5$"

Do not allow a statement on the top limit of *y* as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k = 7$ This can be found by substituting into $(5,2)$

  $x+y=k \Rightarrow k=7$ or substituting $x=5$ into $x^2-7x+(k+3)=0 \Rightarrow 25-35+k+3=0$
  $\Rightarrow k=7$

- or deduces that $k < \frac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for *k*.

Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method.

Eg. Sets $k-x=6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2-7x+(k+3)=0$ The $=0$ may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2=4ac$ oe or $b^2 \ldots 4ac$ where ... is any inequality leading to a critical value for *k*. Eg. one root $\Rightarrow 49-4\times1\times(k+3)=0 \Rightarrow k=\frac{37}{4}$

A1: Range of values for $k = \left\{k : 7 \leqslant k < \frac{37}{4}\right\}$ Accept $k \in \left[7, \frac{37}{4}\right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y=6-(x-3)^2$ equal to -1	M1	3.1a
	$-2x+6=-1 \Rightarrow x=3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of *k*. $y=6-(3.5-3)^2=5.75$ Hence using $k=3.5+5.75=9.25$	M1	2.1
	Range of values for $k=\{k:7 \leqslant k < 9.25\}$	A1	2.5

Q39.

Question	Scheme	Marks	AOs
	$\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\frac{dy}{d\theta}=\frac{3}{2+2\sin 2\theta}=\frac{3/2}{1+\sin 2\theta}$	A1	1.1b
			(5 marks)

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\frac{dy}{d\theta}$ (condone it being stated as $\frac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\frac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\frac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\frac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\frac{dy}{d\theta}$ condoning it appearing as $\frac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\frac{3}{2}$ stated but allow for example $\frac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

(**Q05 9MA0/01, June 2018**)

Q40.

Question	Scheme	Marks	AOs
a	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)$	M1	1.2
	$2\alpha+\frac{1}{2}\left(1-\frac{\alpha^2}{2}\right)=0\Rightarrow 2\alpha+\frac{1}{2}-\frac{\alpha^2}{4}=0 \Rightarrow \alpha=...$	dM1	1.1b
	$\alpha=-0.243$ (3dp) only	A1	2.3
		(3)	
b	$f'(0)=\frac{1}{2}\cos 0\Rightarrow ...\Rightarrow y=...x+3$	M1	1.1b
	$y=\frac{1}{2}x+3$	A1	1.1b
		(2)	
			(5 marks)

(a) **Note on EPEN this is M1A1A1 but we are marking this as M1dM1A1**

Accept to be in terms of α or another variable e.g. x

Note: -0.243 with no working is 0 marks

M1: Fully substitutes $\cos x=1-\frac{x^2}{2}$ into the derivative.

dM1: Attempts to multiply out to achieve a 3TQ ($=0$) **and** attempts to find a value for α. Condone slips. Allow solving the quadratic via any method (usual rules apply).

If they use a calculator then you may need to check this.

A1: $(\alpha=)\ -0.243$ only cao Can only be scored provided a correct 3TQ is seen. If both roots found then the other one must be rejected (or a choice made of -0.243 e.g. underlining it or a tick)

Condone $x=-0.243$

(b)

M1: Attempts to find the gradient of the curve when $x=0$ and achieves an equation of the form $y="f'(0)"x+3$.

$x=0$ must be fully substituted in and a value must be found for the gradient. Do not allow this mark if they attempt to use a changed gradient e.g. the gradient of the normal.

Also allow attempts using the small angle approximation:

$$f'(x)\approx 2x+\frac{1}{2}\left(1-\frac{x^2}{2}\right) \text{ when } x=0,\ f'(0)="\frac{1}{2}"\Rightarrow y="f'(0)"x+3$$

A1: $y=\frac{1}{2}x+3$ or equivalent in the form $y=mx+c$ isw Stating just the values $m=0.5,\ c=3$ without the correct equation is A0

(**Q04 9MA0/01, June 2023**)